Speech by Professor Roger Howe, Chairman of Scientific Advisory Board

It is a delight to be here today to help celebrate the 10th anniversary of the Institute for Mathematical Sciences. It has been a pleasure for me to be part the Scientific Advisory Board (SAB), and wonderful for all of us to work with Louis Chen, whose energy and devotion to the goals and activities of IMS have been inspirational. And also, very successful. Under Louis's direction, the IMS has supported many excellent programs that have enhanced local expertise in a wide variety of areas. Louis has been especially careful to recruit programs related to Singapore's strategic technological goals. It is an enviable record. There have been series of programs supporting the country's efforts in biomedical science, imaging science, and hydrodynamics, and a variety of other programs in pure and applied mathematics that reflect and strengthen the research of local mathematical scientists. You don't have to take my word for it; just go read the already 19 volumes of the IMS Lecture Note Series, published by World Scientific. Here I will use just three for examples: Gabor and Wavelet Frames, Markov Chain Monte Carlo, and Braids.

I would like to relate these volumes to a model of progress in mathematics. It seems useful to distinguish at least three types of mathematical activity. The most dramatic is the big breakthrough that revolutionizes a field and leads to striking applications. Let's call this a type 1 event. We all love this kind of event, but there are two problems with it:

1) it is relatively rare; and
2) it is almost completely unpredictable.

Sometimes of course, a breakthrough doesn't affect theory and applications equally. Sometimes a theoretical breakthrough has little or no practical ramifications, at least not in the near term, and sometimes a breakthrough just uses off-the-shelf mathematics in a new way. Even these one-sided breakthroughs tend to generate a lot of excitement.

The second kind of activity is the feverish period of dissemination, elaboration and consolidation after the first type of event. Lots of conferences, lots of papers with corollaries, or parallel results in a different context, or applications, or elaborations, or refinements.

The third kind of activity is what happens the rest of the time, which in most fields is most of the time. People are working out programs of investigation. They may be working on still unsolved problems, of older or more recent vintage, or trying to refine further or clarify how ideas in their field fit together, or elaborate concepts to adapt them to more complex situations, or investigating examples, or formalizing results, a kind of axiomatization to distill the key ideas in a particular area. During this kind of activity, it may seem to the outsider that not much is happening.

Policymakers and funders of course love the first kind of event. Everybody does. It is obvious that we can now do things better than in the past, or at least understand something much better; and if the breakthrough has an applied aspect, someone (probably not mathematicians!) will save or make a lot of money. Policymakers tend also to feel pretty favorable about the second kind of activity, because the excitement is still there, and they can still remember the difference between before and after the event. These events are natural candidates for IMS programs.

The true test of policymakers is in supporting the third kind of activity. In this activity, which I believe characterizes most of mathematics most of the time, it may seem that mathematicians are wasting their time, becoming overly refined, not keeping their eyes on the main prize. But I submit that this is in fact the most important kind of mathematical activity. It is out of the ferment of exploration, or turning over ideas, subjecting them to critiques and "what if"s, mixing them together and seeing what happens, that the big breakthroughs come. In particular, this type of work is also suitable for IMS programs. I would like to use some IMS Lecture Notes to illustrate this claim, by briefly sketching how complex the history was that led to a few type 1 events.

Volume 10: Gabor and Wavelet Frames

In imaging science, a major revolution came in the 1980s, with the advent of wavelet methods, a superb example of a type 1 event. Wavelets have become an essential part of the toolkit of signal processors, including being incorporated in the JPEG standards. Wavelets have figured strongly in the image processing activities at IMS, and have been a central topic of research of several members of the NUS Department of Mathematics.

What led to the wavelet revolution? The complete history is very complex, but here I simply want to emphasize its long pedigree. Fourier analysis has been a heavily used tool in mathematics and physics since the late 18th century, and the first serious studies of the major linear partial differential equations of physics: the wave equation, the heat equation, and Laplace's equation. Desire to understand what the Fourier Transform does led to intense study. Speaking very loosely, it was learned that Fourier Transform takes spatial information and converts it into frequency information. The standard, spatial representation of a function does not exhibit very plainly its frequency behavior, and vice versa.

The advent of quantum mechanics forced the realization that we cannot arbitrarily specify both the spatial behavior and the frequency behavior of a function. This is the celebrated Heisenberg Uncertainty Principle. Also with the advent of quantum mechanics, we got what mathematicians call the Heisenberg group, since it is the group-theoretic embodiment of the Heisenberg Canonical Commutation Relations. The Heisenberg group embodies both the spatial and frequency aspects of a function at the same time, at the price of being non-commutative.

When these ideas had been sufficiently digested, it occurred to some people to ask if there could be ways to represent functions that are partially localized in both position and in frequency, and also have good properties with respect to scaling. One of the early attempts to create bases of such functions was made by D. Gabor, but it was only in the 1980s, after several decades of development of ideas from the Calderon- Zygmund school of harmonic analysis that Gabor's ideas were combined systematically with considerations based on scaling, giving rise to the bases known as wavelets, and associated multi-resolution analysis. This brief sketch will suggest how much history and patient investigation lay behind the dramatic advent of wavelets.

Volume 19: Braids

Another type 1 event of the 1980s was the discovery by Vaughan Jones of a connection between von Neumann algebras and knot theory. Each of these areas had a long history, with knot theory going back to the 19th century, and von Neumann algebras having their roots in von Neumann's papers on operator algebras in the 1930s. They had been completely separate areas, but in his thesis, Jones found some algebraic structures that eventually led him to a connection with knots. This led to a tidal wave of new results in knot theory, including applications to DNA. (It has been discovered that nature has designed enzymes whose job is to perform basic operations of knot combinatorics!)

A more classical approach to knot theory, developed by Artin, is through the study of braids. A few years ago, a surprising discovery (a type 1 event in pure math) by members of the Mathematics Department at NUS linked braids to some of the classic questions in topology, especially the homotopy groups of spheres. IMS sponsored a program to highlight this new discovery, and followed it up with a broader program to integrate the new discoveries into the already existing fabric of algebraic topology.

Volume 7: Markov Chain Monte Carlo

Finally, let me mention the volume on Markov Chain Monte Carlo (MCMC) methods. Markov chains were invented by Andrei Markov in the early 20th century, apparently motivated by theoretical probability questions. They are a simple model of probabilistic dynamics. They envision a collection of possible states, and a fixed rule governing the chance of moving from one state to another. They have a clean and elegant theory, and are sometimes the subject of a tidy section in a chapter on eigenvalues and eigenvectors in a textbook on linear algebra. However, over the years, many people have applied them to many kinds of phenomena. For example, random walks are Markov chains.

MCMC is part of this roll call of applications, and has itself become a major type of application. MCMC is really not a type 1 event in the strict sense. Rather it is a long, rolling development, in which a particular approach to some hard problems has been found to be applicable in more and more areas. Yet the cumulative effect is like a revolution. Thus we have the article of P. Diaconis in the April, 2009 Bulletin of the American Mathematical Society, with title The Markov chain Monte Carlo revolution.

MCMC started in a 1953 paper by Metropolis, Rosenbluth and Rosenbluth, Teller and Teller. The basic idea, now frequently called simply the Metropolis algorithm, or Metropolis–Hastings algorithm, is to create a Markov Chain to sample approximately a probability distribution which is not easily computable directly. This was a new idea for using Markov chains. Over time, this idea itself has found many variations and new applications, most recently in biostatistics and mathematical finance.

Perhaps the epitome of an applied breakthrough is the rise of Google. Their fabulously successful Internet search engine is also based on a Markov chain, made from all the nodes in the internet! In the last ten years, Google has gone from non–existent to one of the biggest companies on the world in terms of market capitalization. I still cannot quite get my head around the possibility that one can perform a Markov chain on several billion variables, and come out with anything meaningful. Google doesn't prove any theorems, but it shows by example that its methods work, millions of times every day, Few examples show better the power of pure mathematics when used in an opportune way. Let me highlight the fact that Markov chains had been sitting around for a long time, and had been used in a variety of ways that are not so far removed from their use by Google. First their invention, but also the ways they had been applied, reduced the potential barrier to their application to the internet.

These examples underline the importance of maintaining a level of mathematical expertise, so that such technologies can be understood and used adaptively. The IMS has clearly been a positive force in bringing understanding of such new developments to Singapore, and raising the capacity of its mathematical community to adapt to and utilize the new ideas, wherever they arise.

In closing, I would like to say a little about the future of IMS. In its first ten years, IMS has amply demonstrated its value for supporting the mathematical infrastructure of Singapore. I believe I speak for all my colleagues on the SAB, when I marvel at the effectiveness with which Louis Chen has spent the dollars he has been allocated. But I also believe that we have all wished those dollars could have been more. In particular, we have wished that the government had a mechanism for funding IMS, rather than having it be funded as an internal activity of NUS. We are grateful to NUS for taking on that burden, and salute the vision of both Deputy President for Research and Technology Barry Halliwell and Provost Eng-Chye Tan in providing ongoing funding for IMS. But in fact, IMS is, and should be thought of as, a national resource. In a country such as Singapore, with its reliance on technological progress, but with a modest number of mathematical scientists, and where anyone can get anywere in under and hour, it makes eminent sense to have an Institute for Mathematical Sciences, but it does not make sense to have two, or to have one attached to a particular institution, except as providing a physical home. I note that one of the planned programs of IMS is primarily coming from the School of Physical and Mathematical Sciences at NTU, and an earlier program was also. Programs have also been initiated by other institutions, and participants have come from many organizations in Singapore. I regard this as healthy. It is how IMS should work; but when IMS itself is funded through NUS this raises questions of financial responsibility. I hope that, as the challenge of finding a successor to Louis Chen as Director of IMS is addressed, the question of the funding mechanism that is commensurate with Singapore's dependence on mathematical infrastructure and its physical size, will also be addressed.

Thank you.