NUS National University of Singapore

IMPRINTS

INSTITUTE FOR MATHEMATICAL SCIENCES

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Risk Measure, Nonlinear Expectations and Stochastic Calculus under Knightian Uncertainty >>>

Shige PENG

[Editor's note: In June 2013, the Institute hosted the program "Nonlinear Expectations, Stochastic Calculus under Knightian Uncertainty, and Related Topics". Shige Peng, Chair of the Organizing Committee, contributed this invited article to Imprints *as a follow-up to the IMS program.]*

The history of probability theory can be traced back to the communications between Fermat and Pascal in 1654 on the problem of points. Since then, many mathematicians have contributed to the development of the notion of probability, including Huygens, Bernoulli, de Moivre, Laplace, Poisson, Gauss, Bachelier, et al. It is worthwhile pointing out that Louis Bachelier (1900) introduced the notion of Brownian motion to study the option pricing problem. The axiomatic foundation of probability theory was finally established by A. N. Kolmogorov in the twentieth century. The stochastic integral and the corresponding stochastic analysis introduced by Itô (1942) are powerful, beautiful and fundamental, and have been widely applied to finance.

But Frank Knight challenged the feasibility of using probability to treat uncertainty. He said: "Mathematical, or a priori, type of probability is practically never met with in business ..., the concept of an objectively measurable probability or chance is simply inapplicable". The term "Knightian uncertainty" or "ambiguity" is now widely accepted, particularly among economists, to refer to situations where no objective probability or distribution is available for making decisions.

The notion of Knightian uncertainty has deeply influenced the further development of the expected utility theory established by von Neumann and Morgenstern (1944) based on the objective probability and by Savage (1954) based on the subjective probability. Two well-known paradoxes against the expected utility theory are the Allais paradox (1953) and Ellsberg paradox (1961). In order to resolve the Ellsberg paradox, Gilboa and Schmeidler (1989) established the MEU (maximin expected utility) theory. A dynamic version of their theory was provided by Epstein and Schneider (2003). Inspired by the methods of robust control, Hansen and Sargent (2000, 2001) introduced the MP (multiplier preference) theory. All the above revisions of

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the expected utility can be regarded as a nonlinear expected utility, i.e., a utility where the linear expectation is replaced by a nonlinear one.

A more direct motivation for studying nonlinear expectations is the recent rapid development on the monetary risk measure for risky positions in finance. Artzner et al. (1997, 1999) axiomatically proposed the coherent risk measure which challenged the widely used VaR risk measure. Föllmer & Schied (2002) and Fritelli & Rosazza-Gianin (2002) further introduced the notion of the convex risk measure. Both of the risk measures are nonlinear expectations and are closely related to Knightian uncertainty.

From both theoretical and practical point of view, a natural question is whether a new theoretical framework of nonlinear expectations exists, in contrast to the beautiful and powerful modern probability theory. In finance, the unknown probability measures of uncertainty cannot be dominated by one probability measure, and the well-known volatility uncertainty is a typical example. In this case, Peng (2004, 2005, 2007) established a theoretical framework by introducing a fully nonlinear dynamically consistent nonlinear expectation called *G*-expectation. It is theoretically interesting that under such *G*-expectation, the canonical process becomes incrementally independent and stable, namely, it is a *G*-Brownian motion.

Many basically stochastic analysis can be re-established under this new framework, including stochastic calculus of Itô's type and nonlinear martingale theory. From the point of view of statistics, how to obtain parametrical distribution models under Knightian uncertainty is another crucial problem for practical implementations. A typical example is the so-called *G*-normal distribution $N(\mu,[\bar{\sigma}^2,\underline{\sigma}^2])$, which is parameterized by its mean μ, upper variance $\bar{\sigma}^2$ and lower variance $\underline{\sigma}^2$. Like the classical probability theory, such a new type of distributions under sublinear expectation are derived from a new central limit theorem under Knighting uncertainty. Corresponding to this, another typical distribution under sublinear expectation is the so-called maximal distribution $M([\bar{\mu},\underline{\mu}])$, derived from a new type of law of large numbers under Knightian uncertainty. A very interesting open problem is how to estimate these parameters. Similarly under Knightian uncertainty we have typical parameterized

Exchanging ideas on nonlinear expectations
(From left: Freddy Delbaen, Louis CHEN and Shige PENG)

Exploring stochastic calculus under Knightian uncertainty

stochastic processes such as *G*-Brownian (parameterized by $\bar{\sigma}^2$ and $\underline{\sigma}^2$), *G*-Lévy process and the corresponding parameterized stochastic differential equations. All of these need further more systematical developments by researchers from different fields, including probability theory, forward and backward stochastic differential equations, partial differential equations, mathematical finance, statistics, economics, etc.

Shige PENG
Shandong University

People in the News >>>

Jian-Shu Li, Newly Elected CAS Academician

Professor Jian-Shu Li of the Hong Kong University of Science and Technology has been elected as Academician of the Chinese Academy of Sciences in 2013. Professor Li was Co-chair of the Organizing Committees of the IMS activities "Representation Theory of Lie Groups" (July 2002 - January 2003) and "International Conference on Harmonic Analysis, Group Representations, Automorphic Forms and Invariant Theory" (9 - 11 January 2006).

New NUS Appointments

The National University of Singapore (NUS) has recently made the following appointments with effect from 1 April 2014: Professor Andrew Wee as NUS Vice President (University & Global Relations), Professor Zuowei Shen as Dean of NUS Faculty of Science, and Professor Chengbo Zhu as Head of NUS Department of Mathematics.

Professors Andrew Wee and Zuowei Shen are serving members of the IMS Management Board. Professor Shen was also an organizer of several IMS programs and activities. Professor Chengbo Zhu was an organizer of the IMS programs/events "Representation Theory of Lie Groups" (July 2002 - January 2003), "International Conference on Harmonic Analysis, Group Representations, Automorphic Forms and Invariant Theory" (9 - 11 January 2006) and "Branching Laws" (11 - 31 March 2012).

Weizhu Bao honored with the Feng Kang Prize

Professor Weizhu Bao of the NUS Department of Mathematics has been awarded the 2013 Feng Kang Prize of Scientific Computing by the Institute of Computational Mathematics and Scientific/Engineering Computing, Chinese Academy of Sciences for his significant contributions in numerical analysis for Bose-Einstein condensation and computation of solutions to the Schrödinger equation. He received the award in a presentation ceremony at Changsha, China in October this year.

Professor Bao was an organizer of numerous IMS programs, workshops and spring/summer schools, including "Workshop on Nonlinear PDEs and Applications" (25 September 2013), "Multiscale Modeling, Simulation, Analysis and Applications" (1 November 2011 - 20 January 2012), "Workshop on Nonlinear Partial Differential Equations: Analysis, Computation and Applications" (7 - 10 March 2012), "Spring School on Fluid Mechanics and Geophysics of Environmental Hazards" (19 April - 2 May 2009), "Mathematical Theory and Numerical Methods for Computational Materials Simulation and Design" (1 July - 31 August 2009), "Moving Interface Problems and Applications in Fluid Dynamics" (8 January - 31 March 2007).

Past Programs in Brief

Nonlinear Expectations, Stochastic Calculus under Knightian Uncertainty, and Related Topics (3 June – 12 July 2013)

…Jointly organized with Centre for Quantitative Finance, NUS

Website: http://www2.ims.nus.edu.sg/Programs/013wnlinear/index.php

Chair:
Shige Peng, *Shandong University*

The program advanced collaborative exploration and provided an opportunity to present in Singapore the latest development in mathematical technology, quantitative finance, and risk measures. There were a total of 3 workshops, 4 tutorials and 65 invited talks in this 6-week program.

The first workshop on "Knightian Uncertainty and Backward Stochastic Differential Equations" (10 – 14 June 2013) started with a tutorial by Shige Peng (Shandong University) and had 17 invited talks. This was followed up by a 2-day tutorial by Jianfeng Zhang (University of Southern California). The second activity, "Institute of Mathematical Statistics Workshop on Finance - Probability and Statistics" (19 – 21 June 2013) had a total of 32 invited talks. The following week featured a 6-part tutorial lecture by Peter Forsyth (University of Waterloo) and a 4-part tutorial by Marco Frittelli (Università degli Studi di Milano). Last but not least, the program had its third workshop (1 – 5 July 2013) which consisted of 14 invited talks.

The program provided an ideal platform for presenting, discussing and understanding a new field (Path-dependent Partial Differential Equations) from several different viewpoints. The subject was the main topic in two tutorials and several talks within the program. Through these lectures,

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Programs & Activities >>>

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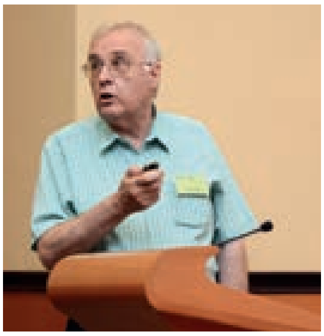
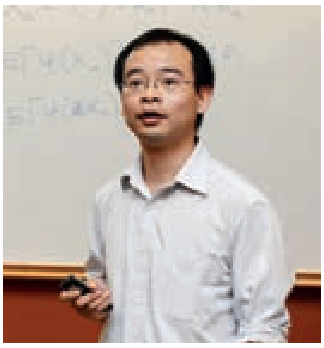
the graduate students and young scientists learnt elementary and advanced topics in this fast-growing interdisciplinary field and interacted with the world's leading experts. The program had a total of 130 participants, and among them were 35 graduate students.



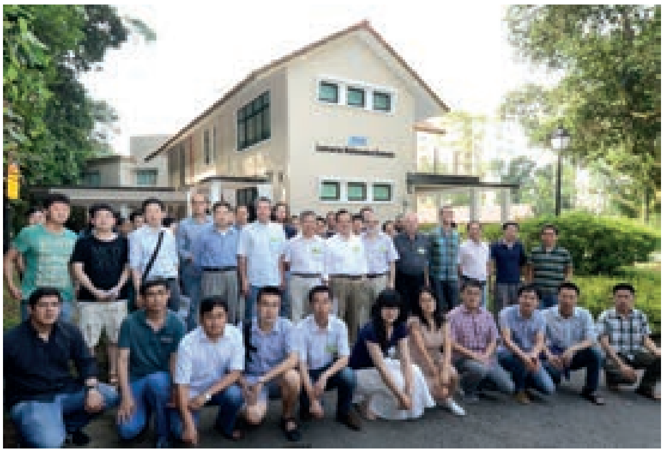
Freddy Delbaen:
Extending time consistent concave monetary utility functions

Marco Frittelli: Measuring risk



Xianhua PENG:
The econometrics of asset pricing with spatial interactions

Certain discussion on uncertainty?
(From left: Arnaud Lionnet and Chao ZHOU)



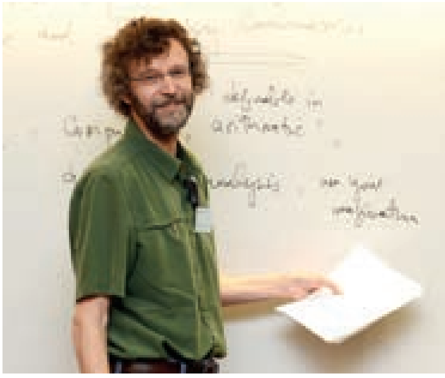
A nonlinear alignment of stochastic calculus theorists

Asian Initiative for Infinity (AII) Graduate Summer School (15 - 26 July 2013)

... Jointly funded by the John Templeton Foundation

Website: http://www2.ims.nus.edu.sg/Programs/013aiiss/index.php

The objective of the AII Graduate Summer School was to bridge the gap between a general graduate education in mathematical logic and the specific preparation necessary to do research on problems of current interest in the subject. The summer school consisted of 25 hours of lectures conducted by Theodore Slaman and Hugh Woodin (University of California at Berkeley, USA and National University of Singapore), each of whom gave 12.5 hours of lectures over a period of one week. The summer school was attended by 49 participants, and among them were 32 graduate students.



Theodore Slaman:
Infinitary combinatorics and mathematical logic

Hugh Woodin: Extender models

Students sharing thoughts amidst refreshment breaks (From left: Ethan Joseph McCarthy and Noah David Schweber)

An initiation to the realm of infinity

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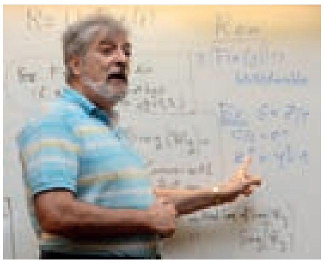
Complex Geometry (22 July - 9 August 2013)

Website: http://www2.ims.nus.edu.sg/Programs/013complex/index.php

Organizing Committee:
Lawrence Ein, *University of Illinois at Chicago*
Ngaiming Mok, *The University of Hong Kong*
Wing-Keung To, *National University of Singapore*
De-Qi Zhang, *National University of Singapore*

The participants of this program consisted of groups of complex analysts, complex differential geometers and complex algebraic geometers. A main part of the program was an eight-day workshop, where there were a total of 30 invited talks. Through these talks, the participants explained to the other groups their methods and techniques, disseminated their recent research findings and surveyed latest research developments. There were also lots of informal research discussions and explorations among the participants. At least 10 research collaborations/papers were initiated or worked on during the program. The program had a total of 51 participants, and among them were eight students who were given a great opportunity to interact with some of the leaders in the field.

Yum-Tong SIU: Very ampleness part of Fujita's conjecture



Fabrizio Catanese: Moduli spaces of automorphism marked varieties

Jun-Muk HWANG: Isotrivial VMRT-structures

A geometric conversation (From left: Mihai Paun and Steven LU)

A union of complex geometers

Mathematical Horizons for Quantum Physics 2 (12 August - 11 October 2013)

…Jointly organized with Centre for Quantum Technologies, NUS

Website: http://www2.ims.nus.edu.sg/Programs/013mhqp/index.php

Programme Coordinator:
Reinhard Werner, *University of Hanover*

Co-chairs:
Berthold-Georg Englert, *National University of Singapore*
Leong Chuan Kwek, *Nanyang Technological University and National University of Singapore*

Quantum theory is one of the most important intellectual developments in the early twentieth century. Since then there has been much interplay between theoretical physics and mathematics, both pure and applied. The first program on Mathematical Horizons in Quantum Physics (MHQP) was held at IMS in 2008, under the overarching theme of operator theory and operator algebra theory. In this second installment of the MHQP series, the program brought together mathematicians, whose work has a bearing on quantum physics, with researchers in mathematical physics and theoretical physics, whose work will benefit from the mathematical progress.

The program consisted of four overlapping three-week sessions, each devoted to a selected topic. Session 1 on Quantum Information Theory (Organizers: Burkhard Kümmerer (Technische Universität Darmstadt) and Hans Maassen (Radboud University Nijmegen)) ran from 12 – 30 August 2013 and started with presentations by the discussion leaders and was followed by close collaborations and discussions among the participants. Session 2 on Information-Theoretic Approaches to Thermodynamics (Organizers: Stephanie Wehner (National University of Singapore), Renato Renner (ETH Zurich), and Jens Eisert (Freie Universität Berlin)) ran from 26 August – 13 September 2013 and had 17 invited talks and three two-hour tutorials. Session 3 on Many-Particle Systems (Organizer: Jakob Yngvason (Universität Wien Vienna, Austria)) ran from 9 – 27 September 2013 and consisted of 14 invited talks, while Session 4 on Open Quantum Systems (Organizer: Robert Alicki (The University of Gdańsk)) ran from 23 September – 11 October 2013 and had five two-part invited talks and four invited talks amidst this session. Four NUS students

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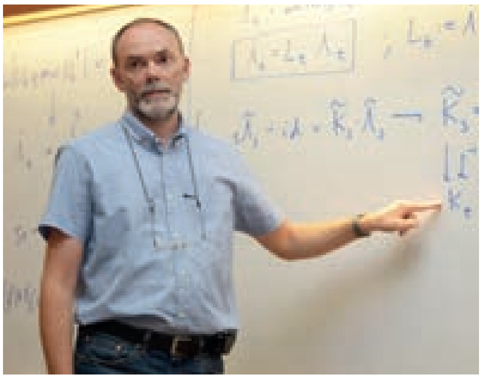
gave presentations during this session. There were a total of 131 program participants, and among them were 39 graduate students.



Dariusz Chruscinski: Witnessing non-Markovianity of quantum evolution

Masahito Ueda: Thermodynamic work gain from entanglement

Bruno Nachtergaele: Gapped ground state phases of quantum lattice systems

An intense discussion session in the lounge

Quantum physicists in horizontal formation

Workshop on Nonlinear PDEs and Applications (25 September 2013)

... *Jointly organized with Department of Mathematics, NUS*

Website: http://ww1.math.nus.edu.sg/misc/Workshop-NLPDEapps-2013.pdf

Chair:

Weizhu Bao, *National University of Singapore*

This one-day workshop on nonlinear partial differential equations and their applications featured six invited talks. Some of the speakers were also participants of the IMS program Mathematical Horizons for Quantum Physics 2 (12 August - 11 October 2013). They were Jakob Yngvason (University of Vienna), Robert Seiringer (Institute of Science and Technology) and Jan Philip Solovej (University of Copenhagen). The workshop was attended by 30 people.

Workshop on Modeling Rare Events in Complex Physical Systems (5 – 8 November 2013)

Website: http://www2.ims.nus.edu.sg/Programs/013wmodel/index.php

Organizing Committee:

Weinan E, *Peking University and Princeton University*

Weiqing Ren, *National University of Singapore*

This workshop brought together researchers working on rare events and related problems, as well as young researchers interested in this area, to foster learning, exchange of ideas and collaboration, and to promote further progress in the understanding of the subject matter.

Two leading experts in the field of complex energy landscape and rare events, Eric Vanden-Eijnden (New York University) and David Wales (Cambridge University) gave a 3-hour tutorial each in the workshop. Eric Vanden-Eijnden's tutorial "Modeling of reactive events" focused on the mathematical theory for rare events, while David Wales's tutorial "Exploring energy landscapes: from molecules to nanodevices" emphasized on the numerical methods and applications in computational chemistry. The workshop also had 20 invited talks which attracted a total of 44 participants, and among them were 10 graduate students from local institutions (NUS, A*STAR and NTU).

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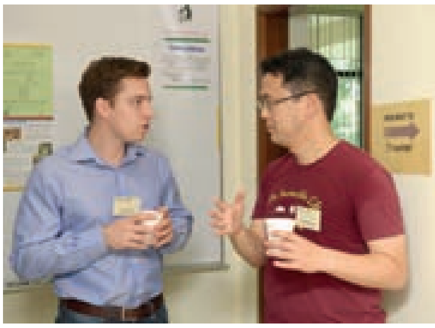
The participants of this workshop were a good mixture of applied mathematicians and practitioners from the areas of computational chemistry, biology and physics. New collaborations were formed, especially between applied mathematicians/numerical analysts and practitioners (computational chemists and computational biologists). Through this workshop, the participants also identified a few new research directions and some important issues which are currently unresolved.


David Wales: Exploring energy landscapes


Eric Vanden-Eijnden: Modeling of reactive events


Experiencing rare events richly (From left: Marco Sarich and Hiroshi Fujisaki)


Mathematicians modeling for a rare event

Public lectures:

IMS co-organized five public lectures between July and December 2013.

Professor Darrell Duffie of Stanford University gave a public lecture, titled "Regulatory Boundaries for the Banking System" at NUS on 4 July 2013. In the lecture, Professor Duffie talked about the appropriate regulatory boundaries of the banking system in the light of the financial crisis of 2007-2009, recently enacted laws and proposed new regulations. In particular, he brought up the issues of the range of financial services to which regulated banks should be restricted and the extent to which "shadow banks" offering these services should be subject to corresponding regulation. He also touched on the closely related issue of access to emergency lending of last resort from central banks by financial institutions that are outside of the regulated banking system. The public lecture was jointly organized with the Department of Economics, NUS, the Centre for Quantitative Finance, NUS and the Monetary Authority of Singapore. The lecture was delivered to an audience of 70 people.


Darrell Duffie: Regulatory Boundaries for the Banking System

Professor Charles Bennett of IBM presented a public lecture, titled "Quantum Physics, Public and Private Information, and the Lost Literature of Antiquity" at NUS on 29 August 2013. Professor Bennett first gave a brief introduction of quantum information theory. In particular, he described the use of polarized photons to carry information, touching on the role of quantum entanglement in quantum teleportation. He also outlined the theory that most classical


Charles Bennett: Quantum Physics, Public and Private Information, and the Lost Literature of Antiquity

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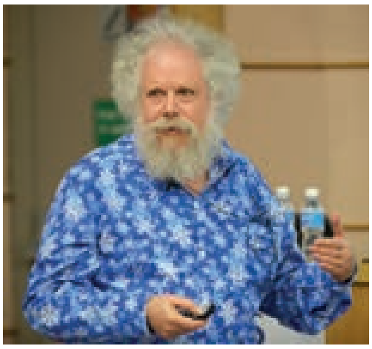
information created on earth is transient, eventually escaping into space as thermal radiation. The public lecture was jointly organized with the Centre for Quantum Technologies, NUS. A total of 75 people attended the lecture.

Renato Renner: Gambling Against the Second Law of Thermodynamics

Professor Renato Renner of ETH Zurich, Switzerland, delivered a public lecture, titled "Gambling Against the Second Law of Thermodynamics" at NUS on 10 September 2013. In the lecture, Professor Renner recalled briefly how Maxwell's demon paradox demonstrated the statistical nature of the second law of thermodynamics. Then he revisited Leo Szilard's argument in his 1929 classical paper on this topic from a modern perspective and exhibited the important role that the concept of "information" plays in our understanding of the laws of physics. The public lecture was jointly organized with the Centre for Quantum Technologies, NUS, and it was attended by 95 people.

Daniel L. Stein: Order, Disorder, Symmetry and Complexity

Professor Daniel L. Stein of New York University gave a public lecture titled "Order, Disorder, Symmetry and Complexity" at NUS on 15 November 2013. In the lecture, Professor Stein recalled briefly the well-understood phenomenon of matter organizing itself into simple ordered structures, such as crystals and magnets. Then he discussed how randomness and disorder, pervasive in the physical world, are, paradoxically, essential for more ordered, complex structures to arise. In particular, he talked about how matter breaks its inherent symmetry to create new information and ever-increasing complexity. The public lecture was jointly organized with the Department of Mathematics, NUS. A total of 49 people attended the public lecture.



Bruce Reznick: A Walk down the Arithmetic-Geometric Mean Streets of Mathematics

Professor Bruce Reznick of University of Illinois at Urbana-Champaign presented a public lecture titled "A Walk down the Arithmetic-Geometric Mean Streets of Mathematics" at NUS on 17 December 2013. In the lecture, he talked about the inequality of the arithmetic and geometric means and described some of its applications in optimization and finance as well as the solutions of some familiar calculus problems without using calculus. He also touched on a famous example of Motzkin which has wide applications in moment problems and which, in turn, has a lot to do with the way integer points appear inside triangles and tetrahedra. The public lecture was jointly organized with Nanyang Technological University. A total of 50 people attended the public lecture.

Current Program

Inverse Moment Problems: the Crossroads of Analysis, Algebra, Discrete Geometry and Combinatorics (18 November 2013 - 25 January 2014)

Website: http://www2.ims.nus.edu.sg/Programs/014inverse/index.php

Co-chairs:
Dmitrii Pasechnik, *Nanyang Technological University*
Sinai Robins, *Nanyang Technological University*

Applications of moments of measures in polynomial optimization led to a number of breakthroughs in optimization and real algebraic geometry, as well as to better understanding of ways to encode measures. Other similar threads are recently seen in the theory of integration on polytopes and counting of integer points in polytopes, as well as in quantum computing. The aim of the program is to further investigate relations between these topics and inverse moment problems, i.e., questions of reconstructing measures from a set of its moments, which are traditionally attacked by purely analytic tools.

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Activities

- Optimization, Moment Problems and Geometry I (Conference/Workshop/Tutorials): 27 November - 6 December 2013 and 11 December 2013
- Quantum Computing Workshop on Inverse Moment Problem (organized by S. Wehner): 9 - 13 December 2013
- Optimization, Moment Problems, and Geometry II (Conference/Workshop): 16 - 27 December 2013
- Graduate Student Winter School/Workshop, Featuring Mini-Courses by A. Leykin, J. Stoyanov and J. Yu*: 16 – 24 December 2013
- Polyhedra, Lattices, Algebra, and Moments (Conference/Workshop/Tutorials): 7 - 16 and 23 January 2014

Next Program

Workshop on Living Analytics: Analyzing High-Dimensional Behavioral and Other Data from Dynamic Network Environments 1 (26 - 28 February 2014)

.... Jointly organized with Living Analytics Research Centre, Singapore Management University

Website: http://www2.ims.nus.edu.sg/Programs/014wliv/index.php

Organizing Committee:

Stephen E. Fienberg, *Carnegie Mellon University*
Ee-Peng Lim, *Singapore Management University*
Wei-Liem Loh, *National University of Singapore*

The aims of the proposed program is to: Introduce current living analytics research activities to other mathematical science, machine learning, and statistical researchers in Singapore; Broaden the statistical research underpinnings of models and computational algorithms for living analytics research and other research activities involving the analysis of large high-dimensional databases; Foster interaction among scientists developing methodology for “big data” problems using living analytics as a focal point; and chart new directions of research; and explore possible collaborations.

Programs & Activities in the Pipeline

Workshop on “IDAQP and Their Applications” (3 - 7 March 2014)

… Dedicated to Professor Takeyuki Hida
… Co-sponsored by RIST, Quantum Bio-informatics Research Division, Tokyo University of Science, and Aichi Prefectural University

Website: http://www2.ims.nus.edu.sg/Programs/014widaqp/index.php

Chair:

Masanori Ohya, *Tokyo University of Science*

In the past few years the fields of infinite dimensional analysis and quantum probability (IDAQP) have undergone increasingly significant developments and have found many new applications, in particular, to classical probability and to different branches of physics. Those fields are rather wide and strongly related in interdisciplinary nature. This workshop aims to make a bridge among these interdisciplinary fields in our workshop. In these fields, the focus will be on quantum information theory and white noise analysis in line with IDAQP.

School and Workshop on Classification and Regression Trees (10 - 26 March 2014)

Website: http://www2.ims.nus.edu.sg/Programs/014swclass/index.php:

Organizing committee:

Wei-Yin Loh, *University of Wisconsin-Madison*
Probal Chaudhuri, *Indian Statistical Institute, Kolkata*
Ben Haaland, *Duke-NUS Medical School*
Tao Yu, *National University of Singapore*

Classification and regression trees are an integral part of the toolbox of data mining, machine learning, and statistics. New techniques have added capabilities that far surpass the early methods. Modern classification trees can partition the data with linear splits on subsets of variables and fit nearest-neighbor, kernel-density, and other models in the partitions. Regression trees can fit almost every kind of traditional statistical model, including least-squares, quantile, logistic, Poisson and proportional hazards models, as well as models for censored, longitudinal and multi-response data. Availability of free software has played a significant role in helping the techniques gain acceptance and popularity in the broader scientific community. The

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purpose of the workshop is to bring together current experts in the field to discuss recent developments and generate ideas for future research. The purpose of the school is to introduce the subject to other researchers and practitioners who are interested to learn the techniques.

Activities

- School (10 - 19 March 2014): Professor Wei-Yin Loh will give a tutorial spread over four days (11, 13, 17 and 19).
  - Day 1: Grand tour of examples and introduction to decision tree methods
  - Day 2: Classification tree algorithms
  - Day 3: Basic regression tree algorithms
  - Day 4: Advanced regression tree algorithms

- Workshop (20 - 26 March 2014): This is a research conference consisting of talks by international experts followed by general discussions each day.

Self-normalized Asymptotic Theory in Probability, Statistics and Econometrics (1 – 30 May 2014)

Website: http://www2.ims.nus.edu.sg/Programs/014self/index.php

Co-chairs:
Ngai Hang Chan, *The Chinese University of Hong Kong*
Xiaohong Chen, *Yale University*
Qi-Man Shao, *The Chinese University of Hong Kong*

Asymptotic theory has played a fundamental role in probability and statistics. The law of large numbers, the central limit theorem, the Edgeworth expansion, the Cramer moderate deviation and the Varadhan large deviation are cores of the asymptotic theory. The classical limit theorems for sums of independent random variables are well-established with necessary and sufficient moment conditions. Driven by the applications in statistics, econometrics, physics, bioinformatics and other subjects, extensions to dependent variables and high dimensional data have been actively studied and various new methods are also developed.

This program will provide the probabilists, statisticians and econometricians a unique platform to discuss interesting fundamental problems and results and explore possible solutions related to asymptotic theory. It is also intended to bring young researchers to the frontier of this fascinating area.

Activities

- Research and Informal Discussions: 5 - 9 May 2014 & 26 - 30 May 2014
- Tutorial on Introduction to Self-normalized Limit Theory: 14 - 16 May 2014
- Tutorial on Introductory Econometrics: 14 - 16 May 2014
- Workshop on Self-normalized Asymptotic Theory in Probability, Statistics and Econometrics: 19 - 23 May 2014

Algorithmic Randomness (2 – 30 June 2014)

Website: http://www2.ims.nus.edu.sg/Programs/014algo/index.php

Chair:
Frank Stephan, *National University of Singapore*

Activities

- Informal Collaboration: 2-8 June 2014
- Ninth International Conference on Computability, Complexity and Randomness (CCR 2014): 9-13 June 2014
- Informal Collaboration: 10-30 June 2014

IMS Graduate Summer School in Logic (23 June – 4 July 2014)

... Jointly organized with Department of Mathematics, NUS

Website: http://www2.ims.nus.edu.sg/Programs/014logicss/index.php

The Summer School bridges the gap between a general graduate education in mathematical logic and the specific preparation necessary to do research on problems of current interest in the subject.

Activities

- Week 1: Lectures in Set Theory by Hugh Woodin, Harvard University
- Week 2: Lectures in Recursion Theory by Theodore A Slaman, University of California at Berkeley and Yu Liang, Nanjing University

The Geometry, Topology and Physics of Moduli Spaces of Higgs Bundles (7 July - 29 August 2014)

Scalar Curvature in Manifold Topology and Conformal Geometry (1 Nov - 31 Dec 2014)

Mathematical Conversations

Shing-Tung Yau: Manifolds, Cosmos, China >>>

Shing-Tung YAU

Interview of Shing-Tung Yau by Y.K. Leong

In a company of three, one will be my teacher.

- Chinese proverb

Shing-Tung Yau made deep and fundamental contributions to differential geometry and partial differential equations with an impact that extends beyond mathematics to many scientific disciplines, notably cosmology and theoretical physics.

The story of Yau from grinding poverty to international eminence reads like a modern fairy tale or the script of a Cantonese movie of the sixties (*Discover Interview* in June 2010 issue of *Discover* magazine). Born as the fifth of eight children during the final stage of the civil war in China, he was only a few months old when the whole family fled from the village of Shantou in Guangdong and sought refuge in the political haven of Hong Kong in the tumultuous wake of the communist takeover of the mainland. Life was hard, even with the pay (which was poor) his father could earn as a college professor and when Yau was 14 his father died of some sickness. Though the seed of his interest in mathematics had already been planted in him by his father, he would spend much time as the leader of a street gang when not reading the kungfu novels that were popular in Hong Kong. It was in Chung Chi College (which later became one of the colleges of the Chinese University of Hong Kong) that he met the proverbial guardian angel, who appeared as a college teacher from the University of California at Berkeley and who initiated, if not played a crucial role in, Yau's successful application for an IBM fellowship to study in Berkeley even before he had completed his undergraduate studies. Two years after arriving in Berkeley, he obtained his doctorate at the age of 22 with the world renowned geometer Shiing Shen Chern (1911-2004) as his advisor. In the following years and before he was 30, by dint of perseverance and his intellectual abilities, he cracked a tough nut known as the Calabi conjecture that had hitherto defied the experts. It was no flash in the pan for the succeeding years saw more contributions, often in joint work with others, to the theory of partial differential equations and general relativity. This corpus of work led to a Fields Medal awarded to him at the age of 33.

Since then, his research output continues unabated, often in collaboration with other mathematicians and physicists. Among the numerous topics and conjectures in differential geometry, algebraic topology, partial differential equations and string theory that his work has resolved and contributed to are geometric invariant theory, holomorphic vector bundles, symmetric spaces, Kähler manifolds, positive mass conjecture, minimal surfaces, Yamabe problem, Smith conjecture, Hermitian Yang-Mills connections, Frankel conjecture in complex geometry, Ricci flows in 3-dimensional manifolds, Monge-Ampère equation, uniformization theorem for Kähler manifolds, rigidity of complex structures for Kähler manifolds and extensions to locally symmetric spaces, minimal submanifolds, harmonic functions with controlled growth, rigidity properties of higher rank metrics for general manifolds, stability of manifolds, mirror symmetry.

While contributing to the resolution of numerous conjectures, partly in joint work with others, Yau's work has been fundamental to the advances made by others. His list of research publications numbering more than 300 is extensive in the range of disciplines and prolific in the collaboration of research. For his contributions, Yau received numerous awards and honorary degrees from universities and learned academies in the United States, Europe and China, notably the Oswald Veblen Prize, John J. Carty Award, Humboldt Research Award, Crafood Prize, US National Medal of Science, Wolf Prize, Sloan Fellow, Guggenheim Fellow and MacArthur Fellow.

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Yau has a personal sense of commitment to the development of future generations of mathematicians. On the academic side, he has successfully mentored more than 60 doctoral students. On the pragmatic side, he has helped raise substantial funds to promote mathematics education and research as well as international multi-disciplinary interaction. Perhaps less well known is the fact that he has personally donated funds to the establishment of scholarships and awards for undergraduates in the Chinese University of Hong Kong. Yau's active organization of and participation in activities for high school and college students have earned him a reputation as an ambassador of mathematics. In a move to popularize mathematics, he co-initiated a series of books *Mathematics and Humanities*. More recently, he wrote together with Steve Nadis two books for the general public – *The shape of inner space: String theory and the geometry of the Universe's hidden dimensions* and *A history in Sum: 150 years of mathematics at Harvard (1825-1975)*.

Since the opening up of China in 1979, he has been tireless in his efforts to encourage, nurture and develop mathematical talent in China from the school level to the research level. He was instrumental in establishing the following research institutes of each of which he is the director: the Institute of Mathematical Sciences at the Chinese University of Hong Kong, the Morningside Center of Mathematics in Beijing, the Center of Mathematical Sciences at Zhejiang University and the Shing-Tung Yau Center of National Chiao Tung University, Taiwan. In 2009 Tsinghua University established the Mathematical Sciences Center (MSC) and invited Yau to be its inaugural director. Within 4 years' time, MSC has expanded its composition to 22 full-time members and 9 part-time members and underlines its collaboration with Tsinghua's original Department of Mathematical Sciences with a number of adjunct appointments from the latter. Historically, Tsinghua University was founded more than 100 years ago in Beijing when China threw off her dynastic yoke. In its formative years, Tsinghua formed one of the three legs of the tripod (the other two being the then Peking University and Nankai University) which held the crucible that brewed and distilled the spirit of enquiry and perseverance among the first generation of modern China's influential scientists and mathematicians. This puts into perspective Tsinghua's ambitious plan to create an international environment of on-going conferences and workshops the whole year round for young researchers to "learn from the masters". To this end, Yau oversees the organisation of the Tsinghua Sanya International Mathematics Forum that is held in the city of Sanya on the island of Hainan. On the other side of the straits, he played a crucial role in the establishment of the National Center for Theoretical Sciences in Hsinchu, Taiwan. All these while Yau is actively engaged in research in the United States.

Yau is the first Chinese mathematician to be awarded the Fields Medal and must have been an inspiration to the generation of young Chinese mathematicians who were given the opportunity to pursue studies and careers outside China and strive to reach the pinnacle of excellence. It is true that he does not have the advantage of charisma and mystique that come with achievement fired and tested by the rites of passage of war and revolution. It is also true that he emerged at a time when the Chinese scientific psyche was very much riveted by the successes of the earlier generation that included legendary figures like S.S. Chern, Luogeng Hua (L.K. Hua) (1910-1985) and Chen Ning Yang. But Chern was his PhD advisor and he (Yau) was one of the first young Chinese scholars invited by Hua when China first opened up her academic doors in the 1970s. And he (Yau) has made significant contributions at the interface of mathematics and physics that is close to the heart of Yang. It is hard to question Yau's credentials to qualify as the mathematical successor of Chern. When one looks at Yau's initiatives and efforts in pushing the development of mathematical talent and research in China, it is hard to dispute his influence in and "titanic contributions" (in the words of number theorist John Coates) to the advancement of mathematics in China. The following results and outcomes bear his imprints: International Congress of Chinese Mathematicians, Morningside Medals, Yau High School Mathematics Awards, New World Mathematics Awards, Hang Lung Mathematics Awards, S.T. Yau College Student Mathematics Contests.

Yau's brilliant career is marked by an unusual, if not unfortunate, episode which brought him international attention of a sort that no mathematician would have wanted and which gave mathematics and mathematicians publicity normally heaped on celebrities in the entertainment industry. In events leading to and following the award of the Fields Medal to Grigory Perelman in 2006 for path-breaking work that culminated in the resolution of the Geometrization Conjecture of William Thurston (1946-2012) (Fields Medal 1982) and the Poincaré Conjecture, Yau was unwillingly and

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unwittingly drawn into a controversy ignited and fanned by the American media and which quickly spread throughout the world. In response to a commentary penned with a somewhat sensational and partial slant, Yau set up a public relations website where he posted numerous testimonials of fellow mathematicians and co-workers written in support of him. This brief episode was, however, quickly and amicably concluded with an article in *New York Times* that presented a conciliatory profile of Yau the man and Yau the mathematician and, in particular, the professional motivation behind the efforts of his team of researchers on elucidating Perelman's work.

Having been at Harvard University since 1987, Yau is now the William Casper Graustein Professor of Mathematics and has recently been appointed a faculty of Harvard's physics department. He has visited Singapore a number of times in the past and is a close and respected friend of many mathematicians in the National University of Singapore. On 4 January 2011, he was invited by the Institute for Mathematical Sciences to give a public lecture on *The shape of inner space*, which is also the title of the popular book then recently written by him and Steve Nadis. It is also a personal story of the evolution of the ideas behind Calabi-Yau spaces and its relation with string theory – a theory which many hope will provide the key to the Holy Grail of physics (Grand Unification Theory or "Theory of everything"). On the eve of the lecture, he was interviewed by Y.K. Leong on behalf of *Imprints*. The following is an edited and vetted version of the interview in which he briefly recounts his early years in Hong Kong and Berkeley and gives us a glimpse of the intriguing, if only speculative, connection between geometry (Calabi-Yau spaces) and string theory. He also offers an insight into some of the factors that thwarted the development of mathematics in dynastic China and assesses the state of mathematical development in modern China.

Acknowledgement. Y.K. Leong would like to thank Yat-Sun Poon (University of California at Riverside, USA and Mathematical Sciences Center, Tsinghua University, China) for his critical and useful comments which help improve the presentation of this introduction.

Imprints: After a few years of study at Chung Chi College of the Chinese University of Hong Kong you went to the University of California at Berkeley for graduate study. Why did you choose to go there?

Shing-Tung Yau: Actually I had no choice. I did not have an undergraduate degree at that point, so basically I could not go to any other university. My teacher in Hong Kong at that point came from Berkeley. He had just graduated from Berkeley and knew quite a few faculty in Berkeley who helped arrange this possibility to go to Berkeley. And, of course, Berkeley was one of the best in the world at that point; so I was pretty happy with that. On the other hand, I did not apply to any other place because I had no other choice.

I: So they waived the requirements?

Y: Berkeley was able to waive the requirements, and more importantly, could provide me with a fellowship. I was so poor I did not have much money. It was a well-funded fellowship from IBM through Berkeley.

I: That scholarship must have been very competitive, isn't it?

Y: Oh, yeah. It was extremely competitive. The math department had only two IBM fellowships and they probably gave them to the best applicants. They gave one to me. This shows how flexible they were; [they give it] as long as they feel somebody is good. I was very impressed by their flexibility. Quite likely, my teacher – my later teacher – Chern played a role in this possibility. But I didn't know Chern at the point when I applied. I knew a man who graduated from Berkeley; he was my professor in Hong Kong and is called [Stephen] Salaff. He had just graduated [from Berkeley] a couple of years before. He arranged [it] with Donald Sarason who was in Berkeley; it was very nice of them.
[Professor Shoshichi Kobayashi (1932-2012) was the chairman of the admission committee of the mathematics department of Berkeley when I was admitted. He had been very proud to have admitted me in 1969. – Yau]

I: He [Salaff] must have been very impressed by you academically.

Y: Oh, yes. Actually he tried to get me an early undergraduate degree. There was a big fight at the Chinese University of Hong Kong. Of course, they regret it now because I got a Fields Medal. If I was a graduate from Chinese University [of Hong Kong] it would count as a good point for them.

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I: You mentioned you didn't know Chern before you went to Berkeley.

Y: That's right. When I applied, I didn't know him, but after I was admitted Chern was offered an honorary degree by the Chinese University of Hong Kong. I think in June he came to Hong Kong from America to pick up the degree and I met him in the middle of June. That was after I was admitted. But I'm sure he must have helped somewhat.

I: You worked on the Calabi conjecture shortly after getting your PhD. How did you get interested in the Calabi conjecture?

Y: Actually I was working on it even when I was working on my PhD. I was interested in curvature and its interaction with topology and complex structure and all that. It comes so naturally that it is the most important question I need to solve because that was the only general procedure to construct manifolds with certain properties on Ricci curvature. Up till now it is still the most general way to construct such a manifold. I was fascinated by it and I thought it had to be solved one way or the other because it is basically the building block of the whole foundation. And if I don't go through it, it would not be possible. So I got to solve it.

I: Was your PhD thesis connected with the [Calabi] conjecture?

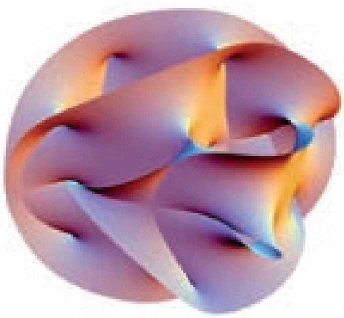
Y: No. After I arrived at Berkeley, I read on some problem relating the fundamental group of manifolds with curvature. I wrote two papers on it and they were published in the second year of my PhD. [In my] first year, Chern was not in Berkeley; in fact, he was on leave. By the time when he came back, I told him that I did something and he was very friendly. Then I asked him to be my advisor. First year, he was not my advisor. He had agreed, but then after one month, he said, ``Well, maybe what you've got is good enough for a PhD thesis." So that was my thesis. On the other hand, I started to change to a different subject. I was influenced by him a lot, of course.

I: You mentioned Morrey [Charles B. Morrey (1907-1984)] in your book *The shape of inner space* . . .

Y: First year, I was taking three different courses. One is by Morrey, which is very important, because I learnt partial differential equations and the non-linear theory from him. The other one is on geometry by [H. Blaine] Lawson; he was teaching elementary differential geometry at that point. I told him what I did, so we started to work on it together. One paper was written with him –- he was working on minimal surfaces, but I was interested in fundamental groups and negative curvature. In fact, he worked on the problem because of my suggestion. The third course I took was by Spanier [Edwin Henry Spanier (1921-1996)] on algebraic topology. They were all good courses. Spanier was a great algebraic topologist. He finished writing his famous book Algebraic topology at that time. You know the quarter system -– you take three courses. [In the] spring quarter, unfortunately, most people went down to the demonstrations because of the Vietnam War. Basically, Spanier's course and Lawson's course got cancelled because [there were] not enough students. But Morrey's, I still went to. Because I was the only student, and he was very faithful, he taught me one-to-one. It was very nice to be able to see him every day.

I: Could you tell us briefly what Calabi-Yau manifolds are?

Y: Well, basically, it is a space which is compact, closed without boundary and yet its Ricci curvature is identically zero, which means in terms of general relativity that there is no matter. Matter distribution is described by the Ricci curvature. So it is a space with no matter, and yet as a compact space we want to have gravity. That means the full curvature tensor is not identically zero. A torus is [a space where] the full curvature is identically zero. We are looking for a closed universe where there is no matter and yet there is gravity. So, no Ricci curvature and yet there is still curvature. These are the things we are looking at. Now Calabi-Yau adds one more thing, namely that there is some internal symmetry behind it. In terms of physics, it's called supersymmetry. In terms of mathematics, it's a Kähler manifold with compact structure behind it. So that's the Calabi-Yau manifold.



Calabi-Yau manifold
(Courtesy Wikimedia Commons, https://commons.wikimedia.org/wiki/File:Calabi-Yau.png)

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I: Were their connections with theoretical physics first discovered by physicists or by mathematicians?

Y: What happened is that I was there with many physicists over the years. One of them, Gary Horowitz was my postdoc, I hired him. He had a PhD in physics, he was in general relativity. I worked in general relativity also. I told him about this work and that it might be useful sometime for physics. In the beginning he didn't know it. And then there was Andrew Strominger who came as a postdoc, but not mine; he was in Princeton. I told him what I had done, but not very clearly. Later the string theorists came up and found out what they wanted was close to what I wanted. So they started to make use of it. Of course, string theory has not been verified. The extra dimensions have not been verified. Many physicists are trying to verify it. If string theory is correct, supersymmetry should be there; then I think it would appear. The question is how to see it. It probably takes quite a long time. In any case, it's interesting. The mechanism may be useful even if string theory were wrong. That's what we hope. It's so beautiful that it is difficult to imagine that it doesn't have any real practical applications.

I: Are you surprised by the connections?

Y: I wouldn't say I was very surprised because when I did the Calabi conjecture I felt that it was so beautiful a statement and the flavor so close to general relativity and if there is a general way to construct such a space – that's what I did – it is difficult to imagine that Nature does not use such a space. I think it has to come up somewhere. Although it was 8 years later that people were excited about it, I am not surprised by it.

I: You have sometimes called yourself a "physicist" (I believe that your wife is a physicist).

Y: My wife is a physicist. I never really call myself a physicist. I don't mind being called a physicist. I do not have enough physical intuition compared with my friends in physics. But, on the other hand, compared with many mathematicians, I have better insight [in physics] than they. That's all I can say. [Yau has been concurrently appointed professor of physics in Harvard's department of physics in 2013. – *Imprints*]

I: How much do you interact with experimental physicists?

Y: I do see them. I go to their seminars and talks. I even listen to astronomy talks and all that. I go to CERN, some of my friends are up there. It's quite interesting. I get some interest and insight from them but I don't understand experiments very much.

I: Do you keep up with the results of the LHC (Large Hadron Collider)?

Y: To the extent that if it has influence on theory, I always listen to them. There are talks in Harvard by people from there. I used to go there to listen.

I: I think that Paul Dirac [(1902-1984)] believed that mathematical beauty is a good guide in choosing or formulating physical theories. Using this kind of guide, do you think that string theory would be the best model available?

Y: Well, I won't say that would be the only guideline because beauty is not the only thing. You can be fooled by it. The way string theory has come about is far more complicated than one single beauty. It is related to many aspects of mathematics –– geometry, topology, representation theory, and many other aspects. Each of them has such a strong influence and beautiful consequences. Some of the problems in math were solved only using the intuition from string theory although the final proof was given by rigorous mathematics. The fact that it can be proved by mathematics demonstrates that the consequence of the physical intuition is fine. And the proof is not trivial. It's not a question of one or two pages or just an operation. It is really tricky otherwise it is difficult to say that there is beauty. It's not just that you draw a picture of an apple and say that the apple is beautiful and conclude it has applications. Here the proof has many consequences, maybe 100 papers were written in mathematics, and the consequences predict some statements and theorems in math which mathematicians did not dream about before and then they go ahead to prove them. This is true beauty, I feel, with structure, and all can be verified. I think this is something that one could not easily give up. I think, as you just mentioned, beauty is not enough. It should have more depth and structure behind it.

I: I think most physicists are not very comfortable with using too much abstract mathematics in their theory.

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Y: There are two kinds of physicists. In the old days, most physicists used combinatorial arguments – Feynman diagrams, calculations and all that. But now string field theory gives far more geometric insight into algebraic geometry, topology and all that. The more old fashioned physicists are not used to it, so maybe that is one reason why they criticize them. But in these modern days the string theorists are comfortable with using this new geometric terminology. They learn quite a lot, they know how to do the calculations themselves. They are very impressive, they know a lot about that. I would say some of them are even better mathematicians than some mathematicians. I don't know whether there is any criticism about that.

I: Stephen Hawking has recently written a book [*The Grand Design*, written jointly with Leonard Mlodinow] advocating M-theory. Do Calabi-Yau manifolds still feature in M-theory?

Y: I didn't read the book. Some of the models from M-theory can be built out of Calabi-Yau models; some of them are not, so far. How closely they are related is quite interesting. The problem about building models in M-theory in 11 dimensions is that the 7-dimensional manifold needed is far more complicated and not understood at this moment, not like the 6-dimensional Calabi-Yau manifolds which are deeper and better understood than the 7-dimensional manifolds. The 7-dimensional manifolds are basically examples but we don't have a clear understanding [of them].

I: Is the 7-dimensional manifold in M-theory related to the Calabi-Yau manifolds?

Y: Some of the constructions are related, but not all of them. So far, most of the constructions can be traced back to the way we construct Calabi-Yau manifolds. Whether the 7-dimensional manifolds are really exotic is absolutely irrelevant and it is not clear also.

I: Why do they add one extra dimension?

Y: Well. That's from physical intuition. They have brane theory and all that. Many physicists are comfortable enough with the Calabi-Yau manifolds in the 10-dimensional space.

I: If I understood it correctly, in string theory you have the 4-dimensional space-time together with a Calabi-Yau manifold, and in this space-time there is time, but in the Calabi-Yau manifold there is no dimension corresponding to time, does it?

Y: The 10-dimensional manifold is a Cartesian product of a 4-dimensional space-time with a 6-dimensional Calabi-Yau manifold. The 4-dimensional space has time there.

I: It seems rather strange that time is not involved in the 6-dimensional Calabi-Yau manifold, doesn't it?

Y: No, no, the time is there; it's in the other part. Even if you look at 4-dimensional space-time, there is one part that has no time – the 3-dimensional space is separate from time.

I: What happens when you do the physics within the Calabi-Yau manifold? Then there is no time involved.

Y: Well, the 6-dimensional manifold sits inside a 10-dimensional space and you do the physics inside this 10-dimensional space. Time is there.

I: Can you observe this 6-dimensional manifold?

Y: It's not impossible to see it. There is speculation that during the Big Bang there may be some strings vibrating and then after inflation there may be some cosmic strings, fundamental strings. People try to observe those strings. If those strings are there, you can actually observe them but it's hard to find them. In astronomy you can actually see cosmic strings like the galaxies. Cosmic strings have some effect on the universe – you can see how light is recessed and all that. That is assuming you are looking at the strings right from the beginning of space and time. On the other hand, you are supposed to see an icon from the Calabi-Yau space that is 10^{-33} centimetres — that's too small to be seen. People try to find ways to understand the consequences of it. That part you cannot see, but you can look at the cosmic strings. Then it is something that is possible to be seen.

I: But at the ordinary level in everyday life, it's not possible to see it, isn't it?

Y: It depends on what you mean by "everyday life". You go up to astronomy, you can actually see cosmic strings. But, of course, in daily life you cannot see a proton or electron either. You don't see them in everyday life even if you see them in experiments. You don't see a quark; a quark can

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never be seen. There are many things you cannot see but it works.

I: In topology and chaos theory, there is a notion of fractional dimension. Has anyone attempted to introduce a "complex dimension" into the study of manifolds (like complex manifolds)?

Y: It's always there. The question is how to make use of it. For example, you were asking a question that there is no time. In quantum field theory usually you have space and time but then people discuss what is called Euclidean quantum field theory. What does that mean? They make time to be imaginary time, $z = it$. Now you move from a Lorentz manifold to an Euclidean manifold. In between you have to go through some analytic continuation. What that really means we don't really know. There's no good intuition in it but it's routinely done in quantum field theory nowadays. Because it's difficult to do Lorentzian geometry, so people do Euclidean geometry first and then quantum field theory and finally conclude something. In many ways it's built in there but what kind of theoretic intuition there is, it's not clear. It's just like the Riemann zeta function [$\zeta(s)$] – you want analytic continuation, but what it continues to, we don't really know. In the Riemann zeta function when s becomes a negative number what does that mean? $1 + 2 + 3 + \ldots$ sum up to a number. It's similar to that. It's out of the ordinary intuition we have. People are still doing it.

I: You mentioned imaginary time. I think Stephen Hawking had something like that in one of his early ideas.

Y: Yeah, Stephen Hawking in the 1970s talked about Euclidean gravity with imaginary time.

I: I believe you have been applying conformal geometry to computer graphics and pattern recognition. Has it been applied to the movie and computer game industry?

Y: It could be done. But we are not good in marketing or whatever. We have done a very good software applying conformal geometry to computer imaging and all that. What comes out is very beautiful and very nice, much better than what we know to exist.

I: Did you get Hollywood to be interested in it?

Y: Well, it's very complicated. Hollywood was interested in it. In fact, before the film *Avatar* became very famous, they started to approach us because we could do a good *Avatar* compared to them. Although we have the original idea, in order to work on it as a package you need to hire many people to do the research, to make it as a finished product. It's too complicated. That becomes an industry. Industry is something I'm not comfortable dealing with. So I decided not to work on that.

I: You have been often described as an "ambassador of mathematics" and have, in fact, devoted much time, effort and money in helping young mathematicians and promoting mathematical education and research in China, Hong Kong and Taiwan. Are you driven by some kind of personal mission?

Y: It's not just in China or Hong Kong. I have been in America for 40 years and tutored many students in America. I consider it a duty of a true mathematician to train younger mathematicians, in China or not. I have many non-Chinese students. Our profession has to continue. In order for it to continue you need to have the younger generation to grow. I feel it's our duty to train them, to make them do well; so I do that. I train many non-Chinese mathematicians but because I'm Chinese China sends a lot of Chinese students to study with me. So naturally I have more Chinese students now, and also quite a lot of the Chinese are very good and they are able to continue. I go back to China, yeah. As I said, I try to help the mathematical community. It's easier for me to contact people I know there, and there's a large group of brilliant young students in China. There are 1.4 billion people in China. Even if 0.001 per cent of students have interest, it would be a big deal. It seems to me for my situation it's more effective for me to train the Chinese students than to go to Europe or other places.

I: How often do you go back to China?

Y: I go back every year, for a few times, to Hong Kong, mainland China and Taiwan.

I: The Chinese-born Swiss geologist Kenneth Hsu wrote, "The scientific revolution did not occur in China, because the truly talented became poets, painters, and creative writers; they chose not to be stifled by the Confucian

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academic tradition." He wrote this in an article "Why Newton was not a Chinese". What is your view on this?

Y: I know Kenneth Hsu. He is a controversial guy, trying to make something exciting by publishing something exotic. Well, there are people who became poets and all that but there are many more complicated reasons than that. People who are driven by Confucian thinking are much too practical. They want to see mathematics to be useful right away. This is not quite the Greek style. The Greeks do not insist that mathematics has to be useful in daily life immediately; they think of beauty and truth much more than practical use. Even now I see in China or even in Singapore that practical use is more important than truth or beauty for mathematics. I think that's one of the problems. For example, the Chinese actually knew what a proof means, I think, around the Tang Dynasty but then they didn't really push it too far. Logic, mathematical logic, and the axiomatic approach had never been something that the Chinese care about because they don't see any reason to be axiomatic. They see a direct application to everyday life. So the Newtonian way of writing Newtonian mechanics, the three laws of mechanics, never came to the mind of the Chinese. They don't see what is the point of a law. They don't see any general law. They just say, "You apply to this, you do this and work it out and it is fine." You look at the books that the [early] Chinese have written in mathematics. Basically they give examples — you do this, you do that, you come up with that. They did know what a proof was at some point, but I don't think they exploited it too much. For example, the translation of Euclidean geometry was done very, very late, about 400 years ago. The complete Chinese translation was done in the 19th century. Four hundred years ago, they translated the first 6 chapters [of Euclid's *Elements*] and then they gave up. Nineteenth century, they finished the translation, but it was too late by then.

I: I think the Chinese way of writing is harder to write for mathematics.

Y: It's also true but I don't think that's a problem. You know, the Chinese had a lot of connections with Arabian countries quite early. Indian mathematicians came to China around 500 AD. They [the Chinese] had contact with the Arabs quite early because a lot of Arabs came to China. The Chinese were exposed to simpler ways of writing but they insisted on their own way. Even when calculus was introduced in the 19th century the symbols of the Europeans were given up because of the ugly way of writing them in Chinese.

I: I think you also once said that it would be a long time, maybe 50 years or even 100 years, before China would be scientifically equal to the US. From your close association with the scientific and sociopolitical establishments in China during the past decades, do you still see remnants of this kind of Confucian tradition that stifles scientific creativity?

Y: Well, this statement, of course, is making the assumption that the current inertia is still going on and [the Chinese] does not want to change. But if Chinese leaders or culture has a sudden jump or change, that will make a lot of difference. For example, there are a lot of Chinese mathematicians now in America. Let's say 10 percent of them want to go back, the very good ones. That will make a good change. That will make the gap smaller immediately but it's not clear they would come back. I made the assessment based on the events that had been going on. You know, if the [Chinese] government does the right thing or if something like a disaster happens in America — for example, in the 20th century when the Germans decided to kick out all the Jews — then, of course, that would make a great deal of difference. I think in the late 19th century American mathematics was nowhere compared with Germany. In a matter of 50 years they are better because most of them came from Germany. After all, America is an immigrant country. A lot of these very brilliant mathematicians in America actually come from all over the world. Yes, there are many local [mathematicians] but there are also many Europeans, Chinese, Indians and others. For example, if the American government suddenly becomes crazy and decides to expel all the foreign scholars because they don't want to use American taxpayers' money to pay foreign scholars, which may happen, that will be good for China and some other foreign countries because they will suddenly get a large group of really outstanding people. American policy right now is great. It's best for the environment for foreign mathematicians to come and try to become outstanding mathematicians. If the American government is stupid enough not to realize the value of this, then that's it. So the statement I made depends on events. If America stupidly changes for the worse and China skilfully changes for the better, then the gap will get smaller.

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I: At the moment are the incentives in China enough to pull back the Chinese mathematicians?

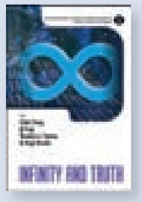
Y: Well, there is a question of culture and inertia, as I've said. As far as money is concerned, China can basically do anything equivalent to what the Americans can do for the best ones. For the mediocre ones it's not so clear what they would do. On the other hand, I think the payment is good enough for them. I think in the next 10 years there will be a much bigger change because they have changed the incentives. They realize the importance of the second group of people.

I: I believe you have a lot of students in the past. Do you still have any students?

Y: I still have five students graduating with PhD this year. It's not easy to have all these students. I'm pleased with these students. It takes a long time to train them.

I: What advice would you give to graduate students who want to do research in mathematics?

Y: First of all, you have to train yourself to know all the basic skills so that when a problem comes you have the skills to work on it and, of course, once you have found a problem, have the curiosity and good interest in the problem, and get good advice from a good mathematician.



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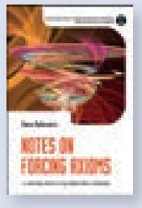

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