

MEETING THE STATISTICAL CHALLENGES IN HIGH DIMENSIONAL DATA AND COMPLEX NETWORKS

From 5 to 16 February 2018, the Institute hosted a workshop on “Meeting the Statistical Challenges in High Dimensional Data and Complex Networks”. The co-chairs of the workshop contributed this invited article to Imprints.

BY JIASHUN JIN AND ZHIGANG YAO

This program was motivated by the recent development in statistics, machine learning, and many other areas, such as social networks, genetics and genomics, cosmology and astronomy. The program aimed at showing the role of modern statistical methods in complex data and served to support interactions among statisticians, mathematicians, engineers and scientists working in the interface of experiment, computation, analysis and statistics. The two-week workshop, which featured two special lectures, was focusing on the development of new statistical methods in high dimensional data and complex networks with their interactions in scientific and social sciences. It has fostered collaboration on all aspects of the effects of the high dimensional data analysis and social networks.

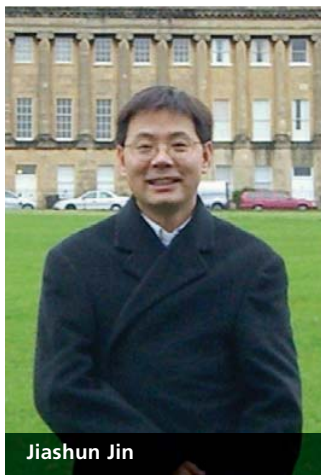
HIGH DIMENSIONAL DATA ANALYSIS – We addressed topics on science frontiers and methodology development. Each of these topics is closely related to the other. The former involves new data type and emerging scientific problems together with challenging statistical issues from recent development in biomedical research (cancer research, brain image), and the latter consists of a series of discussion on classification and clustering, random matrix theory, and recent advancement in machine learning and shape analysis.

Classification and clustering are important problems in biomedical research, especially in genomics and genetics. Exploiting sparsity, a direct result of “large p ”, has become a major strategy for analysing Big Data.

FEATURED	NEWS	OTHERS
Meeting the Statistical Challenges in High Dimensional Data and Complex Networks 01	New Souvenirs 03 Past Activities 04 Upcoming Activities 11	IMS in Numbers 03 Lecture Note Series 24
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Zhigang Yao



Jiashun Jin

The so-called rare and weak signal phenomenon in high dimensional data has been particularly emphasized; that is, the rare signal means that only a small fraction of returns contain tradable signals, others are merely noise; the notion of weak signal means that the signals are individually weak. There have been a series of invited talks centred around survey, theory and applications to Big Data Analytics, on top of one four-hour tutorial focusing on Tukey's Higher Criticism and rare and weak signals, with applications to classification.

Random matrix theory is widely used in many areas of scientific and statistical research, especially in network analysis, where a precise knowledge on the eigenvalues and eigenvectors of very large-size matrices is needed. Unfortunately, existing literature on random matrix theory has been rather under-developed to be used for solutions to many scientific problems.

In this workshop, we have brought together both statisticians and applied mathematicians in random matrix theory to discuss this issue. At the same time, Principal Component Analysis (PCA) and sparse PCA is a research topic that is closely related to random matrix theory.

Recent advancement in *machine learning and shape analysis* has addressed the challenges of analysing data that are lying in non-Euclidean space, therefore on manifolds. Manifold data arises in the sense that the sample space of data is fundamentally nonlinear.

To quantify statistical variation on more complex features such as curves and surface, a strategy of developing statistical tools in parallel with their Euclidean-counterpart is significantly relevant. This workshop has an extensive discussion on statistical challenges with non-Euclidean data, which potentially foster interactions between traditional probability/statistics theory and real applications.

SOCIAL NETWORK – In the last two decades, network structures arise in various fields, such as social media, biology, and technology. Compared to data sets in other areas (e.g., genomics and genetics), social networks data are less expensive to collect, and many interesting data sets can be collected by the researchers through internet. This trend has motivated the statistical analysis in networks.

However, network models, especially for dynamic networks: a challenging issue in the area of network analysis is that it lacks realistic yet tractable models. For example, the block model and degree corrected block models are frequently too idealized, and the exponential random graph models are too complicated to analyse. Modelling is especially needed for dynamic network, where we need to model the evolution over time.

Apart from another four-hour tutorial on large scale networks, the workshop consisted of invited talks in network modelling and estimation. Topics include shrinkage estimation in heteroscedastic hierarchical models, covariate-adjust block model for community detection and brain network topological changes.

There has been an increasing interest on the study of modern statistics and data science in Singapore. Some of those problems deserve closer attention to the science and medical communities worldwide. This program at IMS has brought together some top statisticians in the area of high dimensional data and complex networks. New interdisciplinary collaborations are expected to form, stimulating frontier research both in theory and application. This would be beneficial to the Department of Statistics and Applied Probability at NUS and the local scientific community such as medical research facilities at NUS or other institutions in Singapore.



Participants of the workshop on Meeting the Statistical Challenges in High Dimensional Data and Complex Networks



The workshop's poster session

OUTSTANDING SERVICE AWARD TO IMS DIRECTOR

The Institute's Director, Professor Chong Chi Tat, received the Outstanding Service Award from NUS President Tan Eng Chye during the NUS University Awards ceremony on 14 May 2018. The prestigious award honours professionals within the NUS community who have made exceptional contributions in serving the University and society. The staff of IMS heartily congratulates him on this award.

PERSONNEL MOVEMENTS AT IMS

IMS welcomes Ms Angela Aw, who joined IMS on 25 June 2018. She will provide administrative support in coordinating events hosted at the institute.

NEW SOUVENIRS

IMS is happy to announce the launch of additional ceramic mugs in new colors!

Visit our webpage to view more on our specially designed souvenirs, which are available in different designs, each with its distinctive mathematical theme. Purchases can only be made with cash at the IMS.

For more information, please visit ims.nus.edu.sg
> Resources > Souvenir



IMS IN NUMBERS FROM DECEMBER 2017 TO MAY 2018

NUMBER OF
PARTICIPANTS

473

7

PROGRAMS &
WORKSHOPS
OVER 6 MONTHS

161
TALKS

95
STUDENTS



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Institute for Mathematical Sciences





IMS Distinguished Visitors

STEPHEN H. DAVIS

McCormick School (Institute) Professor and
Walter P. Murphy Professor of Applied Mathematics
at Northwestern University

Professor Stephen H. Davis received all his degrees at Rensselaer Polytechnic, and has been Research Mathematician at the RAND Corporation, Lecturer in Mathematics at Imperial College, London, and Full Professor of Mechanics at the Johns Hopkins University. He is the Editor of Annual Review of Fluid Mechanics, and has authored over two hundred and fifty refereed technical papers in the fields of Fluid Mechanics and Materials Science, and has written a book *Theory of Solidification*.

He has twice been Chairman of the Division of Fluid Dynamics of the American Physical Society, is a Fellow of the American Physical Society, member of the National Academy of Sciences, the National Academy of Engineering, and the American Academy of Arts and Sciences. He is the 1994 recipient of the Fluid Dynamics Prize of the American Physical Society and the 2001 G. I. Taylor Medal of the Society of Engineering Science.

Professor Davis visited IMS from 1 – 31 May 2018 for the program on *Modeling and Simulation of Interface Dynamics in Fluids/Solids and Their Applications*. He gave tutorial lectures on *Thin-domain asymptotics in fluid mechanics* on 9 May 2018.

IMS arranges visits to the Institute by distinguished scientists who are prominent leaders in their communities. The program started in 2015. This initiative aims to enhance the diversity of people participating in our research programs, and provide mentoring/ inspire junior researchers and graduate students. Each distinguished visitor spends at least two weeks in Singapore, and participate in a variety of activities, including lecturing about their own research, give public talks, meet with faculty, and interact with program participants.

Under this program, the Institute has enjoyed visits from a stellar array of distinguished scientists. The list of distinguished visitors can be found on our website.

Ng Kong Beng Public Lecture Series

10 MAY 2018

Professor Stephen Davis gave a public lecture entitled “A History of Moving Contact Lines” at NUS on 10 May 2018. Starting with the simple question how some liquid die within a droplet of mercury moves around within that droplet as it slides down a slanted surface inside a tank of water, he puzzled us with the answer to the question what happens if instead the die is placed on the *outside* of that same droplet — surprisingly, upon reaching the point where the die reaches the surface on which the droplet is sliding, it gets ejected into the water! Professor Davis explained to us the non-trivial mathematical machinery necessary to describe this simple, yet surprising experiment, which in fact is not fully understood in all its details to this day. A total of 60 people attended the lecture.



From left: IMS Deputy Director **Adrian Röllin** and **Stephen Davis**

Workshop on Spline Approximation and its Applications on Carl de Boor's 80th Birthday

4 – 6 DECEMBER 2017

Jointly organized with Department of Mathematics, NUS

ORGANIZING COMMITTEE:

Say Song Goh | National University of Singapore

Hui Ji | National University of Singapore

Zuowei Shen | National University of Singapore

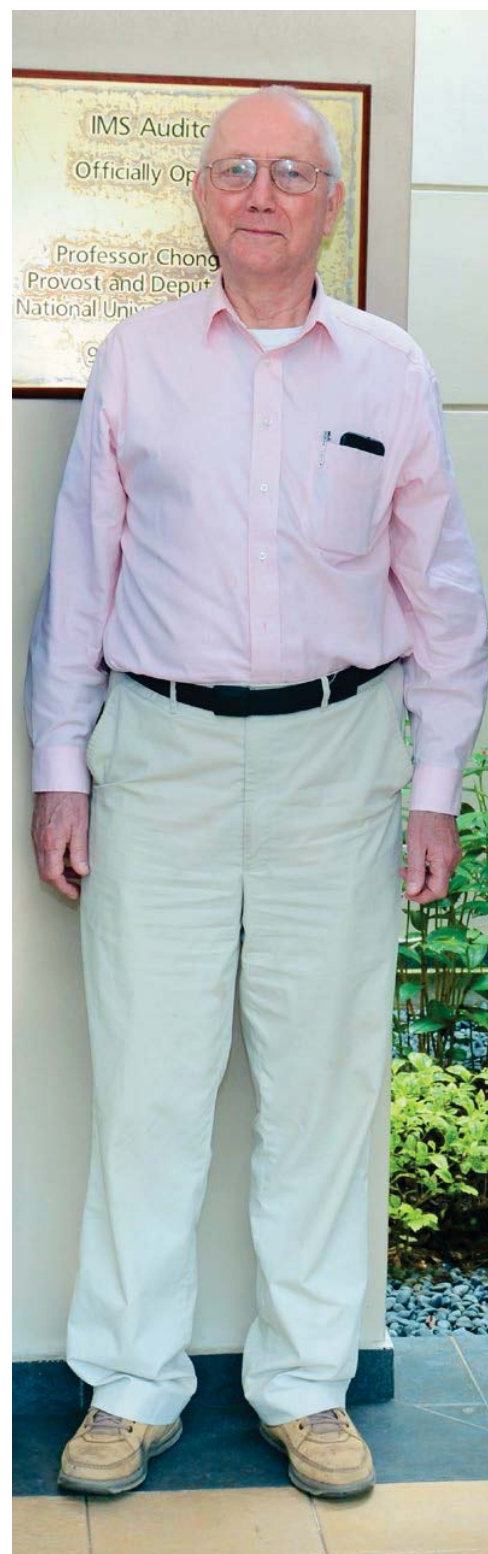
In the late 60s, Carl de Boor embarked on an ambitious program to develop a mathematical foundation for spline functions that would be friendly to computation. The cornerstone of this development was his work on Schoenberg's B-splines – splines with minimal support, and it became clear that spline functions can provide efficient representations of functions, curves, surfaces and digital data. Today, spline functions are widely used in areas such as automotive design, computer aided geometric design, imaging science and data science.

Carl de Boor's contributions to splines, approximation theory, scientific computing, mathematics, and science have not gone unnoticed. In addition to the 2003 National Medal of Science he received in 2005, he has been elected to numerous academic societies in the US and in Europe, including the National Academy of Sciences (1997) and National Academy of Engineering (1993). At the occasion of Carl de Boor's 80th birthday, the workshop brought together a group of mathematicians from many generations to review the glorious history of spline functions, and show new directions of spline functions in both theory and applications.



Party celebration for **Carl de Boor**

A comprehensive review on "From B-splines to box splines" by Rong-Qing Jia (University of Alberta, Canada) provided an excellent overall picture on the contributions of Carl de Boor to spline functions, and fundamental results in this field. Michaël Unser (École Polytechnique Fédérale de Lausanne, Switzerland) discussed the role of spline functions in science and engineering. These discussions have connected applied mathematicians to interesting concepts and applications from engineering. Several other talks on the development of multivariate splines and wavelet frames, for instance the talks given by Amos Ron (University of Wisconsin-Madison, USA)



Carl de Boor

and Zuowei Shen (NUS), have shown new research directions of spline functions and approximation. Topics covered in this workshop also included applications of spline functions in computer aided geometric design, partial differential equations and polynomial approximation. There were a total of 19 invited talks, and over thirty participants.

Representation Theory of Symmetric Groups and Related Algebras

11 – 20 DECEMBER 2017

ORGANIZING COMMITTEE:

Joseph Chuang | City University London

Karin Erdmann | University of Oxford

Kay Jin Lim | Nanyang Technological University

Kai Meng Tan | National University of Singapore

The rich structure enjoyed by the symmetric group algebras, with combinatorics playing a significant role, enables it to be studied as specific examples in the modular representation theory of finite groups, and as algebras of wild representation type. Some of its naturally occurring representations (such as the Lie modules) also have surprising links with other branches of mathematics (such as algebraic topology).

An important message from this workshop is that progress continues to be made on fundamental questions about symmetric groups through a blend of ideas approaching the representation theory from different perspectives.

Highlights of the conference included the talk of Geordie Williamson (University of Sydney, Australia) on his recent breakthroughs in understanding decomposition numbers of symmetric groups, the exposition of Alexander Kleshchev (University of Oregon, USA) on KLR algebras and Turner's conjecture on RoCK blocks, the entertaining introduction of Arun Ram (University of Melbourne, Australia) to his ideas for crystals for symmetric groups, and the lectures of Stephen Donkin and Haralampos Geranios (University of York, UK) on their determination of first cohomology groups for Specht modules. Many of these and other lectures have featured the cutting edge of present knowledge and have suggested the next steps in the developments.

The workshop featured 30 invited talks on the latest developments in the representation theory of symmetric groups and related algebras, preceded by three days of lively informal discussion. It attracted close to eighty attendees, including leading researchers in different approaches to the subject.



Susumu Ariki: On block algebras of Hecke algebras of classical type



Jonathan Brundan: Representations of the oriented Skein category



Alexander Kleshchev: RoCK blocks of symmetric groups and generalized Schur algebras



Andrew Mathas: Content systems and deformations of KLR algebras



Arun Ram: Are there symmetric group crystals?



Geordie Williamson: Computing symmetric group decomposition numbers in the anti-spherical module



Changchang Xi: Lifting stable to derived equivalences with applications to Broué's abelian defect group conjecture



The first of these procedures is straightforward, but what follows are not, as **Stephen Donkin** demonstrates



Linkage principle for symmetric groups

Data Sciences: Bridging Mathematics, Physics and Biology Part II

4 – 12 JANUARY 2018

CO-CHAIRS:

George Barbastathis | Massachusetts Institute of Technology

Hui Ji | National University of Singapore

Patrice Koehl | University of California at Davis

The workshop facilitated productive exchange of emerging computational and instrumentation methods. Key ideas were shared among researchers who used different imaging probes. The inaugural validation task force for X-ray Free-electron Laser (XFEL) based single particle imaging (SPI) has been initiated at this workshop. Represented at this workshop were key experts in various parts of the XFEL-SPI data processing pipeline, who unanimously acknowledged the importance of setting up this validation task force.

There were 20 invited talks in this workshop, with the last day devoted to open discussion on single particle imaging chaired by Dr Duane Loh (NUS).

Emerging issues in bio-imaging sciences that were presented at the workshop include machine learning on imaging systems; wavefront sensing and shaping for bio-imaging; tomography of cells and biological neural networks; reconstructing missing information from sparse, noisy, and incomplete measurements using constraint satisfaction algorithms; practical phase retrieval and ptychography with very sparse and partially coherent conditions; extracting structural information from correlations of fluctuations incomplete measurements; robust simulation of probe-sample interactions.

Participants were impressed with the level of scientific research in the area of computational methods for bio-imaging at NUS. This workshop has strengthened collaborations between NUS and Cornell University (Ithaca, New York, USA), Center for Free Electron Laser (DESY, Hamburg, Germany), Uppsala University (Uppsala, Sweden), University of Melbourne (Melbourne, Australia), and EMBL-EBI (Hinxton, UK).

There were over 50 participants which included 20 students.



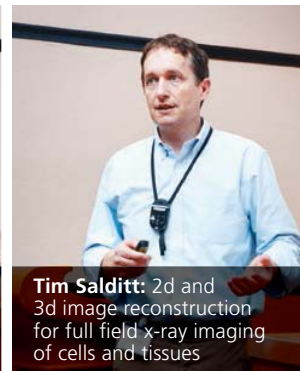
Demetri Psaltis:
Teaching optical systems
by example



Colin Sheppard:
Imaging with illumination
and detection arrays



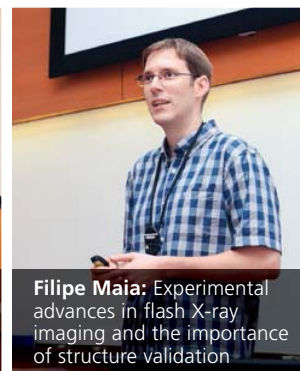
Veit Elser:
50 years of phase
retrieval in 50 minutes



Tim Salditt: 2d and
3d image reconstruction
for full field x-ray imaging
of cells and tissues



Ardan Patwardhan:
EMDB and EMPIAR: public
archiving of cryo-EM data



Filipe Maia: Experimental
advances in flash X-ray
imaging and the importance
of structure validation



Compact representation
of multiple signals



Signal coding in smaller fields



Image classification and assembly



Boosting connectivity analysis

Meeting the Statistical Challenges in High Dimensional Data and Complex Networks

5 – 16 FEBRUARY 2018

CO-CHAIRS:

Jiashun Jin | Carnegie Mellon University
Zhigang Yao | National University of Singapore

The workshop is motivated by the recent developments in statistics, machine learning, and many other areas such as social networks, genetics and genomics, cosmology and astronomy.

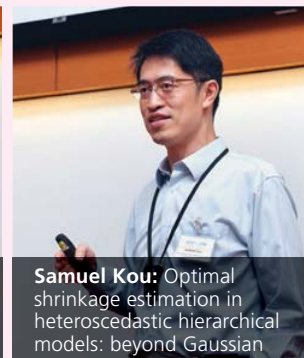
The first workshop addressed topics on the modern statistical analysis from the science frontiers to methodology development. Talks covered on methods for solving new scientific problems such as recent developments in biomedical research and object oriented data. The second workshop mainly addressed recent issues in social networks with the methodology development in various important big data problems.

Highlights of the workshop included a lecture on gradient boosting and its applications on big data analysis by Tze Leung Lai (Stanford University) on 8 February 2018, and another lecture by Samuel Kou (Harvard University) on 14 January 2018 on heteroscedastic hierarchical models. Activities have fostered the interaction and collaboration among mathematicians, statisticians, engineers and scientists who work in the interface of experiment, computation, analysis and statistics.

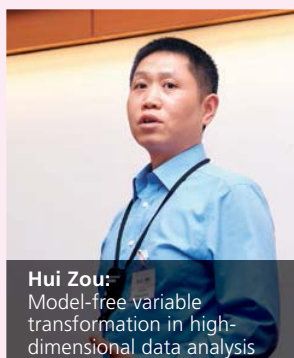
There were over 100 participants, a quarter of the audience were graduate students.



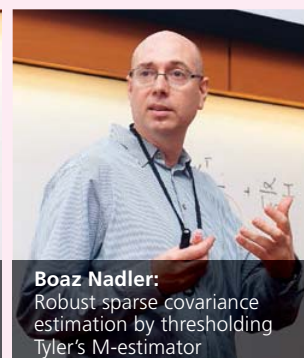
Tze Leung Lai: Gradient boosting: overview, theory and applications to big data analytic



Samuel Kou: Optimal shrinkage estimation in heteroscedastic hierarchical models: beyond Gaussian



Hui Zou: Model-free variable transformation in high-dimensional data analysis



Boaz Nadler: Robust sparse covariance estimation by thresholding Tyler's M-estimator



Anthony C. Davison: Inference for complex extreme events



Van Ha Vu: Low rank matrices with random perturbation and applications



Object oriented data analysis (From left: **Steve Marron** and **Stephan Huckemann**)



Interaction screening for data

Workshop on Particle Swarm Optimization and Evolutionary Computation

20 – 21 FEBRUARY 2018

CO-CHAIRS:

Kay Chen Tan | City University of Hong Kong

Weng Kee Wong | University of California, Los Angeles

This workshop aimed to gather experts in the area and have them share their insights on these nature-inspired metaheuristic algorithms. A focus was on PSO and other evolutionary computational algorithms that have high success rates of outperforming other types of algorithms for solving challenging optimization problems in various disciplines.

The workshop started with a two-hour tutorial on nature-inspired metaheuristic algorithms by Ponnuthurai Nagarathnam Suganthan (Nanyang Technological University). There were a total of eight talks. Local participation from diverse disciplines across various research institutes in Singapore was higher than expected, with over 50 local attendees in the group of participants. There were 16 graduate students.



Ponnuthurai Nagarathnam Suganthan:
Numerical optimization by differential evolution



Kay Chen Tan:
Evolutionary computation opportunities and challenges



A matching-based selection in multiobjective optimization



Opportunities created from streams of data

6th NUS-USPC Workshop on Machine Learning and FinTech

18 – 19 APRIL 2018

ORGANIZING COMMITTEE:

Jean-François Chassagneux | University Paris Diderot

Ying Chen | National University of Singapore

Min Dai | National University of Singapore

Claudio Fontana | University Paris Diderot

Steven Kou | National University of Singapore

Huyền Pham | University Paris Diderot

Chao Zhou | National University of Singapore



Participants of the 6th NUS-USPC Workshop

This workshop is a collaboration between the Laboratoires de Probabilités, Statistique et Modélisation at the University Paris Diderot/Sorbonne Paris Cité and the Centre for Quantitative Finance and Risk Management Institute at the National University of Singapore. It featured talks on machine learning and innovation in financial technology delivered by experts, academics and practitioners from finance, numerics, statistics and engineering/computer science. There were a total of 11 talks and over fifty participants.



Modeling and Simulation of Interface Dynamics in Fluids/Solids and Their Applications

23 APRIL – 25 MAY 2018

CO-CHAIRS:

Weizhu Bao | National University of Singapore

Weiqing Ren | National University of Singapore and
Institute of High Performance Computing, A*STAR

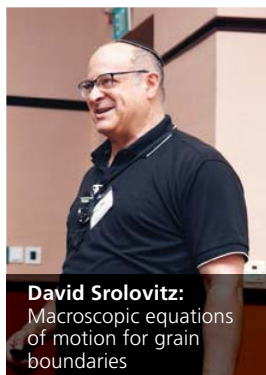
Fluid/solid systems involving interface dynamics are ubiquitous in nature and are found in many engineering applications, such as coating, printing, porous media flows, and micro-fluidic devices. Modelling and simulation of such systems has been challenging, and the problem becomes even more difficult in the presence of a moving contact line, which is the intersection of the interface with a solid substrate. In particular, modelling the boundary condition near the moving contact line is still an issue under debate.

This program hosted a series of interconnected workshops and tutorials. There were 15 talks in the first workshop on modeling and simulation of interface-related problems (30 April – 3 May 2018). The following week had tutorial sessions by Stephen H. Davis (Northwestern University, USA) on Thin-domain asymptotics in fluid mechanics, and Efficient

numerical schemes for gradient flows and multiphase incompressible flows by Jie Shen (Purdue University, USA). There were also two special seminars over 9 and 10 May 2018. The second workshop, scheduled from 14 to 18 May 2018 focused on the modeling and simulation of interface dynamics in fluids/solids and its applications. There were 30 talks.

There were lots of lively discussion and debating in most talks. The program is interdisciplinary, and involved speakers from various disciplines including mathematics, physics, material sciences and mechanical engineering etc. Leveraging on the rich content from the activities in this program, researchers benefitted from focused research discussions, exchange of ideas and identified a few important directions to be worked on. The program activities also trained graduate students and junior researchers.

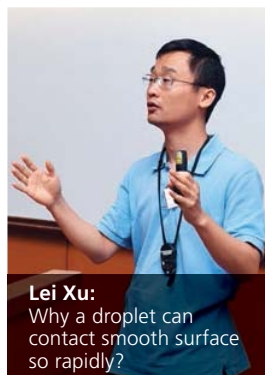
There were a total of 77 participants, which included 15 graduate students.



David Srolovitz:
Macroscopic equations of motion for grain boundaries



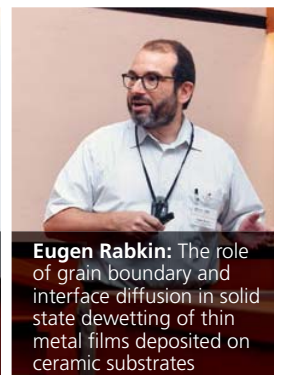
Xiaoping Wang:
The threshold dynamics method



Lei Xu:
Why a droplet can contact smooth surface so rapidly?



Jie Shen:
Efficient numerical schemes for gradient flows and multiphase incompressible flows



Eugen Rabkin: The role of grain boundary and interface diffusion in solid state dewetting of thin metal films deposited on ceramic substrates



Michael Miksis and Amy Novick-Cohen:
Intriguing surface patterns in good ambience



Derivation of the interaction force between multiple particles



Produce early contact at the thinnest gap location

Dynamic Models in Economics

4 – 22 JUNE 2018 & 2 JULY – 3 AUGUST 2018

ORGANIZING COMMITTEE:

Yi-Chun Chen | National University of Singapore

Yeneng Sun | National University of Singapore

ACTIVITIES

- Workshop on Game Theory, 4 – 8 June 2018
- Econometric Society Summer School 2018, 15 – 19 June 2018
- Workshop on Mechanism Design, 9 – 13 July 2018
- Workshop on Matching, Search and Market Design, 23 – 27 July 2018

IMS Graduate Summer School in Logic

18 JUNE – 6 JULY 2018

Jointly organized with Department of Mathematics, NUS

This Summer School bridges the gap between a general graduate education in mathematical logic and the specific preparation necessary to do research on problems of current interest in the subject.

ACTIVITIES

- Week 1: Generalizing Gödel's Constructible Universe by W. Hugh Woodin (Harvard University)
- Week 2: Measure, dimension and computability by Theodore A. Slaman (The University of California, Berkeley)
- Week 3: Model theory of finite and pseudo-finite fields by Zoé Chatzidakis (Ecole Normale Supérieure)

Oppenheim Lecture

22 JUNE 2018

Jointly organized with Department of Mathematics, NUS

The fourth Oppenheim Lecture on "Number of Points Modulo p When p Tends to Infinity" will be delivered by Jean-Pierre Serre (Collège de France).

Pan Asia Number Theory Conference 2018

25 – 29 JUNE 2018

ORGANIZING COMMITTEE:

Wee Teck Gan | National University of Singapore

Lei Zhang | National University of Singapore

Theories and Numerics of Inverse Problems

6 – 17 AUGUST 2018 & 24 – 28 SEPTEMBER 2018

CO-CHAIRS:

Xudong Chen | National University of Singapore

Zuowei Shen | National University of Singapore

ACTIVITIES

- Tutorial on Calderon's Problem: Visibility and Invisibility: 6 – 10 August 2018
- The 9th International Conference on Inverse Problems and Related Topics: 13 – 17 August 2018
- Workshop on Qualitative and Quantitative Approaches to Inverse Scattering Problems: 24 – 28 September 2018

Bayesian Computation for High-Dimensional Statistical Models

27 AUGUST – 21 SEPTEMBER 2018

ORGANIZING COMMITTEE:

Alexandros Beskos | University College of London

Hock Peng Chan | National University of Singapore

Dan Crisan | Imperial College London

Ajay Jasra | National University of Singapore

Kengo Kamatani | Osaka University

Kody Law | Oak Ridge National Laboratory

David Nott | National University of Singapore

Sumeetpal Singh | University of Cambridge

ACTIVITIES

- Opening Workshop and Tutorials: 27 – 31 August 2018
- Reading Groups and Local Seminars: 3 – 18 September 2018
- Closing Workshop: 19 – 21 September 2018

Workshop on String and M-Theory: The New Geometry of the 21st Century

10 – 14 DECEMBER 2018

CHAIR:

Meng-Chwan Tan | National University of Singapore

On the Langlands Program: Endoscopy and Beyond

17 DECEMBER 2018 – 18 JANUARY 2019

CO-CHAIRS:

Dihua Jiang | University of Minnesota

Lei Zhang | National University of Singapore

ACTIVITIES

- Introductory Courses (2 weeks): 17 December 2018 – 4 January 2019
- Research Conference (proposed dates): 7 – 11 January 2019

Statistical Methods for Developing Personalized Mobile Health Interventions

4 FEBRUARY – 1 MARCH 2019

ORGANIZING COMMITTEE:

Bibhas Chakraborty | National University of Singapore

Ying Kuen Cheung | Columbia University

Eric Laber | NC State University

Jialiang Li | National University of Singapore

Susan A. Murphy | University of Michigan

Ambuj Tewari | University of Michigan

ACTIVITIES

- Tutorial on Personalized Medicine, Treatment Regimes, Reinforcement Learning, and Causal Inference: 4 – 15 February 2019
- Workshop on Design of mHealth Intervention Studies: 18 – 22 February 2019
- Workshop on Analysis of Data from mHealth Intervention Studies: 25 February – 1 March 2019

Quantitative Finance

18 – 22 MARCH 2019 & 22 JULY – 31 AUGUST 2019

CO-CHAIRS:

Min Dai | National University of Singapore

Steven Kou | National University of Singapore

ACTIVITIES

- 4th Berlin-Princeton-Singapore Workshop on Quantitative Finance, 18 – 20 March 2019
- Workshop 1: Stochastic Control in Finance, 22 – 26 July 2019
- Workshop 2: Fintech and Machine Learning, 5 – 9 August 2019
- Workshop 3: Asset Pricing and Risk Management, 26 – 30 August 2019

For full list of upcoming events, visit our webpage at ims.nus.edu.sg

JOSEPH BERNSTEIN:

BEAUTY AND REALITY IN MATHEMATICS – D-MODULES, GROUPS, SHEAVES

Interview of Joseph Bernstein
by Y.K. Leong

Joseph Bernstein made deep and fundamental contributions to algebra, representation theory, analysis, algebraic geometry and number theory.



At the age of 14, he participated in the Moscow Mathematical Olympiads; at 17, he won a gold medal in the International Mathematical Olympiad. The following year, he entered Moscow State University from which he obtained his MSc and PhD. After finishing PhD studies he started to work as a junior researcher and remained in that position for 8 years because of the antisemitism prevailing in the former Soviet Union. In 1978 he quitted his research position and waited more than two years before he was allowed to emigrate. And all this while, he made ground-breaking discoveries and collaborated with his fellow Russian colleagues, among them Israel Gelfand (1913-2009), Sergei Gelfand, David Kazhdan, Andrei Zelevinsky (1953-2013), Dmitry Leites and Alexander Beilinson.

In the summer of 1981, he emigrated to the United States, first as a visiting professor to the University of Maryland and then as a professor at Harvard University for 10 years. He has been Visiting Professor and Fellow at the Max Planck Institute for Mathematics, Israel Institute of Advanced Studies, Courant Institute of Mathematical Sciences, Institute for Advanced Study in Princeton and University of California, Berkeley. Having visited Israel several times, he finally moved to Tel Aviv University as full professor in 1993, and became Professor Emeritus in 2014. He continues to be active in research on analytic questions in the theory of automorphic forms such as the meromorphic continuation of Eisenstein series,

bounds for them, automorphic periods and global invariants obtained from them, subconvexity estimates for automorphic periods and L -functions.

The first recognition of his ground-breaking research was the annual prize of the Moscow Mathematical Society awarded when he was officially a junior researcher, an academic position beyond which he never advanced during his academic career at Moscow State University. While he was already well-known within the mathematical community, it was only when he was in his late fifties that he received official recognition: membership of the US National Academy of Sciences and of the Israel Academy of Sciences. In 2004 he was awarded the Israel Prize in Mathematics. More recently, in 2016, he and David Kazhdan were jointly awarded the EMET Prize, which is awarded annually in Israel for achievements in the sciences, arts and culture. This is only the fourth time that the prize for the exact sciences category was awarded to mathematicians since its inaugural award in 2002.

In 11 – 16 June 2017 a group of leading mathematicians gathered in Rehovot, Israel to pay tribute to Bernstein's mathematical contributions and influence as collaborator, teacher and mentor at an international conference on representation theory and algebraic geometry. The invited speakers came from Canada, England, France, Germany, Singapore and the United States.

In an article *Mathematics of Joseph Bernstein* published in *Selecta Mathematica* in 2016, Roman Bezrukavnikov, Alexander Braverman, Michael Finkelberg and Dennis Gaitsgory wrote, "It [Bernstein's approach to mathematics] is guided by the vision of mathematics, however difficult it may be, as governed by ultimately simple and elegant principles that are there for the mathematician to discover. It is this vision that inspired and will continue to inspire generations of mathematicians."

Bernstein had already created the algebraic theory of D-modules in his PhD thesis, written under the supervision of I. M. Gelfand, which proved the meromorphic continuation of P^λ for any polynomial P and complex number λ . The seminal ideas in his thesis (such as the Bernstein inequality for D-modules) were developed further by others and applied to other areas. His years in Moscow produced many fruitful results. For a long period of time, he worked on various problems in the representation of real groups and Lie algebras with I.M. Gelfand and his son Sergei Gelfand. This resulted in important concepts such as O-category, BGG resolution, reflection functors and so on.

He and Zelevinsky discovered important results on the representations of p -adic groups that later led to the (Zelevinsky) classification of irreducible representations of $GL(n, F)$ in terms of cuspidal representations. With Leites, he developed the basic super analogues of linear algebra and manifold theory, which are now at the core of the mathematics of superstrings in physics. He and Beilinson proved the Kazhdan-Lusztig conjecture [George Lusztig] using methods that led to the development of geometric representation theory.

In his last years at Harvard, he and Valery Lunts did important work on non-holonomic D-modules and developed the theory of equivariant derived categories that has found applications in cohomology theory. His years in Israel saw a prodigious collaboration with

André Reznikov in the use of representation theory in analytic number theory.

His work on p -adic groups gave rise to the so-called Bernstein Center which would play an important role in geometric representation theory. He had also discovered a new (Bernstein) presentation of affine Hecke algebras [Erich Hecke (1887-1947)] that would be important in the development of representation theory. Among other things, he (and Mikio Sato independently) introduced the Bernstein-Sato polynomial for differential operators. In his characteristic fashion to understand and give direct and elementary proofs of known results, he gave a direct proof of Harish-Chandra's result [Harish-Chandra (1923-1983)] that described the support of the Plancherel measure [Michel Plancherel (1885-1967)] for reductive groups. Using this method, he derived analogous results for a large class of homogeneous spaces and was able to explicitly compute the continuous part of the Plancherel formula.

Bernstein was in the Institute for Mathematical Sciences, National University of Singapore from 19 March to 1 April 2012 at the invitation of the Institute for its program *Branching Rules* (11 – 31 March 2012), in which he gave a talk on *Some applications of representation theory to estimates of automorphic periods*. On behalf of *Imprints*, Y.K. Leong interviewed him on 29 March 2012. The following is an edited and enhanced version of the interview in which he traced the nonlinear path taken from Moscow to Tel Aviv through Harvard, driven by a mathematical passion undeterred by a racial discrimination in his original home country. Here he shares with us his views on what he considers to be beautiful in mathematics and his belief in the reality of mathematical ideas.

Acknowledgement. Y.K. Leong would like to thank Dmitry Matsukevich of the Department of Physics, National University of Singapore for preparing a raw draft of part of the transcript of the interview.

IMPRINTS

I You have said that your early school years of training for mathematical Olympiads helped to hone your problem solving skills in your research career. In your research, how much of a problem solver are you even as you like to look at structures and theories?

JOSEPH BERNSTEIN

B I should say that this is probably part of the more general question. I participated a lot in Olympiads when I was at high school. Then later on, when I was a student at Moscow University, I participated in organizing the Olympiads in Moscow for many years. This was always controversial, whether Olympiads were useful for future development of mathematicians or not. I don't know the answer. On one hand sometimes the problems were too difficult and thus

contradicted the study of real mathematics. On the other hand, I think it gives you a lot of training.

I know that in my research, in many cases after you study some theory and then you have to prove some things that you are sure are correct. Very often you do this by inventing some tricks. I suspect I was trained to invent such tricks during my Olympiad years. Of course, I cannot prove this. Maybe without it, I also would be able to do this. But nevertheless, it is my suspicion that this helped me. On the other hand, in some sense, in the early stages it may have retarded me. I believed, till the age of 20, that if I cannot solve a problem in 5 hours, then I cannot solve it at all. Only much later, I realized that the problems you solve can take weeks, months, years to solve. So somehow it has both positive and negative effects.

I Do you consider yourself to be a problem solver?

B No, no.

I You build theories?

B Yes, I prefer to build theories. I am a good problem solver, but right now, I don't do it. Probably, when I was very young, this was my main advantage, but now, I think not. I am trying to describe what I see, what I discover. I look at things, see some ideas, some structures. I try to fix things up, to describe them, and usually in order to describe them, you should prove some results. You suspect this; now you should prove it. And in order to prove them, very often it helps if you can actually invent the proof that proves this.

I When you build the theories, do you need to solve specific problems?

B Not always. It happened several times in my life, that I would see some structure, I would formulate some conjecture and I would prove it. But in some sense, I wouldn't know what to do with it. I knew it was important, but did not know how to explain that. For example, this was the case with my result about what is now called the "Bernstein Center". I felt that it is important, but could not invent convincing examples of applications of this result. Later Kazhdan used this result in his work on cuspidal geometry, and this made clear that it is significant.

I often formulate to my students: sometimes you prove a result, but then you should invent how to sell it, how to explain to others that this is interesting. Because this is the result, you proved the theorem, but why it is interesting is a completely different question.

I The skill for building a theory is quite different from solving specific problems, isn't it?

B I think so. I think that these are two different things, and somehow there is some friction between them. Doing one thing develops very much your thinking. Then you realize that it somehow leads to a contradiction with the other one. Somehow you need both.

I You and David Kazhdan were fellow students at Moscow University. You have also mentioned, though he had much mathematical influence on your work, both of you wrote few joint papers. How does his style of research differ from yours?

B I thought about this. He is, in some sense, stronger mentally. I don't know how to say it. Very often he sees some important problem, he understands its significance and just starts solving it – and that is his

method. I usually study some similar problems, get some ideas, then start trying to develop these ideas. Usually if I have a problem, and I don't have some ideas how to solve it, then in some sense, I don't try to solve it. Only if I get some idea, I start developing it. I think that Kazhdan's method is probably better.

I I think, in your PhD work on D-modules, you build up all the machinery on your own, isn't it?

B Yes. It started when I tried to solve a problem which was formulated by Israel Gelfand. I tried to solve it in a very different way using D-modules. I conjectured that using D-modules one can solve this problem for any polynomial. And in some cases I was able to prove this. Then I started to analyze the situation, in some sense building everything from scratch. I did not know algebraic geometry and I somehow built everything just by using very basic things like Hilbert basis theorem.

I Are D-modules now approached by using algebraic geometry?

B Yes, yes. Later on I learned algebraic geometry. Now algebraic geometry is one of the central directions of my work. But at the time I was very ignorant. I used it, but used it on a very elementary level. Everything in this paper is extremely elementary. [*Modules over the ring of differential operators; the study of fundamental solutions of equations with constant coefficients*. Functional Analysis and its Applications 5, No.2, 1-16 (1971)]

I It seems that this is a typically Russian style of solving a problem, usually starting almost from scratch. Is it correct?

B No, I would say "no". If you look at [Vladimir Drinfeld's papers, he has many results based on some very sophisticated things. But he, from the very beginning, learned modern algebraic geometry from [Yuri] Manin. I learned it only much later.

I David Kazhdan also emigrated to Israel after you went there. Was he influenced by your decision to leave the Soviet Union in 1981?

B Not quite. Kazhdan emigrated from Soviet Union in 1975 and myself in 1981. I would say that probably both his emigration and my emigration were influenced by the same reasons, by what is called "zastoi", the stagnation. Life was very unpleasant; it was stagnation. I started to think about moving to Israel very soon after my emigration. It happened that when I went to the States in the summer of 1981, Ilya Piatetski-Shapiro [(1929-2009)] invited me to Israel. I liked what I saw in Israel very much and thought about moving to Israel, but it took me more than 12 years to do so. Kazhdan also thought about this for many years, but his was an independent decision.

I You mentioned this stagnation; it means there were no chances of furthering yourself?

B No, this was also the case because of antisemitism. For instance, I could later defend what is called "*habilitation*" (second doctoral degree). It was not impossible but it would be very difficult since I am Jewish, so I saw no reason why I should do it. This was one of those things, but more generally, life was stagnating and everybody felt this. Many years later I heard on Russian TV this period was described as "*zastoi*" (stagnation), and I was impressed how precise this description was.

I You mean there were a lot of rules and regulations that applied to Jewish people.

B Not only that. It is a long story. For instance, at some moment they stopped accepting Jewish students into the mathematics department of Moscow University. In my year, about one-fourth of students were Jewish; it was like this for several years. After this, there were 2 or 3 out of 400; they just stop accepting them [Jewish students]. But then, what happened was that they stopped accepting students from mathematical high schools. Then they, in some sense, stopped accepting all talented students. One thing led to another. But, in general, not only in mathematics; life in the whole of Soviet Union became stagnated. Nothing happened.

I Why did you choose to go to the University of Maryland at that time in 1981?

B You should understand that communication between Russia and the world was quite difficult. I had no idea, I just emigrated. In fact, I applied for emigration in 1978 and got permission only in two and something years. I had some friends at the University of Maryland.

I You knew somebody in Maryland?

B Yes, a couple of people who emigrated several years before me. That was why I came to the University of Maryland for a visiting position. And then there was a special year at Maryland University in representation theory, so I stayed there for one year to participate in this year, and afterwards I went to Harvard.

I You were in Maryland as an invited speaker?

B Visiting Professor.

I Visiting Professor at the University of Maryland?

B Yes, for a year and a half.

“WHEN I TEACH, I OFTEN FIND NEW POINTS OF VIEW. PRACTICALLY EVERY COURSE THAT I TEACH, EVEN IF I HAD TAUGHT IT SEVERAL TIMES, I PREPARE IT ANEW AND OFTEN DISCOVER NEW THINGS AND IDEAS. FOR ME, I FEEL THAT IT IS VERY IMPORTANT TO TEACH.”

I Was it easy to get a permit or visa to go to the US from the Soviet Union?

B It was very difficult. You just don't understand the situation. It was not permission to go; it was permission to emigrate, something different. In Russia you could not emigrate, but you could apply to connect with the family. So I applied for an emigration visa, in fact, to Israel, because it was the only way. And I waited for two and a half years.

I It took two and a half years to get the visa?

B Yes, to go to Israel, but then I went to United States instead. There were many people who were leaving the Soviet Union this way, but this was the only way to legally leave the Soviet Union.

I During that period, a lot of other people also emigrated?

B Yes, many people emigrated, and many very good mathematicians emigrated. Some of them discovered that if you wanted to emigrate to US you had to spend several months in Europe, because of the way the United States bureaucracy worked. Most of the people who wanted to go to United States had to spend some time in Italy. But some mathematicians discovered that they can spend some time in IHÉS [*Institut d'Haute Études Scientifiques*], and so I also, for instance, spent the summer of 1981 in IHÉS.

I And after that you went from Maryland to Harvard?

B Yes, from Maryland I went to Harvard, and I taught 10 years in Harvard. In 1993, I moved to Israel.

I Was Kazhdan already in Harvard when you went there?

B Yes, he was in Harvard, he emigrated at the end of 1975. He was first a visitor there and then became a faculty member.

I It appears that the research schools in the former Soviet Union were initially built around individuals. For example, your own work on D-modules was influenced by I.M. Gelfand. From your experience in the US, what impression do you have of the style of research in the US compared to that in Russia?

B I don't know, the style is quite different, but in this particular question, "built around individuals" – I think, it is probably the same in the States. I see in many places in the States that the graduate students learn mostly from their teachers and are influenced and shaped by them. For example, in many American universities, it is not so easy to get a good general education. You get a very good education from your teacher, from your professor, but if you want to learn some additional things, there are few courses in some of them. My daughter went to Yale, and she had this problem – no courses in Yale higher than multivariate calculus.

At Harvard, [David] Mumford organized a special program of many courses in analysis and algebra. All graduate students have to take some of these courses to get some general education. In Russia, in Moscow University, the department was divided into several so-called "chairs". I was in functional analysis. As a result, I didn't learn too much algebra. So algebra was something I learned later.

I There were no courses in Soviet Union?

B There were many courses but there were no compulsory courses. For instance, I have never taken a course in algebraic geometry. I haven't had this course because I specialized in the direction of analysis. I was formally in the "*kafedra*" [chair] of functional analysis. There was a *kafedra* of algebra. In this *kafedra*, they had compulsory courses on algebraic geometry. I did not go to this *kafedra*. As a result, in some sense, I actually learned the algebraic geometry much later when I taught in Harvard.

I It seems that after doing significant work in one area David Hilbert [(1862-1943)] would leave that area and get into a new area. In this way he made deep contributions to many areas in his lifetime. Your own research spans a number of areas in algebraic geometry, representation theory, number theory and automorphic forms. Do you have a guiding philosophy in research?

B I'm afraid probably not. Somehow, part of this just happens. For instance, I started to work on p -adic groups, mostly because André Weil [(1906-1998)] came to Moscow and gave a course of lectures on representations of p -adic groups. I was very much interested and started to work with Andrei Zelevinsky. We have written two papers on the subject and later he wrote his famous paper [*Induced representations of reductive p -adic groups. II. On irreducible representations of $GL(n)$* , Annales Scientifiques de l'École Normale Supérieure, Série 4, 13 (2) (1980): 165–210].

As for automorphic forms, I started much later, partially under the influence of Kazhdan. In 1984 I gave a course on Eisenstein series at Harvard, and then I really got interested in automorphic representations. Somehow, I did not have a philosophy. It was just that some things were interesting to me, and then I did them.

I The physicist Paul Dirac [(1902-1984)] attached great importance to beauty in formulating a physical theory. Presumably this kind of beauty could be mathematical beauty. For mathematics, does it make sense to use beauty as a guide in formulating mathematical theories, since one mathematical theory would seem to be as valid as another?

B Probably in the beginning that was not so important for me. But for me now, beauty in mathematics is one of the principal guiding principles. First of all, I disagree with the last statement that one mathematical theory seems to be as valid as another. That is what I am explaining to my students. One cannot argue with the definition. If you give a definition, it is a definition. But you can argue if this definition makes mathematical sense, whether it describes some interesting mathematical object. It is not a mathematical question; it is a meta-mathematical question. I cannot describe what it means by "an interesting mathematical object." Every definition describes some mathematical object. Most of them are, of course, completely not interesting, but some of them are interesting. This is the question, and this question is not mathematical.

The criterion is that you start developing this notion and you see that it is beautiful. So, as a result, beauty is one of the central things. In my work many times I would see some interesting idea and would not know whether it is useful for something. Usually I decide that if it is beautiful, then it will be eventually useful.

I Is there such a thing as ugly mathematics?

B No. In fact, I also thought about it. I give you some examples. What is a real number? I prefer an axiomatic approach to real numbers. You define real numbers axiomatically. Then you work with them, do all the analysis and so on. Now you would like to construct a model of real numbers. What do we mean by a model? It should be something expressed in terms that we know, in terms of integers. You want to describe the real numbers, which have nothing to do with integers, in terms of integers. You can formally do it. For example, you can define real numbers using Dedekind sections [Richard Dedekind (1831-1916)]. However there is no reason to expect that this construction would be beautiful, and it is quite ugly.

Beauty is very important, but on the other hand, some things are ugly, and I am trying to explain that there are some reasons for this. Ugly, because you are trying to prove the existence of some very sophisticated objects in elementary terms. And this is very difficult to do. But in some sense, it does not matter. When you have constructed it, this is beautiful, and it works. But when you want to just prove its existence using very simple axioms, you cannot do it in some simple-minded way.

I In other words, you have to do some sort of not so beautiful calculations?

B Yes. From my point of view, computations are very often needed in order to give you some new idea. If you want to compute some special case, you just compute it. It is often not very pleasant computations. But, in the end, you get some results that you did not know in advance. As a result, you get some new information and you can use it to better understand the picture that you see.

I I remember you mentioned in your biographical notes that often the conceptual meaning is not understood until 10 years, 20 years, or even 100 years later. How do you explain that?

B Mathematics is getting more and more sophisticated, and deals, from my point of view, with more and more central objects which are very far from everyday experience. For instance, in what I wrote in this autobiography, the L -functions first appeared as a technical tool in Dirichlet's work [Johann Peter Gustav Lejeune Dirichlet (1805-1859)]. And now it is clear that these L -functions are very central objects in mathematics. But even to formulate what is an L -function is difficult. We know this thing, we see that it has these properties; so probably this is an L -function. But there is no definition of what is an L -function. So we see that first L -functions were introduced as a technical tool, and it took about 100 years to realize that they, in fact, are central objects of study in mathematics.

I Do you consider yourself to be a Platonist who believes that mathematical reality exists in the universe waiting to be discovered?

B Yes. Here we are on two levels. First of all, there is the physical level. It is obvious that I am not trying to invent things. I'm trying to understand how things exist and they are all somewhere out there. I like to formulate these mathematical structures. They are out there in Nature. In order to discover them you have to dig deeper and deeper. I feel they exist out there. On the other hand, I wouldn't put this in a Platonic world of ideas. I feel they are real; they are part of natural science, a very deep part of natural science. It is not easy to explain this world of ideas. I think that ideas like ideas of groups, automorphic functions and so on are part of natural science, but just deeper and more removed from our world.

I Supposing there were aliens living in another part of the universe, would their mathematics be the same as ours? If mathematical reality is real, shouldn't it be independent?

B I think, yes, but I wouldn't put a strong argument for it. Mathematics, at the deeper level, should be the same.

I How much of your teaching has contributed to ideas in research?

B First of all, in Moscow I didn't teach. When I came to the States, I started to teach. It turned out that I can do this well and that I like to do this. It requires a lot of work, but the effort is very satisfying. Also, as a result of my teaching, I learned a lot of things. Before that I knew many things and how to use them but I didn't know them from the bottom up. When I teach, I often find new points of view. Practically every course that I teach, even if I had taught it several times, I prepare it anew and often discover new things and ideas. For me, I feel that it is very important to teach.

I When you went to Israel did you experience a kind of culture shock?

B No. I have visited Israel several times after I came to US, practically once a year. I moved to Israel because I loved it, and I continue to love it. But, of course, when I moved from Soviet Union to the States in 1981 I experienced a culture shock. Psychologically it was quite difficult. Moving to Israel was quite different.

I It is generally perceived that many distinguished mathematicians and scientists are of Jewish origin. Is there a connection with the Jewish tradition or culture?

B This is a very interesting question that was discussed by many people. The phenomenon is clearly there, but how to explain it is not quite clear. I think that the tradition of high respect for educated people, that has always been one of the basic features of the Jewish community, played a significant role.

Wiener [Norbert Wiener (1894-1964)] or somebody had the following explanation. In Christian Europe the bright boys often would go to church, become priests or monks, and do not have children. But in the Jewish tradition, bright boys would become rabbis; they would marry and usually have many children. In fact most of the rich Jews would be honored to marry their daughters to a rabbi, so these rabbis often were not poor. The result was that there were many very talented Jewish boys. I do not know whether this explanation is correct. In fact, there are also some other explanations. For example, there exists a rather convincing explanation of this phenomenon based on the prevalence of some hereditary diseases specific to the Jewish community.

BEN JOSEPH GREEN:

ADDICTIVE COMBINATORICS, A PRIME OBSESSION

Interview of Ben Joseph Green
by Y.K. Leong

Ben Joseph Green made important contributions to combinatorics, number theory and analysis, and in particular, to the study of the distribution of prime numbers.



He obtained his BA (winning the Senior Wrangler title) and PhD from Cambridge University under the supervision of Timothy Gowers (Fields medalist 1998). As a Cambridge graduate student, he also visited Princeton University for 9 months. There he met Terence Tao (Fields medalist 2006) and started a famous collaboration that led to the landmark theorem on arithmetic progressions of prime numbers. He was a Fellow of Trinity College, Cambridge between 2001 and 2005, during which he was also EU (European Union) postdoctoral researcher at the Alfréd Rényi Institute in Budapest, PIMS (Pacific Institute for the Mathematical Sciences) postdoctoral fellow at University of British Columbia and a Clay Research Fellow. He was subsequently a visiting professor at Massachusetts Institute of Technology (MIT) before returning to Cambridge as Herchel Smith Professor of Pure Mathematics in 2006. He was a member of the Institute for Advanced Study at Princeton and Radcliffe Fellow at Harvard University. Then, in 2013, he moved to University of Oxford as Waynflete Professor of Pure Mathematics.

In his PhD thesis, he had already solved (and independently by Alexander Sapozhenko) the Cameron-Erdős conjecture

[Peter Cameron, Paul Erdős (1913-1996)] on the upper bound of sum-free subsets of integers. He was awarded the Smith Prize by Cambridge University for part of his graduate work. In a seminal paper of 2005, he improved on a result of Klaus Friedrich Roth (1925-2015) (Fields medalist 1958) and proved that any set of primes with relative positive density contains arithmetic progressions of length 3. Using ideas in this paper and in collaboration with Terence Tao, he extended the result to arithmetic progressions of length 4. Finally, they proved that there exist arbitrarily long arithmetic progressions of primes by building on results of Endre Szemerédi, Daniel Goldston and Cem Yıldırım and using various ideas from combinatorics, ergodic theory and the theory of pseudorandom numbers. This work has given rise to a new area (additive combinatorics) with a fruitful interplay of ideas from many classical areas like harmonic analysis, ergodic theory, analytic number theory, analytic combinatorics, Ramsey theory, random graph theory, group theory and discrete geometry. Green has singly and, in collaboration with others like Emmanuel Breuillard, Kevin Ford, Robert Guralnick, Sergei Konyagin, James Maynard, Imre Ruzsa, Terence Tao and Tamar Ziegler, considerably enriched and advanced this field.

His diverse contributions have earned him numerous awards such as the Salem Prize, Whitehead Prize, Ostrowski Prize, SASTRA (Shanmugha Arts, Science, Technology and Research Academy) Ramanujan Prize, European Mathematical Society Prize, Sylvester Medal, Clay Research Award, Fellow of Royal Society and Fellow of American Mathematical Society. Notable among the more than 200 invited lectures he has given around the world are the following distinguished lectures: Calderón-Zygmund Lectures, International Congress of Mathematicians (ICM) Plenary Lecture, Ramanujan Lecture, Stanford MRC (Mathematical Research Centre) Distinguished Lecture Series, Gauss Lecture, Hadamard Lectures, Weierstrass Lectures and Simons Lectures. He has also given a number of public lectures on mathematics and featured on BBC Radio 4.

In addition to serving on his own college committees, he has actively organized workshops in the United Kingdom and United States. He is Managing Editor of Proceedings of Cambridge Philosophical Society and serves as an editor of Glasgow Mathematical Journal and Journal de Théorie des Nombres de Bordeaux. He has contributed more than 130 reviews to MathSciNet Reviews.

Green was in the Institute for Mathematical Sciences (IMS), National University of Singapore from 14 – 30 May 2016 and was an invited speaker in the Institute's program *New Directions in Combinatorics* (9 – 27 May 2016). He gave a colloquium lecture on *Permutations and number theory* and the *Mini-Course on finite field models in additive combinatorics*. On behalf of *Imprints*, Y.K. Leong interviewed him on 23 May 2016. The following is an edited and enhanced transcript of this interview which traces the path he took from the British and International Mathematical Olympiads to an illustrious career in Cambridge and Oxford. We also get a feel of the passion, if not addiction, behind the search for meaningful patterns among prime numbers, initiated by the ancient Greeks more than 2,000 years ago.

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IMPRINTS

I You were in the British team to the IMO [International Mathematical Olympiad] when you were in school. At what age was your interest and talent in mathematics discovered? Did your IMO training influence your choice of research area for your PhD?

BEN GREEN

G Okay, there are a few questions there. The first question: I was only in the British team (the Olympiad team) in 1994-95 but before that, I think, as early as 1991, I took part in the domestic competition which is called the British Mathematical Olympiad. And I think in the 1991 version of that competition I did much better than my teachers expected. I think I had some mathematical talent when I was much younger in primary school. The second question about whether it influenced my research direction. Not really although if you do Olympiads you do tend to get a taste for problems that are somewhat elementary to state. Perhaps that influenced the way the courses I was most interested in as a first year undergraduate, but probably not so much in my research direction.

I Are the Olympiads geared towards problem solving kind of situations?

G Yeah, I mean, it's quite different from research mathematics in several ways. First of all, the problems tend to have an elementary statement. They tend to avoid concepts even like calculus because some countries don't teach that at school. And, more importantly, if you're given an Olympiad problem you know that somebody at least can solve it in an hour and a half, in principle, and that's very different from research in mathematics.

I When you were doing your PhD, did you have in mind any particular person you wanted to work with?

G Well, I decided to work with Tim Gowers and that was for two reasons really. First, I liked his personality (I had been taught by him as an undergraduate), and second, he had just won a Fields Medal. So it seemed like a very exciting time to be choosing to work with him. Actually, just before I started my PhD I really felt I was making a big choice between two different areas. I thought about working in algebraic number theory but in the end, I decided to work with Gowers.

I It seems that Paul Erdős was in Manchester for a postdoctoral appointment in 1934. And he also spent time unofficially in London, Cambridge and Bristol. I bring this up because Bristol is quite an important university in England, isn't it?

G It's probably one of the top ten universities for mathematics. Right now, it's got a very strong mathematics department. It hasn't always been so.

I The famous physicist Paul Dirac [(1902-1984)] was born in Bristol. He was an undergraduate at the University of Bristol. Though you were born in Bristol you went to the University of Cambridge for your BA and PhD. But then you later took on a professorship of mathematics at the University of Bristol for about one and a half years (2005-2006). Were there people working in your field at that time?

G Not particularly although that was a point in the history of mathematics at Bristol where there was a great deal of changes in Bristol. They appointed a lot of people, all at that same time, just after me, in areas like number theory. For example, I arrived in 2005 and then shortly after that they appointed [Harald Andrés] Helfgott who later solved [in 2013] the ternary Goldbach conjecture [Christian Goldbach (1690-1764)] and Jonathan Pila who was later an ICM plenary speaker, and two other people (Tim Browning and Andrew Booker) who are now both professors in number theory there. They really were trying to build up number theory at that time. Before that, they did have a couple of people like Jon Keating and Nina Snaith, who had been there a little bit longer and they did some very nice work on the Riemann zeta function. But a little bit further back than that, Bristol, I think, really wasn't a place that people would go to, to do number theory or combinatorics. I think Erdős was there because Hans Heilbronn [(1908-1975)] was a professor or reader there.

I Did you have some kind of sentimental or social attachment when you went to Bristol?

G I mean, my mum and my dad (who was still alive then) both lived in Bristol. My mum still lives in Bristol. I like Bristol as a city but it was really just a coincidence actually. I just got offered a job there.

I You were the first Herchel Smith Professor of Pure Mathematics at the University of Cambridge for seven years, which is quite long actually, before you moved to Oxford in 2013. Why this move?

G I had spent really, with the exception of a short period in Bristol, more or less nearly twenty years in Cambridge. And I felt I'd like a change but also I thought that mathematics in Oxford was moving in a very positive direction. There're a lot of extremely good young people at Oxford doing exciting work. Even more have arrived recently; we have Peter Keevash (one of the other speakers who arrived here [IMS] after me), James Maynard who did some pretty sensational work on prime numbers and Andrew Wiles as well. [Wiles gained world-wide fame for his proof of Fermat's Last Theorem in 1995.] They've got a brand new building which is extremely impressive and beautiful. So there are many reasons to move there, I felt.

I What's the difference between Oxford and Cambridge from the point of view of research?

G They're both very good research universities. In terms of the nature of being a professor there it's quite similar. In some ways, Oxford is bigger because it's something to do with the way that the university and colleges share their positions. There are more faculty positions in Oxford. Probably, on the whole, the undergraduates at Cambridge are a little bit stronger because most of the Olympiad people tend to go to Cambridge but we also have very good undergraduates at Oxford.

I Has the British system changed very much? I remember that in the old days the PhD students are usually left very much on their own.

G I think that certainly was also the case when I was a student, but then that's what I wanted. I think it was probably also the case for some students who didn't want that. At Oxford we're quite professional about making sure that our students are progressing. They have to jump over various hurdles and take various extra courses. So it's not really possible for students to sort of "disappear".

I Are there sort of formal courses which PhD students should take?

G They are supposed to take a few extra courses; they can choose them. And then there are a couple of intermediate examinations; they have to check that they're making good progress.

I How did you get to collaborate with Terence Tao on the famous theorem of arithmetic progressions of primes of any length?

G Well, I think, first of all, I was lucky to be around in the early days before Terence Tao was super famous. I think nowadays it's probably quite hard to start a collaboration with him. But in those days, I think I first met him when I was at Princeton for a year as a visiting graduate student and he was visiting for a couple of days or something and I chatted to him. Then I later asked him for a letter of recommendation for a job (actually a lectureship at Cambridge) which I didn't get, but somehow in the course of that email correspondence I had with him, we started talking about some mathematical ideas that eventually led to this theorem.

I You were actually working on that already earlier on, isn't it?

G That was our first joint paper. We have now finished writing that paper up. Yet it was a very nice kind of collaboration because what we found essentially is that we both had separate ideas that could be combined. But it didn't give the result immediately. But we both brought separate things in and then added some new things to get this result.

I Was part of this collaboration done by email?

G A little bit but actually I did visit him when the collaboration happened. I was in Vancouver for a year, in UBC [University of British Columbia]. I did get out to visit Terry for a couple of weeks and that was really when things started. So it's very hard to start a collaboration by email. You can continue a collaboration and write papers by email, but to really just start something, that's a bit tricky.

I How was it like to work with him?

G Well, I mean, it's a very interesting experience. He is obviously extremely smart. Sometimes it can be hard to keep up. Also, papers can tend to get written very, very quickly. But, you know, other people can make a contribution but I do feel I've made a decent contribution to our collaboration. A lot of his other co-authors are also serious collaborators. One shouldn't assume that because Terry ties up the paper, he wrote the paper. That's definitely not true.

I I think your name came first in the paper.

G Well, I have to sometimes explain this to people in the biological sciences and so on because, you know, in some sciences (chemistry or biology) the order of the authors is not alphabetical. In pure maths it's always alphabetical. But there the first author is sometimes the person who has the grant or maybe sometimes he's the most important author. Yes, I'm the first author, and as in all of my papers, that's purely an alphabetical phenomenon.

I Does your proof of the existence of arithmetic progressions of primes allow one to construct them?

G Absolutely not because we don't even have a way of constructing primes.

I So it's sort of an existential proof?

G Yes. I mean, forget about arithmetic progressions of primes. Just suppose I want a million primes – not even in arithmetic progression – how do you construct that?

I I think that Euclid's original proof of an infinite number of primes is also non-constructive.

G That's right. I mean, you don't have to construct something to prove that something exists and there are many, many examples in mathematics like that.

I Has there been any application of the result on arithmetic progressions among primes to other areas of mathematics or computer science?

G Well, the techniques have resonated particularly in theoretical computer science. The actual result, probably not. Most results in number theory don't have direct applications although curiously, if we have proven the result in the 1970s ... I'm not sure about the details of the story ... there was a point in time with the proof of Hilbert's 10th Problem [David Hilbert (1862-1943)], which

is the statement that there's no algorithm for solving all Diophantine equations ... before that was solved unconditionally, it relied upon the existence of arbitrary long progressions of primes. There was a paper of Julia Robinson [(1919-1985)] but subsequently that [the proof] was made unconditional. So it's like a curious fact.

I Supposing you were to communicate with somebody in outer space, assuming there's another civilization in outer space, do you think it is a good idea to send them a message in terms of a list of primes?

[In the science fiction novel *Contact* by the astronomer Carl Sagan (1934 – 1996), an extraterrestrial message consisting of prime numbers was received on earth.]

G That is an interesting point. It's probably a very good message to send somebody, to prove we are an intelligent race, a list of primes in binary.

I I believe there are people who look at the distribution of primes and their gaps from a purely statistical point of view. What are some resulting conjectures? Has there been any progress in such conjectures?

G There's a big theory of gaps between primes – various different questions you can ask about whether there are small gaps, and there is something called the twin primes conjecture which says that the gap between consecutive primes is two infinitely often. I think maybe what you're asking is about large gaps where people assume that the primes behave somewhat randomly and speculate about what the size of the largest gap is. There's a conjecture called Cramér's conjecture [Harald Cramér (1893-1985)] which was actually modified a little bit by [Andrew] Granville and says that the largest gap between primes up to x should be around about $(\log x)^2$. And that's a hopelessly difficult problem to actually prove rigorously, either lower or upper bound. The upper bounds are miles and miles and miles away. The lower bound is really quite a long way away as well although there was some relatively recent progress on the lower bound.

I But you are assuming that the primes are random, is it? How do we know?

G Well, we don't know that the primes are random but when we make conjectures about the primes, there are various ways of guessing what should be true for the primes by assuming that they exhibit some random-like behavior. It doesn't make mathematical sense really to say the primes are random. But you can look at them from different angles and sort of say "Well, they behave in this way like a random sequence would behave and let us make some conjecture."

“I GIVE VARIOUS BITS OF ADVICE TO MY PHD STUDENTS. ONE OF THEM IS TO NOT TO GET FIXATED ON ONE PROBLEM, BUT TRY TO WORK ON A FEW DIFFERENT PROBLEMS, AND JUST FOLLOW YOUR NOSE IN TERMS OF WHAT IS INTERESTING. BUT ALSO, ONE SHOULD TRY TO GET A SENSE OF WHAT OTHER PEOPLE THINK IS INTERESTING. THERE’S NO POINT JUST WORKING ON THINGS THAT ONLY YOU THINK ARE INTERESTING.”

I Purely empirically, if you look at the sequence of primes, can it be sort of reasonably shown that statistically they are actually random in distribution?

G In a certain sense, yes, but it depends what you mean by “random”.

I Sometimes the gaps can be so big.

G We can’t say too much about the gaps but we might be able to say things like roughly what the density of the primes is in long intervals, for example. We can say things about what remainder the prime leaves when you divide them by 7, for example. That looks fairly random.

I There are so many tantalizing problems in combinatorics and number theory. How do you pick the problems to work on?

G For me, first of all, the problem has to be simple to state. I’m usually not interested in problems with a very complicated and artificial statement. But the other thing is that I try to pick problems to work on that I think I have some chance of actually saying something about.

I Do you believe in choosing the hard and important problems?

G Not always. This is something I try to explain to my students. Sometimes you can’t just work on the hard problems. You sometimes have to work on some problems that might appear like puzzles or just very specific smaller versions of problems that you actually care about. They have to try to advance a tiny bit at a time. Very often, what I do is try to figure out what’s the phenomenon I don’t understand, and then I try to find a really simple problem that just captures that phenomenon. And then, maybe, I can generalize it to other situations.

I Traditionally, there is the well-known discrete-continuous dichotomy in mathematics. Recent developments have, however, seen the successful use of

probabilistic methods in shedding light on and resolving various problems in combinatorics. Do you think probability might be the vital link in connecting the discrete with the continuous?

G Probability isn’t necessarily continuous. Often when you talk about a probabilistic argument in combinatorics you have a discrete sample space and everything is really just a language to talk about counting discrete sets. However, often you are thinking about a large number of random variables and you might think about how that behaves in the limit as the number of variables tends to infinity and then what you’re really doing is you’re passing to a continuous limit of the problem. So, yes, it is a kind of a bridge between the discrete and the continuous sometimes.

I Probability itself also has a continuous concept in analysis and all that.

G Often continuous things arise as limits of discrete things. So it’s certainly a useful point of view when working with discrete objects to try to form, if you’ve got increasingly large discrete objects, some limit of them which can sometimes be a continuous object. That can be a useful way to argue.

I To solve problems in the discrete is usually very difficult, isn’t it?

G It depends on the kind of problems; sometimes it’s very easy. There are many examples where if you prove a theorem about continuous functions or about differentiable functions, really underlying it is a statement that’s actually very easy for discrete functions.

I Traditionally, a lot of discrete problems are re-interpreted in terms of continuous things like differential equations. Do differential equations play a part in combinatorics?

G Differential equations arise in parts of mathematics that I care about, particularly, for example, in what’s called sieve theory in prime number theory, but I don’t really think of them as a conceptual way to understand how the mathematics that I care about fits together.

I Combinatorial methods are widely used in other mathematical areas such as number theory, group theory, set theory, topology and geometry. Yet they are often accepted grudgingly as a necessary evil to be avoided, if possible. How has this perception changed in recent years?

G Well, that’s interesting. I think you may be right in a sense. I mean, of course, it’s hard to know what is meant by combinatorial methods. Sometimes when people speak of combinatorial methods what they really mean is just some sort of hands-on elementary methods.

But I do think that it's certainly the case that the power of combinatorial methods has become more evident in the last twenty years or so. I think it's something that people have suspected for a while. But maybe people haven't really realized that more combinatorial methods could actually give serious results in number theory.

I About the term "additive number theory". Is additive number theory a subset of additive combinatorics?

G Yes, sort of. I think "additive combinatorics" is a really silly name because it doesn't really describe what the subject is about, but yet somehow additive number theory is what one thinks of as a fairly traditional set of problems to do with maybe adding squares, adding primes, questions about bases of the integers and so on. I think additive combinatorics certainly includes those but also includes a lot of other topics that don't really have a lot of relation with those. So I sort of think, sometimes, of additive combinatorics as the study of approximate structures, which is a possible definition.

I I think there is another term called additive prime number theory.

G Additive prime number theory is part of number theory, which focuses on the primes.

I You have given numerous public lectures in the United States, England and Germany, and even appeared in the forum on BBC Radio 4. This is an area of activity which most research mathematicians, especially young researchers, would shy away from. Is there any memorable experience that this kind of activity has given you?

G Well, certainly going on the radio is quite memorable. I quite like giving this kind of lectures. I have given a few of them. Yes, I quite enjoy it but I think it is important to try and explain to the general public what you're doing for several reasons. One is that the public pays for research through taxation. But it's very important whenever possible to give young people the idea that mathematics is something that is interesting to study. We do really need more people to study mathematics. In the UK we have a terrible shortage of people to teach maths at school. And, of course, if you don't teach maths properly at school, there're so many areas that people don't get into, like engineering, computer science and that they really can't do them if they are not properly taught in maths. I don't think you have the same problem here. I think you probably have much better maths teachers.

I It's very hard to get research mathematicians to voluntarily give lectures to the public.

G Well, most people are very bad at it, but then for most people their own research is often on a topic that's not very suitable for the public. So, as it happens, talking about prime numbers is something that's quite easy for a general audience. I could sometimes give talks with no equations being given at all. A lot of research mathematics can be really quite difficult to explain to the general public.

I I believe your appearance on BBC radio was about primes, isn't it?

G I can't remember what I talked about. It's a very strange program actually. It was basically four quite random people who were expected to talk about their vision for the future.

I What advice would you give students who would like to do combinatorics or number theory?

G I give various bits of advice to my PhD students. One of them is to not to get fixated on one problem, but try to work on a few different problems, and just follow your nose in terms of what is interesting. But also, one should try to get a sense of what other people think is interesting. There's no point just working on things that only you think are interesting.

I Do you collaborate a lot with others?

G I have written a lot of joint papers. At the moment I'm not doing that but I collaborated with a couple of my PhD students quite recently.

I I would like to ask you something about the topic you are giving in this program. You were talking about finite fields as a model for additive combinatorics. Are you formulating the problems for finite fields rather than numbers?

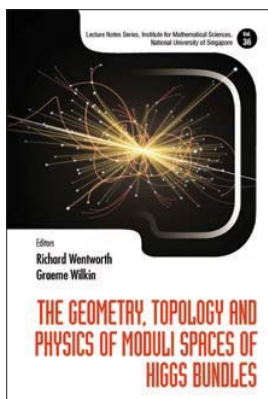
G That's right. The reason for doing that is that often the techniques are cleaner, easier in that setting. That makes for a nice lecture course. But somehow you're not cheating too much because many of the techniques can be modified to work with integers.

I Could you give us an example of a problem that is not solved for integers but is readily solved when formulated for finite fields?

G Yes, there was a recent breakthrough by [Ernie] Croot, [Vsevolod] Lev and [Peter] Pach (it happened just a week before the workshop here in Singapore) where they used methods that are very specific to finite fields, obtaining a hugely better bound on how big a set can be without containing 3-term arithmetic progressions.

VOLUME 36

The Geometry, Topology and Physics of Moduli Spaces of Higgs Bundles

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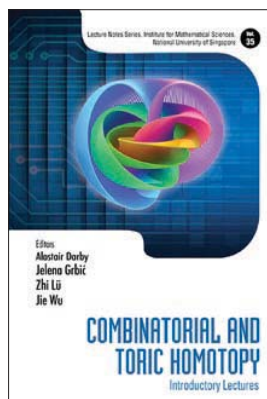
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In the 25 years since their introduction, Higgs bundles have seen a surprising number of interactions with different areas of mathematics and physics. There is a recent surge of interest following Ngô Bau Châu's proof of the Fundamental Lemma and the work of Kapustin and Witten on the Geometric Langlands program. This volume is a collection of the lectures hosted during the program on *The Geometry, Topology and Physics of Moduli Spaces of Higgs Bundles*, held at the Institute for Mathematical Sciences at the National University of Singapore during 2014.

VOLUME 35

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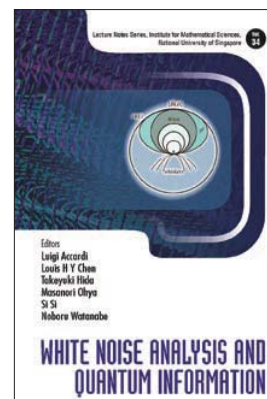
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