NUS National University of Singapore

IMPRINTS

INSTITUTE FOR MATHEMATICAL SCIENCES

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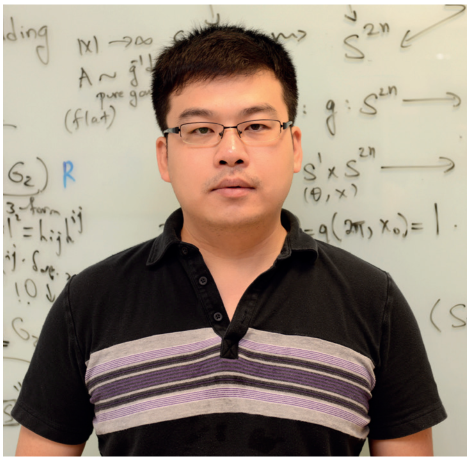
Topological Aspects of Quantum Field Theories >>>



Fei HAN

[Editor's note: In January 2013, the Institute hosted the "Workshop on Topological Aspects of Quantum Field Theories". Fei Han, Member of the Organizing Committee, contributed this invited article to Imprints *as a follow-up to the IMS workshop.]*

In recent years, problems and ideas arising from quantum field theory and string theory have profoundly influenced the development of algebraic topology and geometric topology. Many classical mathematical areas in geometry and topology have been re-examined and understood from the physical point of view. As a result, exciting new subject areas are gradually emerging, some of which I will mention in this article.

The Atiyah-Singer index theory is one of the great discoveries in the mathematics of the 20th century. It relates the Dirac operator (certain differential operators) (analysis) on a manifold to the A-roof genus (geometry/topology) of the manifold. Paul Dirac first introduced the Dirac operator in 4-dimensional space-time when he studied quantization of electric charges. The Atiyah-Singer index theorem unifies several historically famous theorems such as the Riemann-Roch theorem, the Hirzebruch signature theorem and the Chern-Gauss-Bonnet theorem. When one studies the index theory for a family of manifolds, K-theory, an important generalized cohomology theory in algebraic topology, provides the home of the family version of the A-roof genus. The Atiyah-Singer index theorem is related to 1-dimensional supersymmetric sigma models in physics. If one increases to 2-dimensional supersymmetric sigma models, the Witten genus can be discovered. It is a q-deformation of the A-roof genus, which is heuristically the equivariant index of a hypothetical Dirac operator on the free loop space of the manifold. The elliptic cohomology is the home of the family version of the Witten genus. However, unlike the K-theory that can be described in geometric ways, elliptic cohomology has so far been constructed only in the realm of homotopy theory. Many researchers have been attracted to try to give a geometric description for the elliptic cohomology. One of the directions is the Stolz-Teichner program, which proposes to use 2-dimensional supersymmetric quantum field theories to give geometric cocycles of elliptic cohomology. Thanks to the axiomatization of Atiyah-Segal, quantum field theory

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has now been absorbed into mathematics as a concept. In topological quantum field theory, there is an important conjecture, the volume conjecture, relating asymptotic behavior of the Jones polynomial to the hyperbolic volume of a knot. The latter is related to analytic torsion, an important object that has been studied in the index theory.

Certain gauge anomalies in physics have been discovered to be related to the family index theorem. Such anomalies are interpreted as the non-triviality of determinant line bundles of a family of Dirac operators. Closely related to the line bundles is the notion of gerbes. They are in fact "higher" line bundles from the point of view of higher categorical differential geometry. In physics, they are related to "B-fields". There has been a lot of studies in the geometry of gerbes as well as other higher objects. Higher objects can be used to twist classical theories like cohomology theory, K-theory. Many results have been established for the twisting of index theory.

Quantum field theories, according to the Atiyah-Segal axiomatization, are functors from cobordism category to the category of Hilbert spaces. Therefore it's very important to study the cobordism category. Roughly speaking, cobordism category is such a world in which each member is a closed manifold; two members are related by a one-dimension-higher manifold that bounds the union of the two members. Mathematicians have identified the homotopy type of the cobordism category. Another important new topological area motivated by string theory and 2-dimensional topological quantum field is string topology, which studies algebraic structures on the homology of free loop spaces.

The purpose of our workshop was to bring together researchers working on the areas to communicate ideas, dig out connections and stimulate possible research collaboration. Our workshop was able to have five well-known senior mathematicians as keynote speakers, namely, Alan Carey, Michael Hopkins, Ulrike Tillmann, Mathai Varghese and Weiping Zhang. Each one gave the first talk in a day. The workshop also had several series of talks; each series systematically focusing on a concept that is newly developed. The series of talks were delivered by Soren Galatius, Owen Gwilliam, Urs Schreiber and Konrad Waldorf. We felt that such a program structure could help participants be informed of the mainstream of the field and discuss certain topics intensively.

Engaged topologists and quantum field theorists

The theme of the workshop is related to several researchers in National University of Singapore (NUS) working in the areas of topology, geometry and theoretical physics. Several overseas participants had intensive discussion with local participants. Five graduate students from NUS, one graduate student from Nanyang Technological University, one graduate student from Harvard University, one graduate student from UC Berkeley and three graduate students from University of Copenhagen participated in the workshop. The IMS provided the workshop with an extremely supportive working environment, without which the workshop cannot be a successful one.

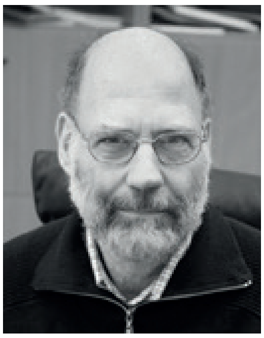
Fei HAN
National University of Singapore

New Scientific Advisory Board Members >>>

The Institute is highly pleased to welcome two new members to the Scientific Advisory Board (SAB) - Professor Douglas N. Arnold (University of Minnesota) and Professor Wolfgang Hackbusch (Max Planck Institute for Mathematics in the Sciences) – as well as a new ex-officio SAB member Professor Teck Hua HO (National University of Singapore (NUS)). In addition, the Institute is very happy that its former Director and former ex-officio SAB member, Professor Louis CHEN (NUS), kindly continues to serve the Institute as a member of the SAB.


Douglas N. Arnold

Professor Douglas N. Arnold is McKnight Presidential Professor of Mathematics at University of Minnesota. His research interests include numerical analysis, partial differential equations, mechanics, and in particular, the interplay among these fields. He served as the Director of Institute for Mathematics and its Applications (2000-2008), and was the President of the Society for Industrial and Applied Mathematics (SIAM) (2009-2010).His honors and awards include the first International Giovanni Sacchi Landriani Prize (1991), Guggenheim Fellowship (2008-2009), SIAM Prize for Distinguished Service to the Profession (2013), and foreign membership in the Norwegian Academy of Science and Letters. He was a plenary speaker at the International Congress of Mathematicians in 2002, and was appointed as SIAM Fellow in 2009, a Fellow of the American Association for the Advancement of Science in 2011 and an AMS Fellow in 2012.


Wolfgang Hackbusch

The second new member, Professor Wolfgang Hackbusch, is Director and Scientific Member of the Max Planck Institute for Mathematics in the Sciences in Leipzig. He is also Honorary Professor at Leipzig University, and Professor at the University of Kiel (on leave). His research work encompasses numerical methods in partial differential equations and integral equations, and efficient solution of large-scale and high-dimensional problems. He was a recipient of the Gottfried-Wilhelm-Leibniz Award of the German Research Council (1994), and he was awarded the Brouwer Medal (1996). He is a founding Member of the Berlin-Brandenburg Academy of Sciences (1993), and has been a Member of the German National Academy of Sciences Leopoldina since 2006.

Professor Teck Hua Ho joins the SAB as an ex-officio member, being the new Chairman of the Institute's Management Board. A brief introduction of Professor Ho is available in the article *New Management Board Members* in the current issue of *Imprints*.

The Institute would like to express its deep gratitude to the outgoing SAB member, Professor Olivier Pironneau. Professor Pironneau joined the Board in 2009, and his advice and suggestions have greatly benefited the Institute, and in particular its scientific programs.

Finally the Institute looks forward to strengthening its scientific programs further under the tutelage of the new and incumbent members of the SAB.

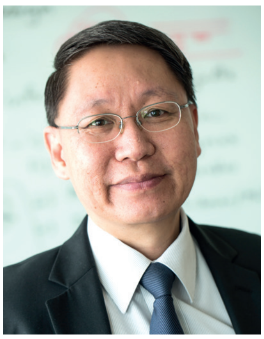
Wing-Keung TO
National University of Singapore

New Management Board Members >>>

The Institute is greatly delighted to welcome the new Chairman of the Management Board, Professor Teck Hua HO (National University of Singapore (NUS)) as well as two new Management Board members, Professor Alfred HUAN (Institute of High Performance Computing, Singapore) and Professor Limsoon WONG (NUS). In addition, the Institute is privileged that its former Director and former ex-officio Management Board member, Professor Louis CHEN (NUS), stays on as a Management Board member. The Management Board's former Chairman, Professor Chi Tat CHONG, remains in the Board as an ex-officio member, having become the Institute's new Director.


Teck Hua HO

The Management Board's new Chairman, Professor Teck Hua Ho, is Vice President (Research Strategy) and Tan Chin Tuan Centennial Professor at NUS. He is concurrently William Halford Jr. Family Professor of Marketing and the Director of the Asia Business Center at the Haas School of Business at the University of California, Berkeley. His research interests include experimental and behavioral economics, quantitative marketing models and marketing and production interfaces. He was a 2006 finalist for the John D. C. Little Best Paper Award, and was a 2005 finalist for the Paul Green Best Paper Award. In 2011, he was a finalist for the William F. O'Dell Award. He serves as the Departmental Editor for *Management Science*; Co-Editor for *Foundations and Trends in Marketing*; and as an Associate Editor for the *Journal of Marketing Research* and *Marketing Science*. At the Haas School of Business, he won the Earl F. Cheit Award for Excellence in Teaching in 2004, 2005 and 2006, and he was honoured with the Berkeley Distinguished Teaching Award in 2010.


Alfred HUAN

Professor Alfred Huan is Executive Director at the Institute of High Performance Computing (IHPC, Singapore) and Professor in the Division of Physics and Applied Physics at Nanyang Technological University (NTU). His research interests include surface and interface science as well as femtosecond dynamics. Professor Huan started his academic career at NUS in 1989 and served as Deputy Head of the Department of Physics at NUS (2004-2005). He joined NTU in 2005, and he served as the Head of the Division of Physics and Applied Physics and the Associate Chair for Outreach and Admissions in the School of Physical and Mathematical Sciences at NTU. He has been on secondment to the A*STAR Science and Engineering Research Council since 2011, and took up his current position at IHPC in 2012.


Limsoon WONG

Professor Limsoon Wong is KITHCT Professor of Computer Science and Professor of Pathology at NUS. He is currently Head of the NUS Department of Computer Science. He previously served as Vice Dean (Research) of the School of Computing at NUS (2007-2008). Before joining NUS in 2005, he was Deputy Executive Director (Research) at the A*STAR's Institute for Infocomm Research. Professor Wong's research areas include database theory and systems as well as computational biology. His honours include the Singapore Youth Award Medal of Commendation (2006), the Asian Innovation Award (Gold Award) (2003) and the Singapore National Academy of Science Young Scientist Award (1997).

The Institute would like to extend its utmost thankfulness to the members stepping down from the Management Board: Professor Andrew NEE, Professor Beng Chin OOI and Professor David Srolovitz. While the three members joined the Management Board in different times (with Professor Nee in 2006, Professor Ooi in 2007 and Professor Srolovitz in 2010), they have all contributed substantially in overseeing the Institute's activities and operations.

The Institute is confident to continue its smooth running and develop further under the guidance of the Management Board with its new composition.

Wing-Keung TO
National University of Singapore

People in the News >>>

David Donoho honored with the Shaw Prize

Professor David Donoho (Stanford University) has been awarded the Shaw Prize in Mathematical Sciences 2013 "for his profound contributions to modern mathematical statistics and in particular the development of optimal algorithms for statistical estimation in the presence of noise and of efficient techniques for sparse representation and recovery in large data-sets". Professor Donoho will receive the award in a presentation ceremony in Hong Kong on 23 September 2013. He was an invited speaker in the IMS program "Mathematics and Computation in Imaging Science and Information Processing" (August 2004), and gave a public lecture at NUS amidst the program. Our congratulations to Professor Donoho for the well deserved honor!

Newly Elected Academicians

The Institute offers its congratulations to Professors Gregory Lawler (University of Chicago) and David Vogan (Massachusetts Institute of Technology) on their election to Membership of the US National Academy of Sciences (NAS) in 2013, to Professor Peter Hall (University of Melbourne) on his election as Foreign Associate of the NAS in 2013, and to Professor Bin Yu (University of California at Berkeley) on her election as Fellow of the American Academy of Arts and Sciences in 2013.

Professor Lawler gave a colloquium lecture titled "Geometric and fractal properties of the Schramm-Loewner evolution" at NUS in February 2011, which was co-organized by the IMS. Professor Vogan was an invited speaker of the IMS programs "Branching Laws" (11- 31 March 2012), "Representation theory of Lie groups" (July 2002 – January 2003), and the IMS event "International Conference on Harmonic Analysis, Group Representations, Automorphic Forms and Invariant Theory" (9 - 11 January 2006). Professor Hall was Co-chair of the Organizing Committee for the IMS program "Meeting the Challenges of High Dimension: Statistical Methodology, Theory and Applications" (13 August – 26 October 2012) and an invited speaker of the IMS program "Stein's Method and Applications: A Program in Honor of Charles Stein" (28 July – 31 August 2003). Professors Hall and Yu were members of the Scientific Program Committee for the 7th World Congress in Probability and Statistics (14 - 19 July 2008) held in Singapore and co-organized by the IMS.

NUS Mathematicians to Deliver Invited Lectures at ICM2014

Weizhu Bao, Wee Teck Gan, Weixiao Shen and Shih Hsien Yu of the NUS Department of Mathematics have been invited to deliver 45-minute lectures in the following sections of the International Congress of Mathematicians 2014: Weizhu Bao in the Mathematics in Science and Technology section, Wee Teck Gan in the Number Theory section, Weixiao Shen in the Dynamical Systems and Ordinary Differential Equations section, and Shih Hsien Yu in the Partial Differential Equations section. Congratulations to the four colleagues!

Weizhu Bao was an organizer of numerous IMS programs, workshops and spring/summer schools, including "Multiscale Modeling, Simulation, Analysis and Applications" (1 November 2011 - 20 January 2012), "Workshop on Nonlinear Partial Differential Equations: Analysis, Computation and Applications" (7 - 10 March 2012), "Spring School on Fluid Mechanics and Geophysics of Environmental Hazards" (19 April - 2 May 2009), "Mathematical Theory and Numerical Methods for Computational Materials Simulation and Design" (1 July - 31 August 2009), and "Moving Interface Problems and Applications in Fluid Dynamics" (8 January - 31 March 2007). Wee Teck Gan was an organizer of the IMS program "Modular Representation Theory of Finite and p-adic Groups" (1 - 26 April 2013). Weixiao Shen was an organizer of the IMS event "Workshop on Non-uniformly Hyperbolic and Neutral One-dimensional Dynamics" (23 - 27 April 2012). Shih Hsien Yu was an organizer of the IMS program "Hyperbolic Conservation Laws and Kinetic Equations: Theory, Computation, and Applications" (1 November - 19 December 2010) and the IMS event "Symposium on Pure and Applied Analysis" (21 April 2008).

Personnel Movement at IMS

Eileen Tan, senior executive, joined the Institute on 22 March 2013. She is the Institute's new program coordinator, and will provide administrative support to the program organizers in their planning and organizing of IMS programs and activities.

Programs & Activities >>>

Past Programs & Activities in Brief

Optimization: Computation, Theory and Modeling (1 November - 23 December 2012)

Website: http://www2.ims.nus.edu.sg/Programs/012opti/index.php

Co-chairs:
Defeng Sun, *National University of Singapore*
Kim-Chuan Toh, *National University of Singapore*

The 2-month program provided a platform for exchanging ideas in solving large scale conic optimization problems including semi-definite programming (SDP) and symmetric cone programming (SCP), and discussed on the latest exciting developments in complementarity and related themes including optimization under uncertainty. The program consisted of three workshops, which consisted of 7 tutorial lectures and 80 invited talks.

The program's first workshop, "Large scale conic optimization" (19 – 23 November) consisted of 26 invited talks and 1 tutorial lecture by Jean Lasserre (LAAS-CNRS). The program's second workshop, "Optimization under uncertainty" (10 – 14 December 2012) consisted of 20 invited talks and 2 tutorial lectures by Werner Römisch (Humboldt-Universität zu Berlin) and Arkadii Nemirovski (Georgia Institute of Technology). The program's third workshop, "Complementarity and its extensions" (17 – 21 December 2012) consisted of 34 invited talks and 4 tutorial lectures by Roger Wets (University of California), Gesualdo Scutari (State University of New York at Buffalo), Vinayak (Uday) V. Shanbhag (Pennslyvania State University), and Jong-Shi Pang (University of Illinois at Urbana-Champaign).

An assembly of optimists


Shmuel Oren: Modeling oligopolistic electricity market and CO2 permit market


Richard Cottle: Special treatment for linear complementarity problems


Small scale conversation within computational moments (From left: Daniel Ralph, Tom Luo and Gesualdo Scutari)

Partly because of the support from leading experts in optimization and numerous outstanding scientists, this highly successful program attracted a total of 189 participants, and among them were 62 graduate students. The tutorial sessions delivered by some of the leading experts in optimization were very well received. In fact, some tutorial sessions attracted more than 100 attendees.

Workshop on Topological Aspects of Quantum Field Theories (14 - 18 January 2013)

Website: http://www2.ims.nus.edu.sg/Programs/013wquantum/index.php

Organizing Committee:
Ralph Cohen, *Stanford University*
Fei Han, *National University of Singapore*
Stephan Stolz, *University of Notre Dame*
Peter Teichner, *Max-Planck Institute for Mathematics at Bonn and University of California at Berkeley*

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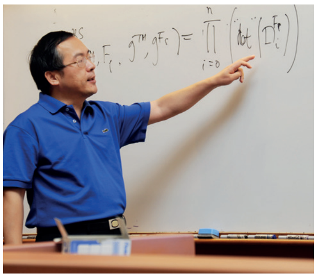
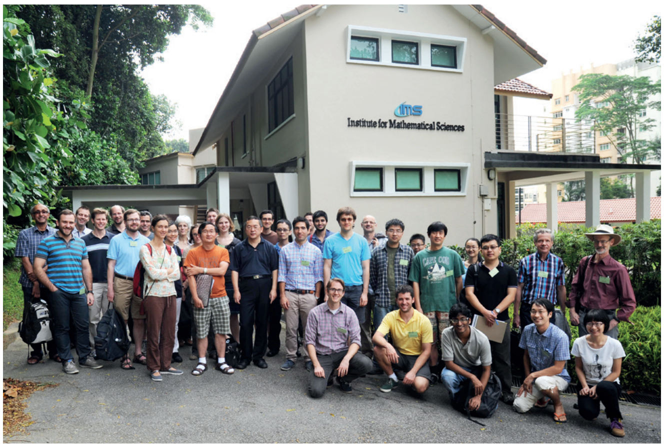
This five-day workshop consisted of 4 three-hour mini courses and 7 invited talks. Five world leading mathematicians gave the first talk in the days of the workshop: Mathai Varghese (University of Adelaide), Alan Carey (Australian National University), Michael Hopkins (Harvard University), Weiping Zhang (Nankai University), and Ulrike Tillmann (Oxford University). The keynote speakers touched on different aspects of the field motivated by quantum field theory and string theory. The 4 three-hour mini courses were delivered by the following visitors: Soren Galatius (Stanford University), Owen Gwillia (University of California at Berkeley), Urs Schreiber (Utrecht University), and Konrad Waldorf (Universität Regensburg).

The workshop had been structured in a way that not only participants were informed of the main stream of the field; graduate students and young researchers had a great opportunity to discuss certain topics intensively with experts as well. There were a total of 46 participants in this workshop, and among them were 13 graduate students.

Michael Hopkins:
Field theories and spheres



Weiping ZHANG:
Asymptotics of analytic torsion



Topologists meeting quantum field theorists

Algorithmic Game Theory and Computational Social Choice (7 January - 8 March 2013)

Website: http://www2.ims.nus.edu.sg/Programs/013game/index.php

Co-chairs:
Ning Chen, *Nanyang Technological University*
Edith Elkind, *Nanyang Technological University*

The objective of the program was to bring together experts on algorithmic aspects of economics and group decision making in order to foster interdisciplinary collaboration in two burgeoning research areas, namely algorithmic game theory and computational social choice. This program provided a meeting point for researchers from different areas that study incentives and collective action, such as mathematics, game theory, theoretical computer science, artificial intelligence, economics, social choice, and operation research, and exposed the participants to a wide variety of tools, techniques, and modeling perspectives.

This program was hosted at 2 different venues, namely Nanyang Technological University (NTU) and IMS. It consisted of one mini-workshop, 2 winter schools, a 2-part tutorial lecture and 2 seminars. There were a total of 13 tutorial lectures and more than 30 invited talks by world class experts in this program. The program started with a 2-day mini-workshop on Mechanism Design which consisted of a tutorial lecture by Jason Hartline (Northwestern University) and 5 invited talks. This was followed by a winter school and workshop on Algorithmic Game Theory which consisted of 6 tutorials and 20 invited talks. The second winter school and workshop on Computational Social Choice consisted of 6 tutorial lectures and 13 invited talks. This activity was followed by a 2-part tutorial lecture by Clemens Puppe (Karlsruhe Institute of Technology). The program concluded with 2 seminars held at NTU which consisted of 5 invited talks.

There were a total of 69 participants in this workshop, with a significant participation from 23 graduate students. At least 9 papers/projects were initiated, worked on or finalized during the program.

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Jason Hartline: Optimizing marginal revenue

Jerome Lang: Graphical languages for preference representation

Participants chatting over coffee (From left: Gerome Lang, Lisa Wagner, Arkadii Slinko and Piotr Faliszewski)

Algorithmic game theorists making social choices

Modular Representation Theory of Finite and *p*-adic Groups (1 - 26 April 2013)

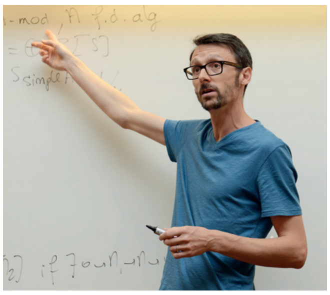
Website: http://www2.ims.nus.edu.sg/Programs/013mod/index.php

Co-chairs:
Wee Teck Gan, *National University of Singapore*
Kai Meng Tan, *National University of Singapore*

This program aimed to bring together leading researchers in the areas of modular representation theory of finite groups and *p*-adic group to discuss the latest developments in the field, chart out new directions for research and explore possible collaboration.

There were 6 two-hour tutorial lectures, 22 invited talks and 1 colloquium talk presented during the program. The tutorial lectures were delivered by: Marc Cabanes (Université Denis Diderot - Paris 7), Andrew Mathas (University of Sydney), Raphaël Rouquier (University of Oxford and UCLA), Vincent Secherre (Université de Versailles Saint-Quentin), Florian Herzig (University of Toronto) and Alexander Kleshchev (University of Oregon). Many of the invited talks were on the recent developments in categorifications and the *l*-modular or *p*-modular Langlands program.

An important message that came out of the program is that any further progress in the modular representation theory of *p*-adic groups has to rely on similar progress for finite groups of Lie type. This should stimulate the participants who work on finite groups of Lie type about the potential applications of their work and urge them to consider certain problems which will play a crucial role in the modular Langlands program. On the other hand, the success of the technique of categorification in advancing the knowledge of the modular representation theory of symmetric groups suggested that such techniques might eventually play a role in the modular representation theory of *p*-adic *groups*. There were a total of 53 participants, and among them were 11 graduate students.



Raphaël Rouquier:
Categorification in representation theory

Alexander Kleshchev:
On Khovanov-Lauda-Rouquier algebras

Sabin Cautis and Joseph Chuang: A dyadic discussion

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A finite group of representation theorists

Public lecture:

Philip Protter: What Really Happened in 2008, and Why?

Professor Philip Protter of Columbia University gave a public lecture, titled "What Really Happened in 2008, and Why?" at NUS on 13 June 2013. In the lecture, Professor Protter examined the causes of the collapse of the bubble in the U.S. housing market in 2008 and how they interacted to create the massive economic disaster. He started with a brief discussion on some earlier financial crisis, namely the Great Depression in 1929 and the Savings and Loans Crisis which peaked in the early 1990's. Then he proceeded to describe the events and circumstances that sustained the long U.S. housing bubble - which began around 1970 in California and turned wild in the last decade - until its collapse in 2008. In particular, he discussed the deregulation of controls for writing mortgages, the repackaging of housing mortgages as Asset Backed Securities (ABS) and especially Collaterized Debt Obligations (CDOs), the rating agencies using outdated methodologies to model ABS, the incentive conflicts for rating agencies, the relaxation of capital requirements for banks by regulators, the insurance of CDOs by insurance companies, and the climate of excessive greed due to out-of-control executive compensation levels. The lecture was delivered to an enthusiastic audience of 68 people.

Current Program

Nonlinear Expectations, Stochastic Calculus under Knightian Uncertainty, and Related Topics (3 June – 12 July 2013)

…Jointly organized with Centre for Quantitative Finance, NUS

Website: http://www2.ims.nus.edu.sg/Programs/013wnlinear/index.php

Chair:
Shige Peng, *Shandong University*

This program will be focusing on, but not limited to, the two areas (i) nonlinear expectations, backward stochastic differential equations and path-dependent PDE; and (ii) nonlinear expectations, risk measures and robust controls. The program is intended for leading researchers working in these areas to exchange ideas and hopefully to inspire new mathematical concepts and results. It is also intended to bring young researchers and investment banking practitioners in the related quantitative areas to the frontier of these two fascinating areas.

Activities

- Workshop on Knightian Uncertainty and Backward Stochastic Differential Equations: 10 - 14 June 2013
- Tutorial on BSDE, PDE, Nonlinear Expectation and Model Uncertainty: 10 - 11 June 2013
- Tutorial on Viscosity Solutions of Path Dependent PDEs: 17 - 18 June 2013
- Institute of Mathematical Statistics Workshop on Finance - Probability and Statistics: 19 - 21 June 2013
- Tutorial on Numerical Methods for Hamilton-Jacobi-Bellman Equations in Finance: 24 - 25 June 2013
- Tutorial on Risk Measures: 28 June 2013
- Workshop on Knightian Uncertainty and Risk Measures: 1 - 5 July 2013

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Next Program

Complex Geometry (22 July - 9 August 2013)

Website: http://www2.ims.nus.edu.sg/Programs/013complex/index.php

Organizing Committee:
Lawrence Ein, *University of Illinois at Chicago*
Ngaiming Mok, *The University of Hong Kong*
Wing-Keung To, *National University of Singapore*
De-Qi Zhang, *National University of Singapore*

Complex manifolds/varieties arise naturally in and are deeply connected to other branches of mathematics, including hyperbolic geometry, symplectic geometry, number theory and mathematical physics such as string theory. In the study of complex manifolds, there are two important approaches. The first one is the analytic approach, which involves the use of complex analysis, PDE and differential geometric methods. The second one is the algebraic approach, which is the subject of complex algebraic geometry. Often these two approaches complement each other, and there are important problems in the area which may be solved by different approaches with varying degrees of algebraic and analytic flavor. The aim of this program is to bring together groups of complex analysts, complex differential geometers and complex algebraic geometers to: explain to other groups their methods and techniques; survey recent developments and disseminate their own research findings relating broadly to the area of complex geometry; chart new directions of research; and explore possible collaborations, especially among the different groups.

Activities

- Informal discussions: 22 - 24 July 2013
- Workshop on Complex Geometry: 25 July - 5 August 2013
- Informal discussions: 6 - 9 August 2013

Programs & Activities in the Pipeline

Asian Initiative for Infinity (AII) Graduate Summer School (15 - 26 July 2013)

... Jointly funded by the John Templeton Foundation

Website: http://www2.ims.nus.edu.sg/Programs/013aiiss/index.php

The AII Graduate Summer School in Logic bridges the gap between a general graduate education in mathematical logic and the specific preparation necessary to do research on problems of current interest in the subject. The main activity of the Summer School will consist of two tutorials offered by leaders in the field; namely Theodore A Slaman and Hugh Woodin from the University of California at Berkeley, USA.

Activities

Week 1: Infinitary combinatorics and mathematical logic by Theodore A Slaman, University of California at Berkeley

Week 2: Extender models by Hugh Woodin, University of California at Berkeley

Mathematical Horizons for Quantum Physics 2 (12 August - 11 October 2013)

Workshop on Modeling Rare Events in Complex Physical Systems (5 – 8 November 2013)

Inverse Moment Problems: the Crossroads of Analysis, Algebra, Discrete Geometry and Combinatorics (18 November 2013 - 25 January 2014)

Workshop on IDAQP and their Applications (3 - 7 Mar 2014)

School and Workshop on Classification and Regression Trees (10 - 26 March 2014)

Self-normalized Asymptotic Theory in Probability, Statistics and Econometrics (19 – 24 May 2014)

Algorithmic Randomness (2 – 30 June 2014)

Caroline Mary Series: Pearl of Hyperbolic Manifolds >>>

Caroline Series

Interview of Caroline Mary Series by Y.K. Leong

In each and every dust mote are infinite bodies
With cloudlike transformations pervading everywhere …
-- Avatamsaka Sutra (The Flower Adornment Sutra)
Volume 7, Book 4 (B.T.T.S.)

Caroline Mary Series has made important contributions to the theory of Kleinian groups, hyperbolic 3-manifolds and dynamical systems.

Educated in Oxford University, Series won a Kennedy Scholarship that took her to Harvard University where she wrote her PhD thesis on "Ergodic actions of product groups" under the supervision of George W. Mackey. Immediately after that, she did short stints at the University of California, Berkeley and Newnham College, Cambridge, and then moved to University of Warwick in 1978, became full professor in 1992 and went on to establish a distinguished career until the present time. In July 2011, a workshop on *Aspects of Hyperbolicity in Geometry, Topology and Dynamics* was held in University of Warwick to celebrate her 60th birthday.

Series was awarded the Junior Whitehead Prize of the London Mathematical Society in 1987. She has been invited to deliver lectures at numerous conferences, workshops and important meetings in the UK, Europe, US, Japan, New Zealand and Singapore; among them the International Congress of Mathematicians, a Royal Society Meeting on Chaos and the British Mathematical Colloquium. She has also actively popularized mathematics and promoted mathematical awareness in the UK by giving many public lectures to school students, undergraduates and the general public. She was featured several times on Radio 4 and in the BBC documentary *The True Geometry of Nature*. She was President of the Mathematical Sciences Section of the British Science Association in 2011.

In addition to making deep research contributions, she has organized many conferences and rendered professional and advisory service to the London Mathematical Society, the European Mathematical Society, the Isaac Newton Institute, the British Science Association, RAE 2008, Oxford Mathematics Institute, EPSRC Mathematics College and the Association of Commonwealth Universities. She is an editor of the journals *Mathematical Research Letters* and *Conformal Geometry and Dynamics*.

She is a co-author of the famous book *Indra's Pearls: The Vision of Felix Klein* published in 2002 after a 10-year protracted collaboration with David Mumford (Fields Medal 1974) and David Wright (and the computer). It is one of the few books which lead one through the labyrinths of hard mathematics into the inner visual world of fractals associated to 3-dimensional hyperbolic geometry.

Series is well-known for her relentless and tireless efforts in promoting the professional interests of women mathematicians around the world. In particular, she is a founder member of the organisation European Women in Mathematics in 1986 and is currently Chair of the European Mathematics Society's Women in Mathematics Committee.

Well before the Intergovernmental Panel on Climate Change sounded the alarm on global warming in its by-now famous report in 2007, Series started an initiative for the formation of an environment committee in the University of Warwick in 2000, followed by a climate change seminar and related activities. She continues to be on the University Environment and Amenities Committee.

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She was co-chair of the IMS (Institute for Mathematical Sciences) program on *Geometry, topology and dynamics on character varieties* (18 June-15 August 2010), which was sponsored by Global COE (Center of Excellence) of the Tokyo Institute of Technology, Compview and the National Science Foundation (USA). During her visit to the Institute, Y.K. Leong interviewed her on 12 August 2010 on behalf of *Imprints*. The following is an edited transcript of the interview in which she talks about her passion for mathematics, her fascination with the beautiful and tantalizing world of fractals and her personal commitment to women mathematicians and to the environment.

Imprints: At which point in your education did you decide to choose mathematics as a career?

Caroline Series: When I was about 14 years old and we had recently studied geometry in high school, we were given a problem for homework. I spent the whole evening trying to solve it. I succeeded and later I discovered that nobody else in the class had done it. From that time I resolved that I would always try to solve every problem which we were given in mathematics. I had a real passion for mathematics. My ambition then was to go to university and study mathematics as an undergraduate. Beyond that life ended and I never thought about it anymore until I was about to graduate from Oxford University. Then I wondered what to do next. I thought that maybe I would be a schoolteacher. But almost accidentally and not knowing what else to do, I was influenced by a friend to try for a master's in the US and perhaps go on to a PhD. Until I had finished my PhD I was uncertain about having a career as an academic mathematician.

I: You obtained your BA in mathematics at Oxford University but you did your doctorate at Harvard University. Wouldn't it have been expected for you to continue your postgraduate studies at Oxford? Was there any reason for not doing so?

S: Actually I was born and grew up in Oxford. My father [George William Series, FRS (1920-1994)] was a physicist in Oxford University and I had always intended that as an undergraduate I would move to Cambridge. However for various reasons I ended up studying in Oxford. After that I thought it was time I should change and I had a close friend who was recommended to go to Harvard to study for a PhD. So I decided to try to go to the US to study. It was more or less accidental. For my career it was an extremely important choice and I have recommended it to several students since. I just recommended a good undergraduate student to go to the University of Illinois in Chicago. I think it is very good to have international experience.

I: Were you on a scholarship at Harvard?

S: Yes. I won a very nice scholarship for the first two years, called the Kennedy Scholarship. It was a fund set up in memory of President Kennedy to allow some British students to go to either Harvard or MIT. After a couple of years, I was given a teaching assistant position. By the end of my PhD, I had no money left.

I: How did you choose your supervisor at Harvard?

S: When I arrived in Harvard, I hadn't really done enough research on the type of mathematics done there. I had imagined that because it was in the US, the graduate school would be enormous, but the mathematics department was actually quite small. Most of the professors were doing either algebraic geometry or number theory and I was more interested in analysis at that time. So there were really not many professors to choose from. One was about to retire and another I felt too intimidating. I chose George Mackey because I thought he was more geometrical, more analytical than others.

I: I notice that the topic of your PhD thesis is something about ergodic theory. Isn't it a bit probabilistic?

S: Yes, it is a bit probabilistic. George Mackey was, of course, a great expert on representation theory of Lie groups. But at that time, he was very interested in groups acting on measure spaces because this gives good examples of group representations. He had rather interesting and very original ideas. His other students at that time, including Bob Zimmer, worked on his ideas about what he called virtual groups, which turned out to have relations with many other topics. Also, early on I was very uncertain if I could complete my PhD. I thought that I could perhaps instead get a master's degree in statistics and become a statistician. I didn't quite do that, but I did take some statistics course. Studying ergodic theory fit in and would have made it easier to switch.

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I: You taught at University of California for only one year immediately after your PhD before you returned to England to pursue and develop your academic career. Was this due to some kind of cultural pull?

S: I always had the intention of returning to the UK in the long term. After I finished my PhD I was in a fortunate position. I was offered a temporary lectureship in Berkeley which academically was my first choice. But I was also offered a position in Newnham College, a women's college in Cambridge (UK). It was a research fellowship but it was pretty certain it would become a permanent college teaching job. I thought I couldn't give up the opportunity of a likely permanent job in a Cambridge college. So it was agreed that I would go to Berkeley for one year and then go back to England. In fact, I only stayed in Cambridge for one year. At that time there were very few people there who had related mathematical interests. I was very lucky I got a job in Warwick where there was a big group in ergodic theory.

I: If I'm not mistaken, Cambridge was rather male dominated at that time.

S: At that time, it was exceedingly [male oriented], socially and academically. But I did make a few good acquaintances, particularly S. J. Patterson; we found our work had an unexpected scientific overlap and our connection has continued over the years.

I: According to David Mumford, the book *Indra's Pearls: the Vision of Felix Klein* which you wrote with him and David Wright took 20 years in the making and writing. Mumford and Wright were in the US while you were in England. How did this collaboration come about?

S: Of course, I knew Mumford when I was a student in Harvard, but this has nothing to do with our book. What happened was that some years after I left, Mumford became interested in making computer generated pictures of limit sets of Kleinian groups, which are rather like Julia sets. David Wright was at that time a graduate student in Harvard, and very interested in computing. Together they made a large number of astonishingly beautiful pictures. They wanted to create an elegant coffee table type book of the pictures but it never took off because every time they met they would spend the time creating more pictures. I saw some of their pictures and they fascinated me. I began trying to prove some facts about them. Mumford came across a popular article I had written about the pictures and he asked me to join them writing the book. It took a long time, a full 10 years, but we finally succeeded. It was difficult partly because we didn't very often have the opportunity to meet. One of us would write a long piece and send it to the others. The others, of course, wouldn't answer because they were busy with other things. Finally we would meet and decide we didn't like what had been done and we would start all over again. This went on for years. It was really our publisher, David Tranah at Cambridge University Press, who had faith in the project and wanted it to succeed. He began to give us more and more pressing deadlines and finally the book took shape. Without him it would never have been written.

I: The patience paid off.

S: Yes, I'm very happy with it. David Tranah gave us constant encouragement and support.

I: I must say that the pictures are really very nice. How did you do the colours?

S: David Wright did the colours. It's not hard to add colours; for example, you are plotting things at different levels, level n, level n+1, and so on, and you can cycle the colours. David Wright enjoyed playing with it. Since then some people have taken it up as a much more artistic enterprise, particularly Jos Leys who is a wonderful mathematical graphics artist. He modified our programs to create very beautiful pictures; one has to pay attention to colour, arrangement and background. It's a great skill.

I: The book *Indra's Pearls* is largely computational in approach. Does this represent an alternative, if not new, approach to research in geometry? Has this approach contributed significantly to research in geometry?

S: It's not just our book. Computational pictures have made very important contributions to geometry in recent years. The whole theory of complex dynamics has been inspired by the computational aspect. The pictures Mumford originally liked actually had a lot of influence because when you see them, you understand that something that conjecturally occurs really exists. You begin to see intricate structures and this inspires people to go and try to prove things. It's a two-way thing. The computer generated pictures have inspired a lot

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of research and then the research suggests we make further pictures. They go hand-in-hand.

I: Did any of those pictures suggest new theorems?

S: Yes. They suggested theorems of mine. I believe they also contributed to a very important result proved by Yair N. Minsky called the Ending Lamination Conjecture. Perhaps you have a theorem in mind but seeing the pictures makes you feel more certain there must be something in it and you become more determined to prove it.

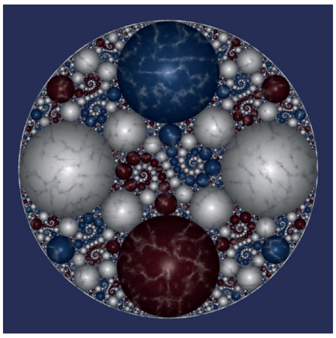
I: Did it suggest any counterexamples?

S: I think it showed unexpected phenomena. You see something that you don't understand fully and you realize you have to pay attention. The pictures in our book are 2-dimensional. Eventually I learn that in order to prove results about them one needs to use 3-dimensional hyperbolic geometry. It took me a long time to understand this. The 3-dimensional geometry is very important.

I: It must be hard to imagine 3-dimensional space.

S: It's hard to draw pictures in 3-dimensional space. Now I'm looking at parameter spaces in higher dimensions. One difficulty is how to present computations; all you can do is to show two dimensional slices and really you want to see the full picture.

I: In the *Avatamsaka Sutra*, which you quoted in several places in your book, there is mention of an infinite web of pearls which mutually reflect each other and all the reflections in each pearl. This is an amazing analogy with a religious world view of the mutual interconnectedness and dependence of all things. Do you see any parallels in such viewpoints and your own mathematical viewpoints? For example, do you have a Platonist viewpoint that fractals exist "out there" and not just as mental constructs.



Kleinian Groups Orbit Trap 01
(Courtesy Ross Hilbert)

S: The last part is a very fascinating question. Mathematics exists in our minds and yet it gives us such a powerful way of dealing with the external world. One reason I like working with these computer generated pictures is that you start with an abstract mathematical construct but as you plot it by computer the reality seems to grow. All the things that you have proved you see vividly in front of your eyes. And somehow one can't argue with the pictures; they are so concrete. As for fractals, I find it astonishing that so many natural objects have outlines or structures that are fractal-like. It seems to me that the concept of fractals captures something very important in nature. I imagine that fractal geometry and its applications will become more and more important as time goes on.

I'm not sure about the religious part. But one thing that strikes me is that different cultures have different concepts of infinity. In the western tradition, we think of infinity as counting: 1, 2, 3, 4, going on forever in a linear way. The concept of infinity in Buddhist and Hindu writings is much more to do with objects subdividing into parts and then each part subdividing again and again forever. This leads to an uncountable infinity of limit parts. It's a very different way of thinking. I think it must really colour the world view of these cultures. Western culture is very goal-oriented and likes to measure everything whereas the Eastern culture is more about inner structures and realities. Another example is the Maya in South America – their calendar worked in cycles. That is another idea of infinity. I suspect these different viewpoints profoundly affect people's attitudes to life. If I were more of a philosopher or an ethnologist, I would like to study this more.

I: Can I ask you if you have read the *Avatamsuka Sutra*?

S: I can't say I have read the whole thing. I bought a translation (it's translated into English now) and I did read a considerable part of it while we were writing the book. I was always looking for suitable quotations. The book is extremely long. One thing is amazing. At various points are descriptions of large numbers of things: buddhas, jewels, worlds within worlds, and so on. The numbers described are huge, gigantic numbers. The people who were writing this ancient text must have had the concept that infinity was truly unimaginably huge. (Of course, I am speaking here about what I found in the English translation, not the Sanskrit or Chinese.)

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I: Now that Thurston's Geometrization Conjecture has been recently solved by Perelman [Grigori Perelman (Fields Medal 2006)] and others, is it fair to say that everything of importance is now known about hyperbolic 3-dimensional manifolds?

S: It is true that the Geometrization Conjecture has been solved by Perelman and others. But actually there has been another extremely important development that happened more or less at the same time, done by Yair N. Minsky and his co-workers. This is the Ending Lamination Theorem I mentioned above. They managed to carry out a program initiated by Thurston [William Thurston (1946-2012), Fields Medal 1982] to classify infinite volume hyperbolic 3-manifolds using some rather simple invariants associated to their ends. From the knowledge of the end invariants, they show it is possible to reconstruct the entire geometry of the manifold. This is transforming the subject. Previously people were always trying to understand whether there could be any "wild" hyperbolic manifolds lurking about somewhere. Now we know that in a precise sense they are all "tame". We know how to handle all the infinite geometry. In that sense, it is true that the big questions were solved within the recent past. I think it is true the field has changed since this happened. But there are still many things to do, which one can now tackle using the manifold structure. I'm currently thinking about how limit sets evolve as the manifold is deformed. Minsky's theory is just what is needed. People in the hyperbolic geometry community are also now looking at different kinds of geometry and geometric structures, sometimes related to physics. Our current program at NUS brings together different kinds of geometry in a broad way.

I: Is it possible for one single person to read through all those proofs of the classification results? (I'm thinking of the situation in the classification of finite simple groups.)

S: It's not bad as that. There is a group of people who understand in principle how it [the proof] works. I can't say that I have read it all. I think it is perhaps more conceptual. There are some very important geometrical and topological principles, and once you understand the principle and the outline structure of the proof then you can go and check the details. It is a reasonable thing to hold in your mind. I doubt that there are many people who have gone through everything but it is feasible. There have been some simplifications and no doubt it will be cleaned up further – it's a matter of time.

I: Is the next step in geometric research on manifolds to go on to dimension 4?

S: There is a community of people who work on dimension 4, but it has a quite different flavour. For hyperbolic geometers it's not so much dimension 4 but investigating different kinds of geometric structures. This could be in dimension 4, but it could be much broader.

I: You have been very active in promoting and supporting opportunities for women in pursuing careers in mathematics for many years. What have been the achievements of the organization "European Women in Mathematics" [EWM] of which you are a founder member?

S: When we set this up, it was not really to be a campaigning organization. Rather, we wanted to create a network of women to exchange ideas, share problems, encourage each other, meet at conferences and so on. I believe we have had some influence. It was set up almost 30 years ago now, and there have been a considerable number of programs in different countries and different institutions to encourage and give more recognition to women. This could be inviting women to give prestigious lectures, or special meetings or grants. Some universities have made it a point to make sure that they have women on committees. Particularly in Germanic countries, it used to be very difficult for women, but now the climate has become quite favourable. We were a group of women from many different countries and this gave us a wide perspective. We gathered some statistics and it was very noticeable that there are a lot more women mathematicians in some countries than others; it is partly a cultural issue. I think we probably encouraged a number of young women who without us would have taken up some other job. Women usually have to carry the burden of family life. We had some successful older members who could say, "Well, it's difficult but I've managed to succeed and this is how I did it." This has been encouraging to other women.

I: Is this organization centred in England?

S: No, it is pan-European. There was already a big organization in the United States. Our idea was that we should have a counterpart in Europe. We have a main meeting once every two years somewhere in Europe, which attracts 70 to 80 people. We also have a number of members from outside Europe. Our structure is set up so that women

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anywhere in the world can come along and say they want to establish a group in their country. Just before the ICM [International Congress of Mathematicians, 19 – 27 August 2010, Hyderabad] there's going to be a meeting which was initiated by EWM and which is being organized in collaboration with a group of Indian women. We hope there will be a group set up in India and perhaps other countries. There is a group in Korea and Japan now. Our structure is very flexible and informal.

I: Could you tell us something about your interest and involvement in activities concerning environmental issues and climate change?

S: This is something I have become increasingly concerned about in recent years. It began when I became involved in our university's recycling. We were not recycling anything very successfully. The system was reliant on charitable and voluntary actions, and it didn't really work. Eventually I came to realize that you needed something in the infrastructure of the University to take care of environmental issues. It's no good just relying on people's goodwill. So around 2000 I managed to set up a committee in the university. I made contact with a professor from the Business School and together we approached the university administration. We set up an informal group of some academics and some administrators. For several years we were not very successful because we had no proper resources. Finally, the university decided to hire an environmental officer whose job was to be a focal point for environmental issues within the university. He was responsible for handling issues like recycling, energy consumption, water consumption, and transport problems. Now we have an official university committee with an environmental officer, transport officer, recycling officer, energy officer, and the university is making very substantial efforts to reduce its carbon footprint. We have a cost effective recycling scheme. I'm very happy with it. I was right; such things have to be integrated into the university infrastructure.

On a personal level, about 10 years ago I put hot water solar panels on my house. Recently, our government introduced a subsidy if you generate electricity using solar electric (PV) panels. The panels are quite expensive to put it up, but they generate quite a lot of electricity even in the UK. You get quite a lot of money for the electricity you generate, so the panels will pay for themselves in about 10 years. I put up PV panels and it's like a long-term investment. In the UK environment groups were trying to convince the government to do this for many, many years, because previous policies were not working very successfully. The last Labour government made a lot of glamorous commitments but didn't actually do very much practical. But in the last couple of years, things have changed because the European Union has become quite serious about reducing its carbon footprint. It's now becoming almost a legal requirement and serious money is involved. There is recycling in the UK now instead of disposing rubbish by burying it underground. The practical cost of environmental actions can be high and until serious economic incentives are brought in so that it makes economic sense, they don't really happen in a mainstream way.

I: I remember hearing over BBC one year ago that some company in England was trying to recycle plastics. I thought that was a good idea. Do you know anything about it?

S: Oh, yes, there's a lot of recycling of plastics. In fact, our university collects not just bottles, but all kinds of plastics, mixed plastics. For example, people wear a lot of fleece jackets in colder countries, and a lot of them are made of recycled plastics. Recycled plastic is used to make insulation materials. People are working on developing the technology to make the recycling process less difficult (there are many different kinds of plastics) so that you don't have to separate different kinds of plastics. Plastics are everywhere. They do not biodegrade and last thousands of years. We need to find better ways to deal with them and I'm sure we will.

I: What advice would you give to students, especially women students, who would like to pursue a research career in mathematics?

S: My advice is that first of all, you must really like doing mathematics. It may not be a glamorous career. But if you really want to do it and love mathematics, and you are persistent, then you will find a way to succeed. What I did and what worked for me is, at the beginning of your career try to go to the best places you can and meet the leading people in your field. That way you get a good grounding. Later on, when you have to find a permanent job and you perhaps have family constraints, then you have got a good basis and good contacts, so wherever you settle, you are able to continue research. For me it has been a wonderful career and I would recommend it.

Tai-Ping Liu: Boltzmann Equation, Partial Differential Equations and the Computer >>>

Tai-ping LIU

Interview of Tai-Ping Liu by Y.K. Leong

Tai-Ping Liu made important contributions to nonlinear partial differential equations, shock wave theory and the kinetic theory of gases.

Liu had his undergraduate education in the National Taiwan University at a time when its department of mathematics was in its formative stages. From there he went to Oregon State University for his MS degree and then to University of Michigan for his PhD Immediately after that, he joined the University of Maryland, where from 1973-1988, he established for himself a niche in research on hyperbolic conservation laws and shock wave theory. He then spent 2 years at the Courant Institute for Mathematical Sciences of New York University before moving to Stanford University in 1990. From a distinguished career in applied mathematics, he returned to Taiwan in 2000 as a Distinguished Research Fellow at the Institute of Mathematics, Academia Sinica. Initially maintaining links with Stanford University, he soon took up a full-time position at Academia Sinica and retired from Stanford as emeritus professor.

Since his return to Taiwan in 2000, Liu has focused his research interests on the study of microscopic phenomena; in particular, on the Boltzmann equation in the kinetic theory of gases. He was instrumental in forming a research group at the Institute of Mathematics to work on the quantitative aspects of the Boltzmann equation in a direction (via Green's function) different from the approach of the well-established French School. He began organizing learning seminars on the Boltzmann equation for researchers, graduate students and postdocs. He and his co-workers started research communications with a group of physicists in Kyoto University led by Yoshio Sone and began the quantitative study of the Boltzmann equation.

Liu's research output consists of more than 130 single-author and joint papers. Among his important contributions in shock wave theory for hyperbolic conservation laws are the introduction of the Liu entropy condition for the admissibility of weak solutions, the deterministic version of the Glimm scheme for the construction of solutions, and with Tong Yang, the Liu-Yang functional for the well-posedness theory. In recent years, Liu and Shih-Hsien Yu (of the Department of Mathematics, NUS) initiated the Green's function approach in finding quantitative pointwise estimates for the Boltzmann equation. This work has contributed to the mathematical understanding of physical aspects of the Boltzmann equation.

Liu was elected an Academician of Academia Sinica. He is Honorary Professor of various universities and an elected member of the Academy of the Developing World, TWAS (The World Academy of Science). In 2009 he was awarded the *Cataldo e Angiola Agostinelli* International Prize by the august Accademia Nazionale dei Lincei of Italy.

He has been invited to give talks at numerous conferences, scientific meetings and universities around the world; in particular, at the Institute of Advanced Study, NTU and Institute for Mathematical Sciences (IMS), NUS. He was actively involved in the IMS program on *Hyperbolic Conservation Laws and Kinetic Equations: Theory, Computation and Applications* (1 November-19 December 2010). During this visit, he was interviewed by Y.K. Leong on behalf of *Imprints* on 2 December 2010. The following is an edited and enhanced version of the transcript of the interview in which he traces the unlikely path from an obscure village in Taoyuan, Taiwan to centres of active research at University of Maryland and Stanford University in the US – a path that completes a full circle at Academia Sinica in his native country. On the way, he takes us on a brief tour of nonlinear partial differential equations with a glimpse of the impact of computers on his field. In a parting thought, he reflects briefly on the cultural differences between Chinese and western attitudes in learning.

Imprints: When was your interest in mathematics first formed? Did the school environment in Taiwan play an important role in shaping your interest in mathematics?

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L: My interest may be a little bit unusual. It was not really the schools that shaped my interest in mathematics. My interest in mathematics was really initiated by my mother. My mother could not read or write but she is talented in mathematics. One time my elder brother was six (I was 4 at that time), went to school and then the neighbor from the village came back from the parents' meeting and said that the teacher told them that my brother could not count. My mother found that incredible. For her mathematics was so interesting. How could my brother not be able to count? Of course, my brother never learnt [to count]; my mother was so busy with the farming. So she took time off from her farming chores and began to teach my brother and me how to count and how to read the clock. We learnt counting that afternoon but I don't think we got the clock precisely. We counted to 100 and then backward to one. A few days later, my mother told her neighbor in the village, "My Tai-Ping can be an accountant." For her the only thing she knew about mathematics was about accountants. She is a farm woman but she is very talented. We went to market and she would begin to teach the farm women how to count the price of chicken – you develop a certain scheme by interpolation and so on, all this by herself. So she had a very genuine interest in mathematics. That really had a very deep impression on me. I was 4 at that time and my mother had that curiosity and interest in mathematics. That was how I got started.

I: Where in Taiwan was it?

L: It was in quite a poor village in Taoyuan. Taoyuan is where the [international] airport is in. That part of Taiwan was very poor because there was no irrigation. Anyway, all the farm women were very surprised that my mother was helping them to deal with the city people and with selling their chicken and vegetables.

I: Was your childhood spent mainly in the farm?

L: All the time until I went to college.

I: What about in school? Any particular teacher …

L: No one in particular. You probably know, like in Singapore, we had exams all the time. Everyone wanted to pass entrance examinations, from junior high to senior high, from senior high to university. I had difficult times in passing examinations. I did pass but that was not the fun part of my education.

I: You went from a BS degree in National Taiwan University to a PhD degree in University of Michigan. Tell us how you took this path.

L: In 1968 I went into military service, and in 1969 I went abroad. There was no graduate school for PhD in Taiwan, not even in Taipei, at that time. In fact, there was no department of mathematics until nineteen forty something. Since Japanese time [1885-1945] there were classes on mathematical teaching and elementary mathematics but there was no department of mathematics. The faculty in [National] Taiwan University, which had the best department in mathematics, had only 4 PhDs. That was pretty good; in the physics department they had even less. We had no choice if we wanted to pursue mathematical graduate study. My undergraduate record was poor; I couldn't get to a good school, so I went to Oregon State University. After one year I transferred to [University of] Michigan.

I: Were you on a scholarship?

L: It was a teaching assistantship. At that time in the late 1960s the economy in US was good, the student population was growing and they needed teaching assistants. Even though my English was very poor I had to teach a class and grade the final exam in English. I learned English in one semester or so and then I was okay.

I: Was your PhD research work crucial in shaping your subsequent research interests?

L: Yes, it is. When I was an undergraduate there was a professor, Wang Ju-Kwei. He would come to our dormitory to chat, and one day we were talking. I said I was reading Paul Cohen's booklet on the continuum hypotheses and I couldn't understand it. (I liked set theory. When I went to Oregon State I even had a conjecture, and someone from Cambridge [University] remarked that the conjecture was still open.) In any case, when I was an undergraduate, Professor Wang told me that "set theory will make you famous but nonlinear PDEs (partial differential equations) are difficult and important." These were two good adjectives, and so I started [to study] PDEs. In Michigan University Joel

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[Alan] Smoller was the one studying PDEs. But Courant Institute in New York University would be the stronghold of PDEs, not Michigan, at that time. But I wanted to study PDEs, and Joel Smoller was a good advisor. He gave me one problem on uniqueness and well-posedness. I could not solve it at that time. I came back to it some 25 years later and eventually solved it. The one that I solved in my thesis was about entropy conditions.

I: Where did you go to after your PhD?

L: I went to University of Maryland. I was very lucky. Joel Smoller was very helpful. There was Avron Douglis (he co-wrote the important Agmon-Douglis-Nirenberg paper). He was in Maryland and he read my application. With Joel Smoller's help I got the one and only one job in April when the hiring season was already over. I got a tenure track position. In1973 it was very difficult to find jobs, even for engineers. So to get a tenure track at Maryland was the envy of everyone. They have an institute called IPST [Institute for Physical Science and Technology], which is a good place for computation, finite element method and so on.

I: In your work on kinetic theory and shock wave theory, what is the guiding light – the physics or the mathematics?

L: At first I tried to solve any problem I could find and could solve. Those were very analytical. Eventually I try to find a problem and formulate something which is physically relevant. Now looking back, I think I'm more interested in physical phenomena.

I: Wouldn't that have something to do with physics?

L: I'm not very knowledgeable in physics but I want to understand the physical meaning behind the mathematical model, and so I did a couple of things in modeling, something called "nozzle flow" and so on. I'm interested in physical phenomena but I'm a pure mathematician. I prove theorems and I'm interested in the basic mathematical patterns and more of the solution properties.

I: After Maryland you went to Stanford, isn't it?

L: I was in Maryland for 15 years. And then I spent 2 years (1988-1990) in New York University, Courant Institute [of Mathematical Sciences]. Then in 1990 I moved to Stanford [University].

I: It is understandable that applied mathematicians are only interested in those nonlinear partial differential equations that are of relevance in nature and applications. Is there any work on nonlinear PDEs done from a more purely mathematical and general aspect without being physically motivated?

L: This is a very good question. It's very difficult to say what is applied mathematics; for example, the things that applied scientists find appealing are not the kind of mathematics that is basically using the existing tools to solve, justify their models, to prove existence and things like that. It's not like that. It is the mathematicians, motivated by the physical concern, who come up with some basic mathematical techniques or basic mathematical framework and mathematical idea. And that basic mathematical idea may help to solve this physical problem or may help to understand the general case or what not. So in a way the pure mathematics part comes after one takes into consideration what are the physical phenomena one is thinking about. This line between pure and applied [mathematics] is very blurred. One would suppose that if the mathematician proves existence and uniqueness theorems, the engineer would be somewhat interested in them because this will give him confidence in the model and because it works well. But it would be even more interesting if a mathematician motivated by the question comes up with a new formulation, a new technique and something which is fundamentally new in mathematics, and they will find it very nice because then it can tell them what are the things they could look into, what are the possible experiments.

I: Are there any cases where the pure mathematician looks at certain nonlinear PDEs which are not considered by the physicists?

L: Yes, yes. This happens a lot. A classical example in shock wave theory would be the following. The engineer would look at the gas dynamics, the set of Euler equations for the shock wave and do all the right computations and one can see them in the classical book of Courant and Friedrichs [Richard Courant (1988-1972), Kurt Otto Friedrichs (1901-1982)] on shock wave theory. However, mathematicians

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always find the questions very difficult. In spite of all the efforts by applied mathematicians of the first rank like Prandtl, G.I. Taylor, von Neumann, [Ludwig Prandtl (1875-1953), Geoffrey Ingram Taylor (1886-1975), John von Neumann (1903-1957)], mathematicians find it difficult to go on; instead they go back to simpler models, something like Burgers' equation [Johannes Martinus Burgers (1895–1981)]. People like Hopf [Eberhard Hopf (1902-1983)], Oleinik [Olga Arsenievna Oleinik (1925-2001)] and Peter Lax would try to do a more general theory for a certain class of PDEs which have a certain general pattern, entropy conditions, Riemann problem and things like that. After this general study, the next generation would go back again to the Euler equations for the compressible flow. So this took a very large effort by many very good mathematicians. After the general theory is done to some level, one would go back to the Euler equations and try to see if one can solve the problems that could not be solved during the classical period. This is the challenge for the new generation. There are some preliminary successes.

I: Are the Euler equations hyperbolic? I believe there is a lot of work done on elliptic equations, isn't it?

L: Yes, the elliptic PDE community is a much bigger community and has a very long and illustrious history.

I: Is it true that the elliptic case is easier to treat than the hyperbolic?

L: Yes, but if it is easier, then it goes deeper and eventually it is hard. Ordinary differential equations are easier than partial differential equations but it doesn't mean that ordinary differential equations are easy. It could be a deeper theory. You have KAM [Kolmogorov–Arnold–Moser] theory and so on. So you have a simpler situation but you go deeper.

I: Why are there more people interested in elliptic rather than hyperbolic PDEs?

L: Well, I think elliptic has more tools, and in mathematics there is also the habit that if you go into an area and you can do something then you tend to stay in that area. This is a kind of inertia or habit. People would say that hyperbolic [PDE] is a very exciting area, even more exciting are equations of mixed type, but in academia you want to be able to do something. So a lot of people stay in elliptic PDE. Of course, it is also very important.

I: For applications, which one comes up more often – the elliptic or the hyperbolic?

L: That is difficult to say. Both come up. Hyperbolic PDE, if you look at specific problems, can be reduced to elliptic PDE. If you look at waves of certain frequency or simplification of flow, then it is elliptic. In a way, elliptic PDE is more basic.

I: I'm always intrigued by these two terms hyperbolic and elliptic; they are geometric terms. Is there a geometry of PDEs?

L: It is a geometric term – hyperbola, right? I don't really know who first invented this term. The simplest hyperbolic PDE is $u_{tt} - c^2u_{xx} = 0$. If one simply pretends not to learn calculus very well, this is like $(1/t^2) - c^2(1/x^2) = 0$, or $x = \pm ct$. So this is a hyperbola. The elliptic PDE would be $u_{xx} + u_{yy} = 1$, say. Then it's like $x^2 + y^2 = 1$; this is like an ellipse.

I: Does this mean that the name has no real geometric significance?

L: Yes and no. One time I was in Paris and I began to just test around, and I said, "Hyperbolic is *yang* and elliptic *yin* (the Chinese term *yin-yang*)" but in English "elliptic" seems to mean "mellowed, smoother;" while "hyperbolic" means "wild". Mathematically this is true. Elliptic PDEs are always smooth and nonlinear hyperbolic PDEs result in shock waves. So maybe without knowing it, this "hyperbolic-elliptic" terminology makes sense.

I: How much has the computer helped in the "complete" solution of nonlinear PDEs in general, and the Boltzmann Equation in particular?

L: The computer is very, very important. In fact, the computer is one of the tools, which makes the general applied mathematics program complete. In old times, the Chinese had the "*suan-pan*" (abacus). This was the computer at that time; great mathematicians even wrote treatises on this. With it you can do a lot more computations and so

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on. This can lead to other thinking in the hands of great mathematicians such as the Japanese Takakazu Seki [(1642-1708)] in advancing Japanese arithmetic (*ho-suan*) [*wasan*]. There is the algorithm for matrices and for manipulating systems of linear equations like the Gaussian elimination. The computer is definitely very important but it should be combined with traditional analysis or thinking.

Last time I was working on the Euler equations in multidimensional shock reflection and a former student of mine Volker Elling did some computation and the result was very surprising. The computer generated an interesting problem and we finally did something. The computer is an integral part [of research].

I: Is it the simulation part?

L: The computer is used for simulation. But in this particular instance, the computer was asked to analyze a certain problem. Sometimes the computer is used to find out what the solution looks like and whether the model gives the right phenomena and things like that. That is the use of computers in general but there are also situations where the computations lead to a different mathematical thinking. The most famous example is soliton theory.

I: Was it simulated by computer?

L: Yes, yes. They [Enrico Fermi (1901-1954), John R. Pasta (1918-1984), Stanislaw Ulam (1909-1984) and Mary Tsingou] had this computation. The paper on the computer simulation was published in 1955 after Fermi's death. The physicist Fermi was so surprised by the result of the computation. But even in doing the computations you need to have a deep analytical thinking and Fermi had a deep analytical thinking. So when he saw this solitary phenomenon, he was surprised. Otherwise some people doing this computation would not be surprised by the result and it would not stimulate new thinking. So analytical thinking has to precede computation and new analytical thinking would come after the computation. You have to have that interaction.

I: In that case the computer has actually led to new theorems?

L: That's right, new theorems. In that case, solitary waves are really a revolution in mathematical sciences.

I: If it had not been for the computer, would solitary waves have been discovered?

L: That is hard to say.

I: I understand that the Boltzmann equation can be used to study galaxies but not the development of the cosmos in the early stages of the Big Bang. Can the equation be modified to incorporate quantum effects?

L: This is a hard question to answer. In one aspect there is a belief, in fact, there are models which incorporate quantum effects into the Boltzmann equation. However, it is more urgent to study the early stage of the Big Bang where quantum effects are very important. Afterward, you have a lot of stars, galaxies and there are so many of them. Your question perhaps is: what happens in the time period between these two [stages]. That is a very difficult question because there are other effects, relativistic effects and so on. Let me turn this question around. Studying the development of the cosmos is a huge program. This consists of all the physics because at the beginning it was so dense and at the end there are so many stars. You have to have fluid dynamics coming into play, kinetic theory, relativistic theory and what not. We are in a situation where there is explosion of human knowledge. Exactly how much we need to learn, what direction of research we should go into – that is a very serious question. Ideally we need to understand all the physics of the basic things. But our time and energy are finite. So we have to be selective in what we learn. You mention this cosmos – a hard – question. This is one example where perhaps we need to rethink what we need to teach our students, what our students need to learn. You are talking about quantum effects and the Boltzmann equation, and there are many things in between. It could be very exciting research in the future, but incorporating quantum effects into the Boltzmann equation by itself would not solve the equation; I don't think so. That's too much to hope for.

I: Will a complete solution of the Boltzmann equation lead to a clearer understanding, if not the solution, of the Navier-Stokes equations?

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L: I guess the answer has to be yes, except for the fact that the Boltzmann equation is so much harder because it contains much more phenomena and has many more different scales. When we say the Boltzmann equation is much harder than the Navier-Stokes, people could misunderstand me because if you want to prove existence of the solution of the Boltzmann equation, it might be easier than the Navier-Stokes equations. But the point is not to construct the solution. The point is to understand the properties of the solution. In fact, your question is very good; it says "a clearer understanding". The Navier-Stokes equations represent a certain physical situation, and in that situation, the Navier-Stokes equations are good approximations of the Boltzmann equation. So the answer is yes, but a complete resolution of the Boltzmann equation is a difficult program.

I: Is the Boltzmann equation more general than the Navier-Stokes equations?

L: Yes, the Boltzmann equation is more general.

I: Does that mean that if the Boltzmann equation is solved, then the Navier-Stokes equations will be solved?

L: The thing is what do we mean by "solved it"? To solve it, we have to have a good understanding of the solution on various scales. The Navier-Stokes equations are mainly at certain scales. In order to understand the solution of the Boltzmann equation in that sense is extremely difficult. Another thing is that the Boltzmann equation is more general than the Navier-Stokes equations around the boundary for example and there are very rich phenomena.

Shih-Hsien Yu here and I worked on the boundary and nonlinear waves for the Boltzmann equation. Around the year of 2000, Shih-Hsien and I tried to generalize the techniques developed for the conservation laws to the Boltzmann equation. We had some successes. However, we soon learned from the Kyoto School that the most interesting aspect of the kinetic theory is that it can model physical phenomena that the fluid dynamics equations such as the Navier-Stokes equations cannot. So we started from the basics, the construction of the Green's function, which allows us to gain quantitative understanding of the Boltzmann solutions. We are now able to study the bifurcation phenomena of the transonic condensation/evaporation, a problem of interest to the Kyoto School. Of course, within the whole scope of the kinetic theory, we have barely scratched the surface.

I: In that case, they should give the (US) one million-dollar prize to the Boltzmann equation rather than the Navier-Stokes equations.

L: I would agree with that. [*Laughs*] The Boltzmann equation is derived from more first principles in physics than the Navier-Stokes and is more basic.

I: The Boltzmann equation is much later than the Navier-Stokes equations.

L: Much later. The Navier-Stokes equations are phenomenological ones. The Boltzmann equation, thanks to the great genius of Boltzmann, is derived from very first principles.

I: In 2000 you went back to Taiwan (Institute of Mathematics, Academia Sinica) after 27 years of distinguished careers in the US. I believe you still have close links with Stanford University. How do you maintain the dual roles in two places that are geographically so far apart?

L: This question is very easy to answer. The answer is that now I am full-time in Taiwan. I'm professor emeritus at Stanford. I tried to have a dual role, as you put it, for a while (three or four years), but then I realized that it is not so much about giving your expertise to and teaching students. Doing research, I realized that the cultural and general attitude of people is really very important. There must be a reason why modern science was founded in the west and not in the east. For whatever reasons there was a different culture and different attitude. In order to have some effect, you need to be in one place most of the time and try to change the cultural attitude a little bit. For example, in traditional Chinese culture if you are dean or president or, in old times, if you were a *chu-jen (juren), chin-shih (jinshi) and hanlin* and so on, then it is written on the front of your house and this is the most important thing. Never mind what exactly you have done; it's this title that is important. So it is not so much the curiosity driven kind of thinking. Like my mother, she never thought that she would be called the number one mathematician. Yet she has a natural interest and curiosity in mathematics and wanted to share her knowledge with other villagers.

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I: If she had the education and opportunities, she might have become a mathematician.

L: She might have. She was talented. She could have been good in school and become a medical doctor or whatever. In spite of her lack of formal education, she had interested herself in mathematics. But the prevailing culture is not favorable to science from the point of view of traditional Chinese culture. This cultural thing and general attitude towards research demands a full time interaction and living together, and so I say I will retire from Stanford.

I: Do you think there is a difference in attitude towards learning between the east and west?

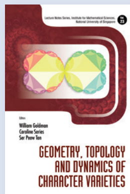
L: It is too big a question for me to answer. Even for myself, I feel that I am very much a product of the traditional Confucian system. We have taken some classics. When we look at some expert mathematicians, in a way we take them as the saints and therefore we want to solve some problems which they propose and their famous open problems. But that is much less so in the west, I think. In the west, young people, even though they may not know a lot, think they can be the equal of anyone. I think that's a different attitude.

I: If a student is interested and shows equally good potential in both pure and applied mathematics, what acid test would you give him or her to help in deciding whether to be an applied mathematician?

L: This is a very difficult question. For example, some of my friends are very good mathematicians and they were actually graduates or undergraduates from engineering school but they decided to go to graduate school in mathematics. It turns out that they are the purest kind of mathematicians; they are born to like mathematics in spite of the fact that as undergraduates they were engineers. Then, people like me, I was an undergraduate and graduate in mathematics but deep in my heart I like to understand physical phenomena. Also, these days there are a lot of possibilities – mathematical physics, mathematical chemistry, mathematical biology and so on. In general, I would say that the thesis advisor is very important because that is when you start your research. In general, I tell the student, "Never mind what you studied for your undergraduate or master's degree. When you go to a good school you take the courses, and the one who everybody says is a good mathematician and the one whose courses you can understand, that person is your advisor." Otherwise I don't have a more precise answer to your question.

I: Do you have many graduate students doing PhD under your supervision?

L: Not many. I have somewhere between 10 and 15 so far who had a PhD from me. I have some very good students and I think the credit should go to their parents and others. Some eventually very quickly became my teacher.



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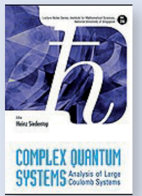

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