IMPRINTS

NUS National University of Singapore | IMS Institute for Mathematical Sciences

NEWSLETTER OF THE INSTITUTE FOR MATHEMATICAL SCIENCES, NATIONAL UNIVERSITY OF SINGAPORE

JULY – DECEMBER 2017 ISSUE 30

DATA SCIENCES: BRIDGING MATHEMATICS, PHYSICS AND BIOLOGY

From 29 May to 16 June 2017, the Institute hosted the program "Data Sciences: Bridging Mathematics, Physics and Biology". The program organizers contributed this invited article to *Imprints*.

BY SAY SONG GOH, HUI JI AND PATRICE KOEHL

Advances in technology and the ever-growing role of digital sensors and computers in science have led to an exponential growth in the amount and complexity of data that scientists collect. We are at the threshold of an era in which hypothesis-driven science is being complemented with data-driven discovery. This alternative way to pursue research is especially visible in modern biology, with the advent of genomics and the development of multiple imaging techniques to visualize living organisms at multiple time and length scales. The data collected are complex in size, dimension, and heterogeneity. These data provide unprecedented opportunities for new discoveries, but also come with challenges that need to be addressed. Solving those challenges requires expertise from multiple disciplines. There is a need to develop new mathematical models for formalizing the information content of data, and a need to develop novel, efficient algorithms for dimensionality/complexity reduction, tools for statistical analysis, as well as approaches to data exploration and visualization. The two workshops on "Frame Theory and Sparse Representation for Complex Data" and "Geometry and Shape Analysis in Biological Sciences" held amidst the program from 29 May to 16 June 2017 at the Institute for Mathematical Sciences aimed to illustrate and promote such an interdisciplinary framework. The workshops followed the full workflow of modern data analysis, including topics on advanced signal processing techniques for analyzing experimental data, topics on geometric and statistical data analyses, and applications to biological problems.

From left: **Hui Ji, Say Song Goh** and **Patrice Koehl**

Sparse representations of high-dimensional data sets are of fundamental importance in many aspects of data sciences [1]. A succinct expression of a large and complex data set not only increases the amount of practically accessible data but also offers insight to information that may be hidden in the raw data. A sparse representation of input data requires decomposition of the data under an appropriate system to reduce its size by orders of magnitude and yet without negative impact on its important information. Such a dimension reduction is often more likely to occur when a redundant system, in particular a frame, is used. Frames, especially wavelet frames [2], have been one of the main systems used in signal and image processing for compact representations of signals, with many successful applications [3]. In recent years, various new types and quantities of complex data sets have emerged in scientific research [4], such as high-dimensional data from the bio-imaging sciences, large graph data from social networks, and data on manifolds from brain imaging. This emergence raises new challenges in frame theory and sparse representation. The workshop on Frame Theory and Sparse Representation for Complex Data (29 May – 2 June 2017) provided a platform for the interaction among mathematicians, computer scientists and applied scientists, with a focus on representation and computation related to data sciences. It covered topics such as wavelet theory and its applications, representation and computation of graphical data, dictionary learning and its applications, optimization techniques for non-convex problems, imaging sciences and robotics, neural network and deep learning, and artificial intelligence in medicine.

A highlight of this workshop was the mathematical understanding of deep neural network. Deep neural network is one of the highly promising tools in data sciences, owing to its modeling power of big data [5]. However, a thorough understanding of deep neural network is still lacking, and it is often treated as an art with tremendous engineering effort for good performance. There were several presentations in the workshop on mathematical results advancing towards the understanding of deep neural network from different perspectives. In data sciences, many frequently encountered problems are formulated as non-convex problems. The design of new algorithms to find global minimizers for these problems is critical [6]. Another highlight of the workshop were the presentations on the optimization techniques for solving certain large-scale non-convex problems, providing both algorithms and analyses.

In any scientific experiment, devices are often used to provide insight into cause-and-effect between the parameters that control a system and the observations that are made on this system. The data generated by modern experimental techniques that are now ubiquitous in science come with a high level of complexity and heterogeneity, usually providing indirect measurements of hidden, albeit essential, processes that are keys to the systems being studied. Topological data analysis is a recent and promising approach to the analysis of such data, using techniques from geometry and topology [7, 8]. Extraction of knowledge from data that are high-dimensional, incomplete and noisy raises many challenges for data scientists. Topological data analysis provides a general framework to analyze such data. It is somewhat insensitive to the metric chosen for comparing the data points. It also provides dimensionality reduction and robustness to noise. The underlying assumption of topological data analysis is that shape matters, i.e., the underlying connections between the data lead to geometric and topological patterns. One part of the workshop on Geometry and Shape Analysis in Biological Sciences (12 – 16 June 2017) was therefore dedicated to recent advances in the field of topological data analysis, including the development of techniques for detecting persistent and dynamic structures within data sets.

The problem of comparing and aligning two shapes is also referred to as surface warping, search for best fit, and shape analysis. It is used widely in radiology, computer vision, biological imaging, brain mapping, target recognition, and satellite image analysis. In molecular biology, the notion that the structure (or shape) of a protein is a major determinant of its function has led to extensive development of methods for representing, measuring and comparing protein structures [9]. In another setting, brain morphometry plays an important role in neurobiology. It is concerned with the measurement of the brain's geometry and the changes that occur during development, aging, learning, and disease evolution [10]. As a result, it relies heavily on recognition and comparison of surfaces in 3D. In evolutionary biology, the quantitative analysis of forms, or morphometrics, has great potential to answer basic questions about the relationship between species [11, 12], complementing the great success of phylogenetics in this field [13]. This ubiquity of applications is based on the observation that we live in a 3-dimensional world in which many interactions between objects are highly influenced by the geometry of their exteriors. The second part of the workshop on shape analysis was consequently focused on analyzing the geometry of shapes described by their surfaces. It included presentations that were related to theoretical and practical aspects of this problem, covering topics such as shape parameterization, shape correspondence, and the construction of a metric in shape space.

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ALICE CHANG
NEW SCIENTIFIC ADVISORY BOARD MEMBER

The Institute is pleased to welcome a new member to the Scientific Advisory Board (SAB) – Professor Sun-Yung Alice Chang (Princeton University).

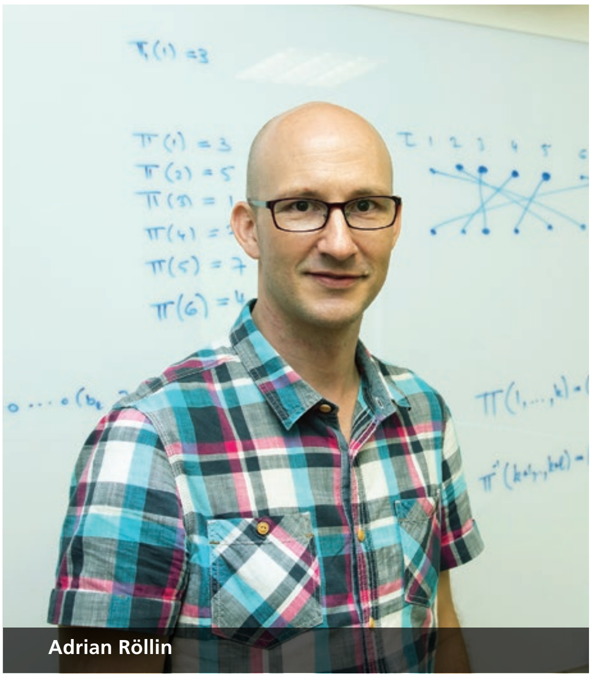
Professor Chang is Eugene Higgins Professor of Mathematics at Princeton University. Her research interests include harmonic analysis, geometric analysis and partial differential equations. Professor Chang was an invited speaker at the International Congress of Mathematics in 1986 and a plenary speaker in 2002. Her honors and awards include the Alfred P. Sloan Foundation Fellowship, Guggenheim Foundation Fellowship, and the Ruth Lyttle Satter Prize in Mathematics (1995) by the American Mathematical Society. In 2013, Université Pierre et Marie Curie (UPMC) awarded Prof. Chang their honorary doctorate (Doctor Honoris Causa). She is a member of the American Academy of Arts and Sciences and Fellow of the National Academy of Sciences (2009), Academia Sinica (2012) and American Mathematical Society (2015).

The Institute looks forward to strengthening its scientific programs further under the new and incumbent members of the SAB.

Kwok Pui Choi
National University of Singapore

Alice Chang

NEW SAB MEMBER


Adrian Röllin

ADRIAN RÖLLIN
NEW DEPUTY DIRECTOR

Associate Professor Adrian Röllin of Department of Statistics and Applied Probability (DSAP) will join the Institute as our Deputy Director from 1 January 2018. Associate Professor Röllin's area of research interests are in the area of probability theory, in particular distributional approximations via Stein's method and mathematical and statistical modeling of infectious disease processes.

Former Deputy Director Associate Professor Kwok Pui Choi, who was with the Institute from 1 January 2015 – 31 December 2017, has relinquished his position to resume full-time duties at DSAP. We would like to express our heart-felt thanks and appreciation to his efforts and contributions for the past three years!

PERSONNEL MOVEMENTS

The Institute's housing officer Lee Jia Ling gave birth to a boy on 17 August 2017. Rajeswri, former operations associate, left IMS in August 2017. The Institute takes the opportunity to thank Rajeswri for her service and wishes her success in her future endeavors.


Baby Zander

IMS DISTINGUISHED VISITOR LECTURE SERIES

Speakers invited to this set of lecture series are prominent leaders in their fields. They were invited to participate in various activities of the Institute. Lectures are intended to highlight important developments in the field.

9 MAY 2017

Professor Bo Berndtsson (Chalmers University of Technology) delivered two lectures "Lelong numbers for singular metrics on vector bundles" and "Curvature of higher direct image bundles".

18 AND 19 MAY 2017

Professor Yum-Tong Siu (Harvard University) delivered a two-part lecture series on "Analytic Methods of Constructing Bundle Sections and their Geometric Applications".

29 AND 30 MAY 2017

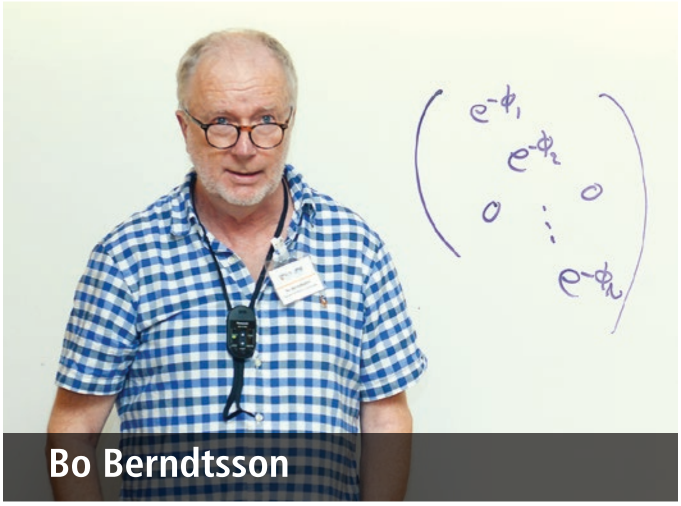
Professor Stanley Osher (University of California, Los Angeles) gave two lectures on "Overcoming the Curse of Dimensionality for Hamilton-Jacobi equations with Applications to Control and Differential Games" and "What mathematical algorithms can do for the real (and even fake) world".

14 AND 15 JUNE 2017

Professor Ingrid Daubechies (Duke University) gave two lectures titled "Mathematicians helping art historians and art conservators" and "Biologically relevant distances between morphological surfaces representing teeth and bones".

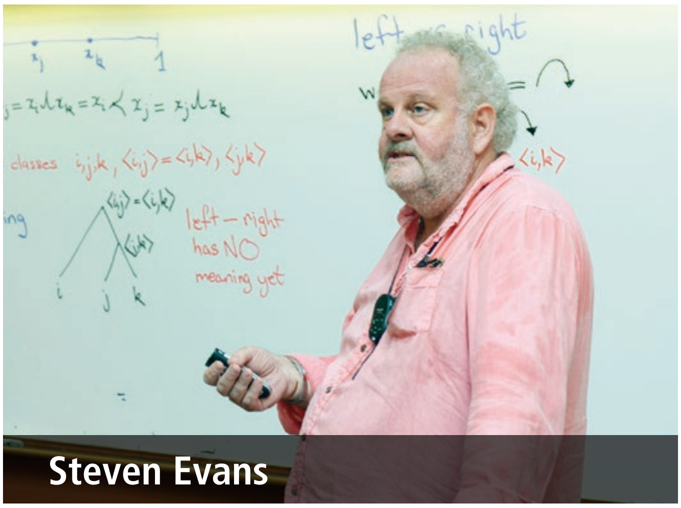
7 AUGUST 2017

Steven Evans (University of California at Berkeley) delivered a lecture on "Infinite bridges for Rémy's algorithm".

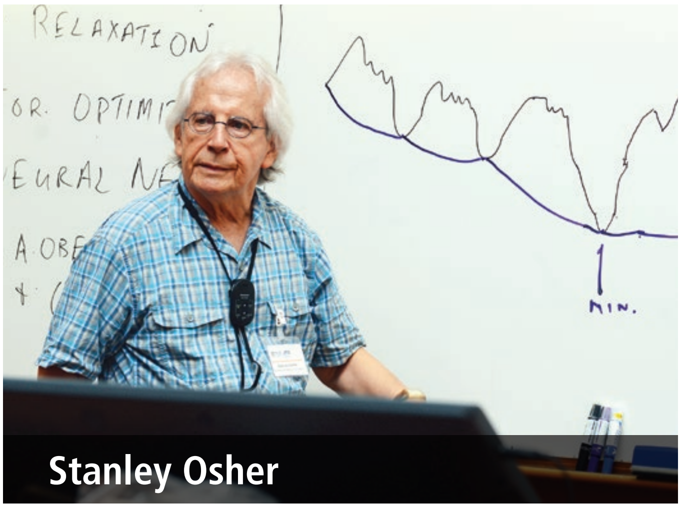


Bo Berndtsson

Ingrid Daubechies



Steven Evans



Stanley Osher

Yum-Tong Siu

Oppenheim Lecture

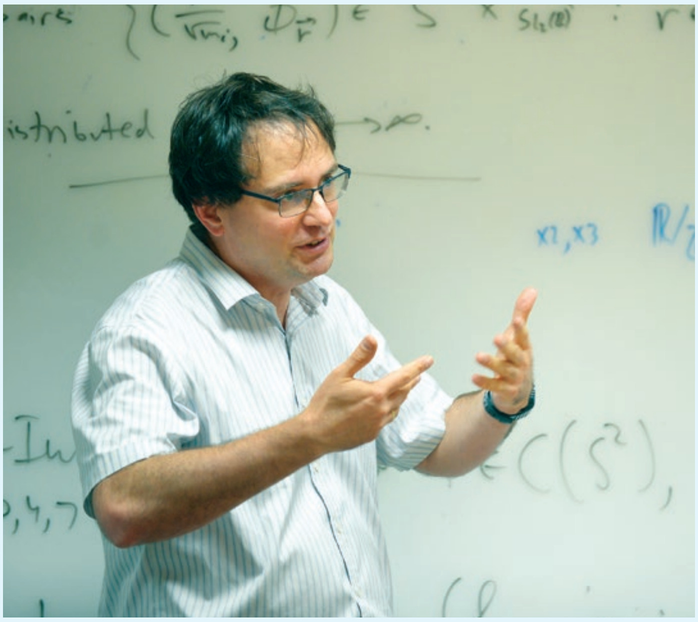
15 FEBRUARY 2017

Jointly organized with Department of Mathematics, NUS

The third Oppenheim Lecture "On an effective proof of the Oppenheim Conjecture" was delivered by Professor Elon Lindenstrauss of The Hebrew University of Jerusalem and Princeton University on 15 February 2017.

Activities held in conjunction with the Oppenheim Lecture included the *Workshop on Ergodic Theory & Dynamical Systems*, which ran from 14 – 16 February 2017 and comprised eight talks, and a two-hour conversation with Professor Lindenstrauss on 16 February 2017.

The Oppenheim Lecture was attended by close to sixty participants and the workshop was attended by fifteen participants.



Elon Lindenstrauss

Conversation with Elon Lindenstrauss

Snacks were wonderfully prepared for the participants at the workshop

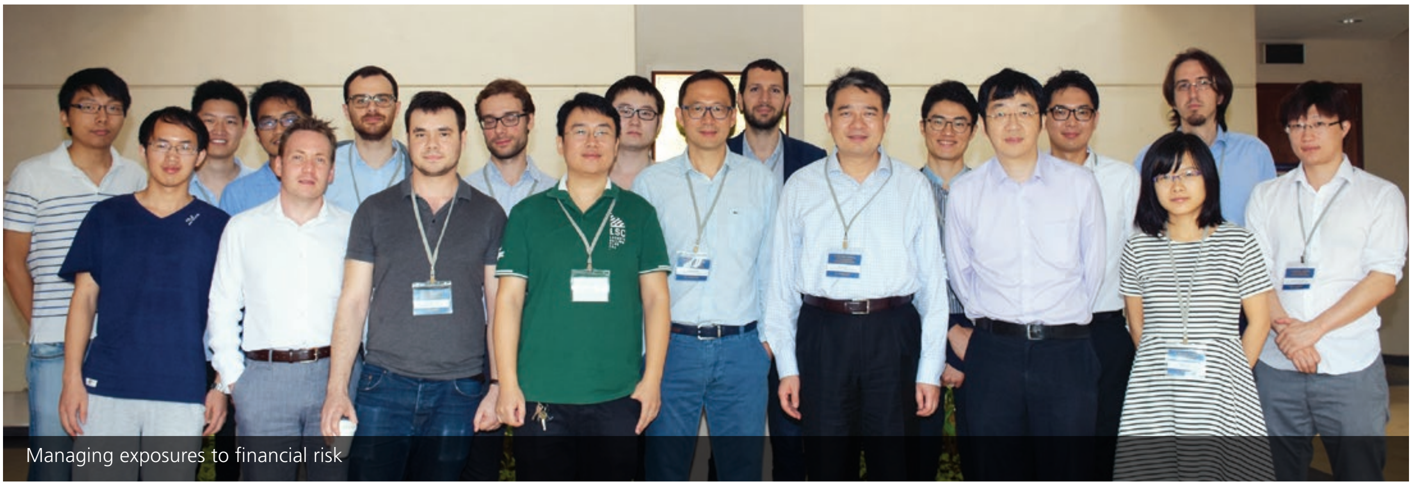
2nd NUS-USPC Workshop on New Challenges in Financial Risk Control

11 – 12 APRIL 2017

Jointly organized with the Centre for Quantitative Finance, NUS

ORGANIZING COMMITTEE

Jean-François Chassagneux | University Paris Diderot
Min Dai | National University of Singapore
Noufel Frikha | University Paris Diderot
Steven Kou | National University of Singapore
Huyên Pham | University Paris Diderot
Chao Zhou | National University of Singapore



Managing exposures to financial risk

This workshop was a collaboration between USPC-University Paris Diderot, the Centre for Quantitative Finance and the Risk Management Institute. It provided a forum for researchers and practitioners to present and discuss issues in financial risk control. There were a total of sixteen talks and well over thirty participants.

PAST ACTIVITIES

Complex Geometry, Dynamical Systems and Foliation Theory

1 – 26 MAY 2017

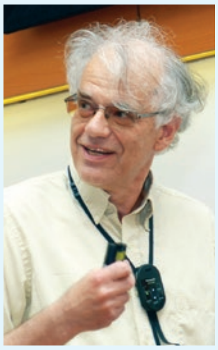
ORGANIZING COMMITTEE:
Tien Cuong Dinh | National University of Singapore
George Marinescu | University of Cologne
Xiaonan Ma | Université Paris Diderot – Paris 7
De-Qi Zhang | National University of Singapore

This program focused on pluripotential theory, a powerful research tool in Complex Analysis, Complex Differential Geometry, Complex Algebraic Geometry, Dynamics and Foliations.

The program started with a mini-workshop on Complex Analysis and Geometry from 3 – 4 May 2017 and had seven talks. The following week (8 – 12 May 2017) consisted of mini courses by Stéphane Nonnenmacher (Université Paris-Sud, France), George Marinescu (University of Cologne, Germany), Min Ru (University of Houston, USA), Xiaojun Huang (Rutgers University, USA) and Yong-Geun Oh (Pohang University of Science and Technology & IBS Center for Geometry and Physics, Korea). There were also three invited talks. The five-day conference, which started on 15 May 2017, had a total of twenty three invited talks.

The week after the conference was set aside for informal discussions. There was also a seminar by Emmanuel Ullmo (Institut des Hautes Études Scientifiques, France) on 24 May 2017.

Eric Bedford: Dynamics of Automorphisms of Complex Surfaces



Misha Lyubich: Dynamics of dissipative complex Henon maps

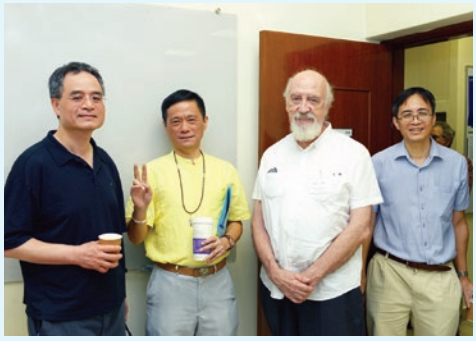
Emmanuel Ullmo: Bi-algebraic arithmetic and bi-algebraic geometry

Results from a feedback survey that at least seven research projects initiated or worked on during or after the program. There were more than eighty participants (69 overseas, 14 local), including 15 graduate students (four local).

Right: **Patrick Ng** and **Erlend Fornæss Wold**: Squeezing value from conjectures

Below: Sharing light moments [from left: **Sai-Kee Yeung, Man Chun Leung, Daniel Burns** and **Wing Keung To**]

Complex dynamic systems analysis



Geometric Structures and Representation Varieties

3 – 5 MAY 2017

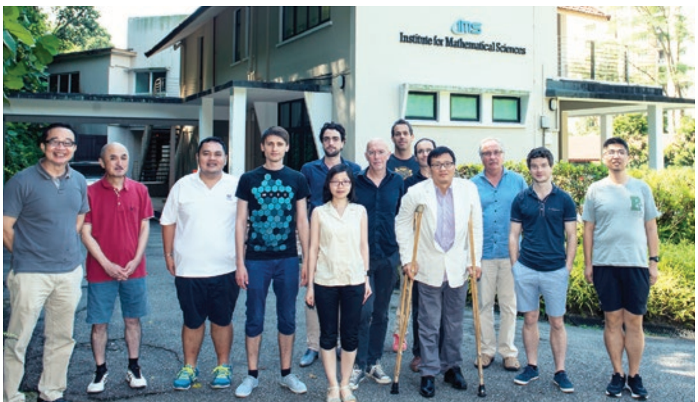
Jointly organized with Fonds National de la Recherche, University of Luxembourg, and Department of Mathematics, NUS

ORGANIZING COMMITTEE:
Jean-Marc Schlenker | University of Luxembourg
Ser Peow Tan | National University of Singapore

The workshop focused on geometric structures on low-dimensional manifolds, representation varieties, and related questions. There were a total of ten talks and more than a dozen participants.

Sadayoshi Kojima: Moduli space of equilateral plane pentagons

Hugo Parlier: Interrogating length spectra and quantifying isospectral finiteness



Meeting of different geometric structures

Data Sciences bridging Mathematics, Physics and Biology

Data Sciences: Bridging Mathematics, Physics and Biology Part I

29 MAY – 16 JUNE 2017

CO-CHAIRS:

George Barbastathis | Massachusetts Institute of Technology
Hui Ji | National University of Singapore
Patrice Koeh | University of California at Davis

Advances in technology and the ever-growing role of digital sensors and computers in science have led to an exponential growth in the amount and complexity of data that scientists collect. The data collected are complex in size, dimension, and heterogeneity – all three generating the generic term "Big Data". These data provide unprecedented opportunities for new discoveries; they also come with challenges that need to be addressed. There is a need to develop new mathematical models for formalizing the information content of data, and novel efficient algorithms for dimensionality/complexity reduction, as well as tools for statistical analysis, and approaches to data exploration and visualization.

Henri Orland and **Patrice Koehl:** Minimum action required for shape description

Discrete uniformization and study of random walks [From left: **Peter Roegen, Feng Luo, Jason Canteralla, Kelin Xia** and **John M. Sullivan**]

The program started with the *Workshop on Frame Theory and Sparse Representation for Complex Data* from 29 May – 2 June 2017. There were 31 invited talks. Moreover, Dong Bin (Peking University, China) gave a tutorial on Mathematical image processing over two days (5 – 6 June 2017), followed by another two-day tutorial (8 – 9 June 2017) on Geometry and Shape Analysis in Biological Sciences by Patrice Koehl and Joel Hass from the University of California at Davis, USA.

The following week (12 – 16 June 2017) was planned with the *Workshop on Geometry and Shape Analysis in Biological Sciences*, and had a total of 23 invited talks and a session on Open Problems in Biogeometry and Biotopology.

There were well over a hundred participants, more than half of which local, including 41 graduate students (37 local).

The second half of the program is currently planned with a *Workshop on Computational Methods in Bio-Imaging Sciences* (8 – 12 January 2018).

Stéphane Mallat: High-dimensional learning and neural networks

René Vidal: Dual principal component pursuit

Herbert Edelsbrunner: Shapes, radius functions, and persistent homology

Reidun Twarock: New insights into virus structure, assembly and evolution

Zuowei Shen: Image restoration and beyond

IMS Graduate Summer School in Logic

19 JUNE – 7 JULY 2017

Jointly organized with Department of Mathematics, NUS

This Summer School bridges the gap between a general graduate education in mathematical logic and the specific preparation necessary to do research on problems of current interest in the subject.

The summer school consisted of 12.5 hours of lectures by Artem Chernikov (University of California, Los Angeles), 6.75 hours of lectures by Steffen Lempp (University of Wisconsin), 10.75 hours of lectures by Theodore A. Slaman (The University of California, Berkeley) and 12.5 hours of lectures by Hugh Woodin (Harvard University).

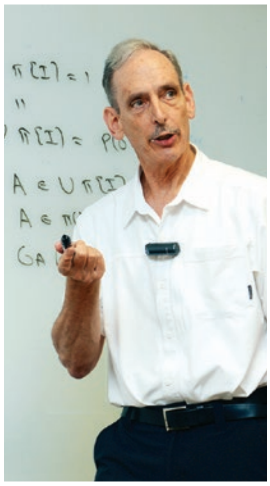
There were more than fifty participants (43 overseas, nine local) and among them 29 were graduate students (28 overseas, one local).

Artem Chernikov:
Geometric stability theory

Huishan Wu:
Reverse mathematics and divisible Abelian groups

Theodore Slaman:
Fragments of arithmetic and their models

Steffen Lempp:
Computable model theory



Hugh Woodin:
The universe of sets and Ultimate L

Above: Sensing the logical strength from a smaller group configuration

Right: Participants of the 2017 Summer School in Logic

Multiple considerations in design, monitoring and analysis of data [From left: **Frank Bretz, Wei-Yin Loh** and **Toshimitsu Hamasaki**]

Quantitative Methods for Drug Discovery and Development

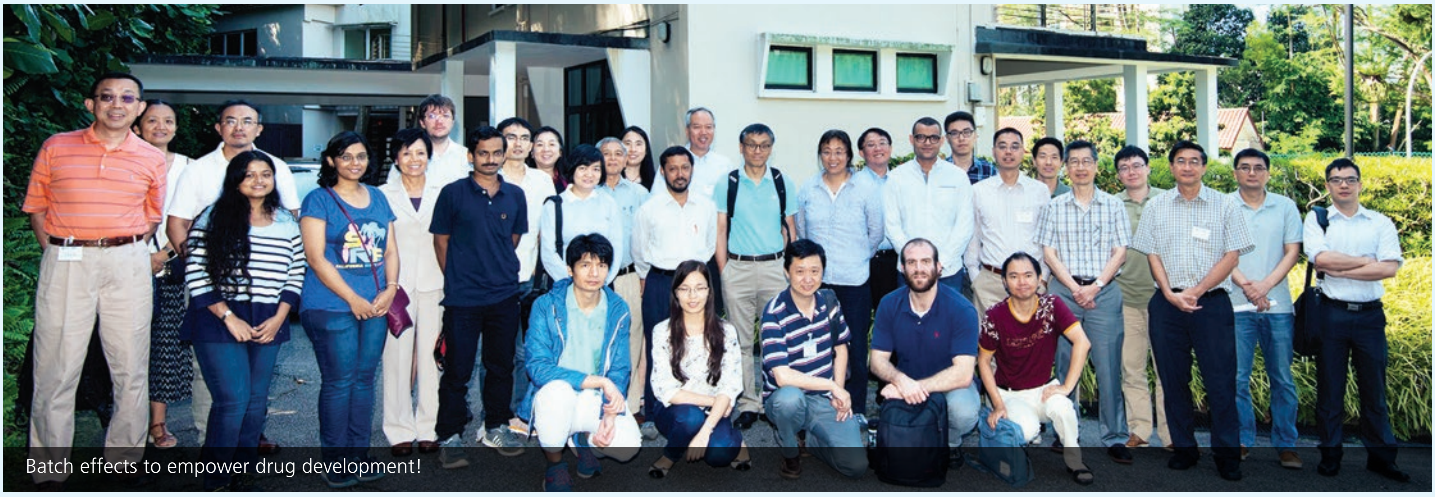
19 JUNE – 14 JULY 2017

CO-CHAIRS:

Wei-Yin Loh | University of Wisconsin-Madison

Weng Kee Wong | University of California, Los Angeles

There has been an increased interest in quantitative methods for discovering and developing new treatments for personalized medicine, which is partly driven by rapid advances in genomics, computational biology, medical imaging, and regenerative medicine. With personalized medicine, there is the problem of subgroup identification: to identify (in terms of patient characteristics) the subgroup of the population for which the drug produces an enhanced effect.



Batch effects to empower drug development!

There is already a U.S. industry working group called "Quantitative Sciences in the Pharmaceutical Industry" dedicated to sharing information for subgroup identification and analysis. The proposed program aims to further this goal as well as to provide the opportunity for academic and industrial statistics professionals to learn from each other. It will also be of interest to health and medical professionals in the pharmaceutical and biotechnology community in Singapore.

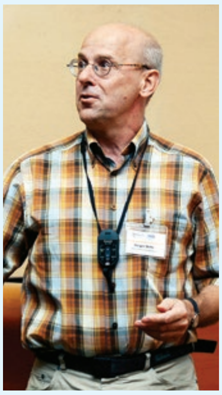
There were two tutorials from 19 – 30 June 2017. Wei-Yin Loh (University of Wisconsin-Madison, USA) lectured on regression tree methods for precision medicine, and Tze Leung Lai (Stanford University, USA) gave a three-hour tutorial on "Medical Product Safety: Biological Models and Statistical Methods".

The theme of the first workshop, *Design of Healthcare Studies* (3 – 7 July 2017) was on the design of experiments for personalized medicine, and had 22 invited talks. The second workshop, *Perspectives and Analysis Methods for Personalized Medicine* (10 – 14 July 2017) focused on the analysis of data from the experiments, with emphasis on identification of patient subgroups with differential treatment effects. It had a total of twenty one invited talks.

There were close to ninety participants (49 overseas, 38 local) including 20 graduate students (seven overseas, 13 local).

Ivan Chan and **Lu Tian**: Opportunities of statistics for innovative methods in drug development

Ying Lu: Designs of dose escalation studies in phase I oncology trials



Holger Dette: Optimal designs for comparing (dose response) curves

Stephen Senn: From hype to scepticism to realism?

Jack Lee: Bayesian adaptive designs in the era of personalized medicine

Weng Kee Wong demonstrating multi-probe applications to functional data

Noah Simon: Adaptive enrichment trials for biomarker-guided treatments

Yik Ying Teo: The promise and peril of healthcare analytics

Yili Pritchett: The utility of adaptive enrichment designs on personalized medicine

Peter Mueller: A nonparametric bayesian basket trial design

Above: Participants benefitted a lot from the unconventional format of the Learning sessions

Left: Building a closer interaction between different particle systems

Genealogies of Interacting Particle Systems

17 JULY – 18 AUGUST 2017

CO-CHAIRS:

Matthias Birkner | Johannes Gutenberg-Universität Mainz
Rongfeng Sun | National University of Singapore
Jan Swart | The Czech Academy of Sciences

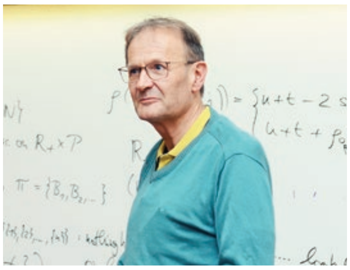
The program aimed to bring together both experts and younger researchers who have worked on or are interested in topics at the intersection of interacting particle systems, population biology, and random graphs.

Apart from planning a workshop with talks by invited speakers, the organizers also planned a new type of activity, which they referred to as *Learning Sessions*, where participants study material outside their own research focus and present it to fellow participants. Participants could thus study the material in-depth, allowing more diversity rather than just listening to a series of individual lectures. There were nine leaning sessions in total from 31 July to 4 August 2017, which covered a diverse range of themes from classical topics such as the continuum random tree and the look-down construction, to more cutting-edge topics such as algebraic duality, Brownian net, etc. Twenty two participants gave presentations in these learning sessions and engaged in lively discussions with many others. Many participants expressed appreciation for this format, stating that they got more out of the learning sessions than the conventional talks.

The following week (7 to 11 August 2017) was planned with a workshop with presentations by 15 participants on their current research. There was an afternoon of activities in the Department of Mathematics on 16 August 2017, which had a colloquium talk by Anton Wakolbinger (Goethe-Universität Frankfurt), a Young Mathematician Lecture by Matthias Hammer (Technische Universität Berlin), and a seminar by Adrian Röllin (NUS).

There were a total of 56 participants (49 overseas, seven local) and among them 12 were graduate students (11 overseas, one local).

Learning session on tree-valued Markov processes [From left: **Matthias Birkner, Jan Swart, Andrej Depperschmidt** and **Andreas Greven**]



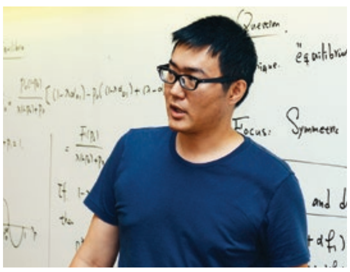
Amandine Véber and **Anton Wakolbinger:** Learning session on look down constructions

Jason Schweinsberg: Rigorous results from a population model

Qiang Yao: Contact process in a static random environment

Stein Andreas Bethuelsen: Stochastic domination in space-time

Federico Sau: Unveiling possible dualities



Yu-Ting Chen and **Matthias Hammer:** Competing species models

Beyond I.I.D. in Information Theory

24 – 28 JULY 2017

CO-CHAIRS:

Masahito Hayashi | Nagoya University and National University of Singapore
Vincent Y. F. Tan | National University of Singapore

This workshop focused on understanding the following question in more detail:

"If one specifies an error tolerance no larger than some error $\varepsilon > 0$ and allows for using n instances of a given resource, what communication rates are achievable?" Having good general answers to various instances of this question would be conceptually rich and directly relevant for practical implementations of quantum communication devices. Such questions are technically challenging and should spark interest from both the theoretical quantum information and mathematical communities.

Researchers (both quantum and classical information theorists) participate in focused discussions

Multi-party interactive coding on a secure network [From left: **Masahito Hayashi**, **Guangyue Han** and **Ning Cai**]

Hal Tasaki: Thermodynamics, statistical mechanics, and quantum mechanics

Michal Hajdusek: Self-guaranteed measurement-based quantum computation

Marco Tomamichel: Rényi divergences as weighted non-commutative vector valued L_p-spaces

The workshop featured 34 talks on a mixture of subjects from classical to quantum information theory, of which 20 speakers were students/postdocs sharing their work. There was also a poster session. Clearly, the workshop helped graduate students and postdocs gain visibility and promote their works in front of international experts.

The talks were broadly grouped under the following subjects: thermodynamics, compression, entropy (information measures), classical-quantum channels, secrecy, entanglement, networks and quantum-quantum channels.

There were a total of 70 participants (48 overseas, 22 local) and among them 29 were graduate students (19 overseas, ten local).

Independent and identically distributed variables

Aspects of Computation

21 AUGUST – 15 SEPTEMBER 2017

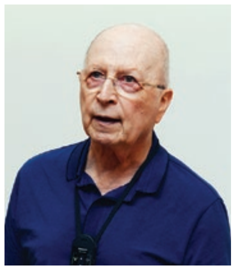
ORGANIZING COMMITTEE:

Noam Greenberg | Victoria University of Wellington
Keng Meng Ng (Selwyn) | Nanyang Technological University
Guohua Wu | Nanyang Technological University
Yue Yang | National University of Singapore

The program started with the *Workshop on Parametric Complexity* from 21 to 25 August 2017. There were nine talks and mini courses by Eric Allender (Rutgers, The State University of New Jersey) on the minimum circuit size problem, Elvira Mayordomo (Universidad de Zaragoza) on effective fractal dimension theory and Henning Fernau (Universität Trier) on FPT-inspired approximation algorithms.

Bakhadyr Khoussainov: Algorithmically random structures

Linda Westrick: Increasing dimension *s* to dimension *t* with few changes



Alan Selman: Disjoint NP pairs and propositional proof systems

Expressing computable structures as a power series [From left: **Julia Knight** and **Wolfgang Merkle**]

PAST ACTIVITIES

The following week (28 August – 1 September 2017) had a *Workshop on Algorithmic Randomness* with seven talks. The third *Workshop on Classical Computability Theory* (4 – 8 September 2017) and the fourth *Workshop on Computable Structures and Reverse Mathematics* (11 – 15 September 2017) had 15 talks. The activities of these two workshops overlapped with another standalone *Workshop on Computability Theory and the Foundations of Mathematics* (8 – 12 September 2017).

The program provided a good environment for participants to have discussions and initiate collaborations. This program also provided an opportunity for research fellows supported by local funding of the local organizers to work with other participants of the program. Twenty six collaborations were initiated or continued during the program. In particular, Julia Knight, Karen Lange, and Reed Solomon made substantial progress on their on-going project on the complexity of finding zeros of polynomials over the Hahn field.

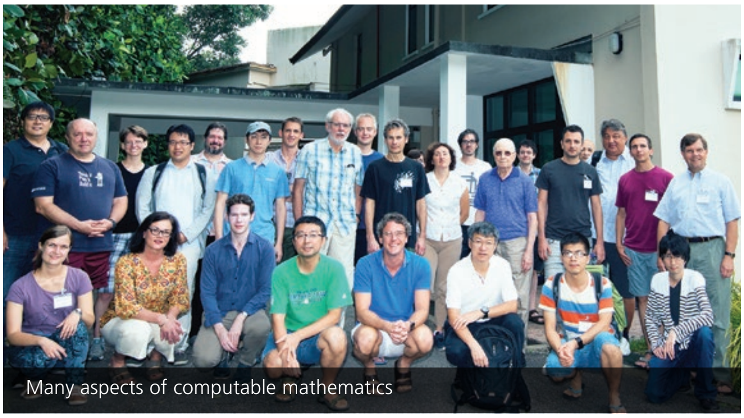
There were a total of 66 participants (55 overseas, 11 local) and among them seven were graduate students (six overseas, one local).

Alexander Melnikov: Computably categorical torsion abelian groups

Valentina Harizanov: Computably enumerable vector spaces

Margarita Marchuk: Autostability relative to strong constructivizations of structures of finite signature

André Nies: Closure of resource bounded randomness notions under polynomial time permutations



Many aspects of computable mathematics

Investigation in various degrees [From left: **Guohua Wu, Michael McInerney, Rod Downey** and **Noam Greenberg**]

Workshop on Computability Theory and the Foundations of Mathematics

8 – 12 SEPTEMBER 2017

ORGANIZING COMMITTEE:
Chi Tat Chong | National University of Singapore
Kazuyuki Tanaka | Tohoku University
Guohua Wu | Nanyang Technological University
Yue Yang | National University of Singapore
Keita Yokoyama | Japan Advanced Institute of Science and Technology

This workshop, jointly sponsored by the Japan Society for the Promotion of Science and the National University of Singapore, is the seventh in the Computability Theory and Foundations of Mathematics (CTFM) series. CTFM 2017 was the first time a meeting in this series was held outside Japan. The first day and the last day of the 2017 workshop focused on classical recursion theory, computable structures as well as reverse mathematics. The activities were held jointly with the program on *Aspects of Computation* (21 August – 15 September 2017). The other two days of the workshop focused on set theory and topics in the foundations of mathematics.

Joerg Brendle: Rearrangements

Ludovic Patey: Can we fish with Mathias forcing?

Thomas Zeugmann: Active learning of classes of recursive functions by ultrametric algorithms

The five-day workshop had a total of ten invited talks. There were 19 participants which included 15 visitors from overseas and four faculty members from local universities. Several local PhD students and tutors also participated in the workshop. Results from a feedback survey indicated ten research initiated or worked on during or after the workshop.

Variations of ground models [From left: **Takayuki Kihara, Yasuhiko Omata, Toshimichi Usuba** and **Takeshi Yamazaki**]

Liang Yu on analysis with other operators

Activities were held jointly with the program on *Aspects of Computation*

NG KONG BENG PUBLIC LECTURE SERIES

Professor Anton Zorich of Université Paris Diderot – Paris 7, France, delivered a public lecture on "Butterflies, Cats, and Billiards in Polygons" at NUS on 11 May 2017. In his lecture, Professor Zorich started with explaining how we could count the number of digits in the 10,000th term of Fibonacci sequence without using a computer and the regular long-run behavior of chaos, then he described the deep connections among wrapping a bicycle tube over itself, playing billiards, Boltzmann gas and many other phenomena to an attentive audience of 110 people.

Professor Denis Hirschfeldt of The University of Chicago, USA, delivered a public lecture on "Waking Up from Leibniz's Dream: On the Unmechanizability of Truth" in NUS on 14 September 2017. Professor Hirschfelt began his lecture tracing the historical development leading to Leibniz's idea of developing a language suitable for precise arguments and a framework for logical calculation. He then expounded how it subsequently impacted the work of Hilbert, Gödel and Turing. His lecture captured the keen interest of an audience of 140 people.

Anton Zorich:
Butterflies, cats, and billiards in polygons

Steven Evans:
Some mathematical insights into aging and mortality

Denis Hirschfeldt:
Waking up from Leibniz's Dream: on the unmechanizability of truth

Mark Wildon:
The liar game – truths & proofs from Euclid to Turing

Professor Steven Evans of University of California at Berkeley, USA, delivered a public lecture on "Some Mathematical Insights into Aging and Mortality" at NUS on 3 August 2017. Almost two centuries ago, Gompertz noted that the mortality rates after maturity increased exponentially with age. With this starting point, Professor Evans pointed out this simple relationship had since been observed in many multi-cellular organisms. He then discussed ongoing effort of developing quantitative framework that explains how these patterns of mortality results from natural selection principle. A total of 70 people attended the lecture.

Dr Mark Wildon of the Royal Holloway, University of London, UK delivered a public lecture on "The Liar Game: Truths & Proofs from Euclid to Turing" at the National Library on 14 December 2017. Starting with an accessible proof of Euclid's beautiful argument that there are infinitely many primes, Dr Wildon explained how prime numbers are applied in codes used by space probes, computers and mobile phones. He ended his lecture with Turing's amazing proof that there are things we can never compute, no matter how hard we try. A total of 162 people were enthralled by his lecture and live demonstration.

Workshop on Spline Approximation and its Applications on Carl de Boor's 80th Birthday

4 – 6 DECEMBER 2017

Jointly organized with Department of Mathematics, NUS

ORGANIZING COMMITTEE:
Say Song Goh | National University of Singapore
Hui Ji | National University of Singapore
Zuowei Shen | National University of Singapore

Representation Theory of Symmetric Groups and Related Algebras

11 – 20 DECEMBER 2017

WORKSHOP ORGANIZING COMMITTEE:
Joseph Chuang | City University London
Karin Erdmann | University of Oxford
Lim Kay Jin | Nanyang Technological University
Tan Kai Meng | National University of Singapore

Data Sciences: Bridging Mathematics, Physics and Biology Part II

4 – 12 JANUARY 2018

PROGRAM CO-CHAIRS:
George Barbastathis | Massachusetts Institute of Technology
Hui Ji | National University of Singapore
Patrice Koehl | University of California at Davis

Meeting the Statistical Challenges in High Dimensional Data and Complex Networks

5 – 16 FEBRUARY 2018

WORKSHOP CO-CHAIRS:
Jiashun Jin | Carnegie Mellon University
Zhigang Yao | National University of Singapore

Workshop on Particle Swarm Optimization and Evolutionary Computation

20 – 21 FEBRUARY 2018

CO-CHAIRS:
Kay Chen Tan | City University of Hong Kong
Weng Kee Wong | University of California, Los Angeles

6th NUS-USPC Workshop on Machine Learning and FinTech

18 – 19 APRIL 2018

Jointly supported with Centre for Quantitative Finance, NUS

ORGANIZING COMMITTEE:
Jean-François Chassagneux | University Paris Diderot
Ying Chen | National University of Singapore
Min Dai | National University of Singapore
Claudio Fontana | University Paris Diderot
Steven Kou | National University of Singapore
Huyên Pham | University Paris Diderot
Chao Zhou | National University of Singapore

Modeling and Simulation of Interface Dynamics in Fluids/Solids and Their Applications

23 APRIL – 25 MAY 2018

PROGRAM CO-CHAIRS:
Weizhu Bao | National University of Singapore
Weiqing Ren | National University of Singapore and Institute of High Performance Computing, A*STAR

Dynamic Models in Economics

4 – 22 JUNE 2018 & 2 JULY – 3 AUGUST 2018

PROGRAM ORGANIZING COMMITTEE:
Yi-Chun Chen | National University of Singapore
Yeneng Sun | National University of Singapore

IMS Graduate Summer School in Logic

18 JUNE – 6 JULY 2018

Jointly organized with Department of Mathematics, NUS

Oppenheim Lecture: Number of Points Modulo p when p Tends to Infinity by Jean-Pierre Serre

22 JUNE 2018

Jointly organized with Department of Mathematics, NUS

Pan Asia Number Theory Conference 2018

25 – 29 JUNE 2018

ORGANIZING COMMITTEE:
Wee Teck Gan | National University of Singapore
Lei Zhang | National University of Singapore

Theories and Numerics of Inverse Problems

6 – 17 AUGUST 2018 & 24 – 28 SEPTEMBER 2018

PROGRAM CO-CHAIRS:
Xudong Chen | National University of Singapore
Zuowei Shen | National University of Singapore

Bayesian Computation for High-Dimensional Statistical Models

27 AUGUST – 21 SEPTEMBER 2018

PROGRAM ORGANIZING COMMITTEE:
Alexandros Beskos | University College of London
Hock Peng Chan | National University of Singapore
Dan Crisan | Imperial College London
Ajay Jasra | National University of Singapore
Kengo Kamatani | Osaka University
Kody Law | Oak Ridge National Laboratory
David Nott | National University of Singapore
Sumeetpal Singh | University of Cambridge

String and M-Theory: The New Geometry of the 21st Century

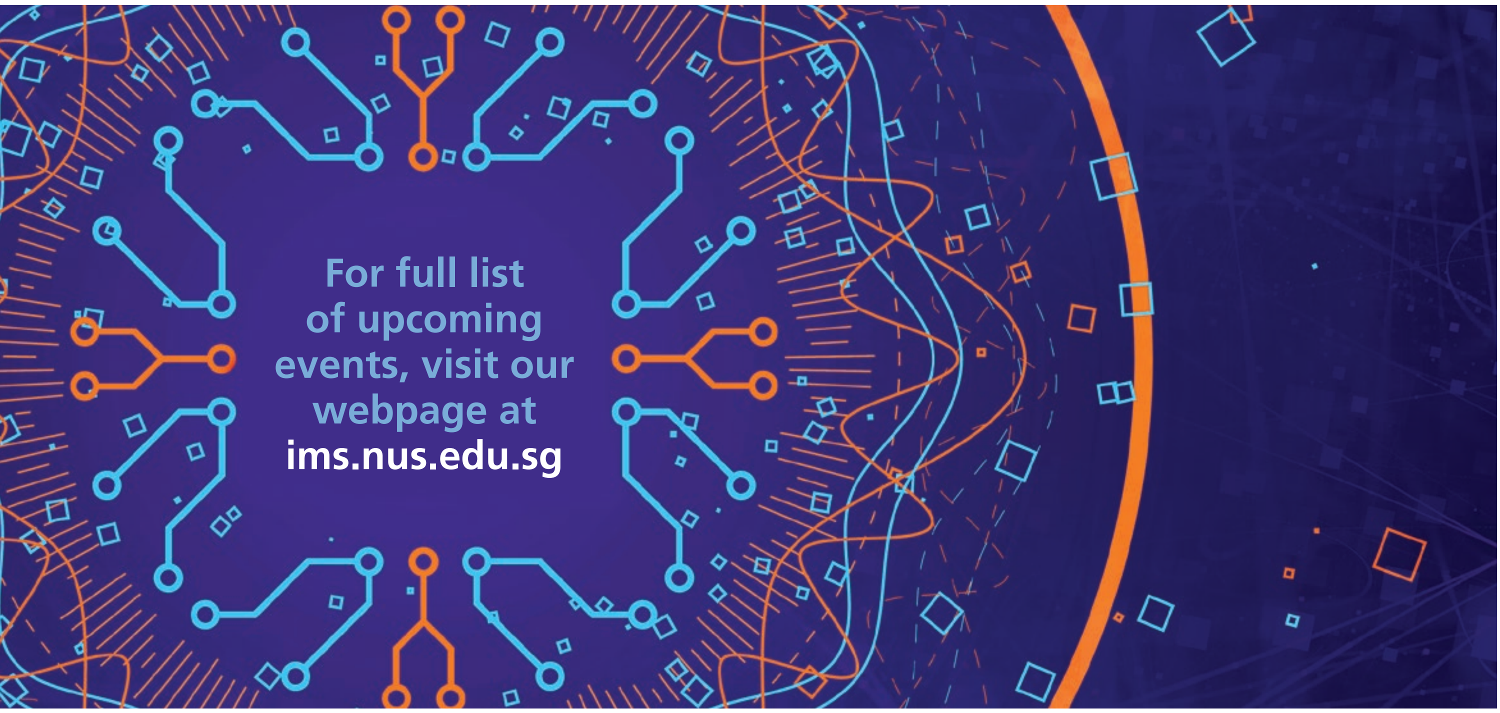
10 – 14 DECEMBER 2018

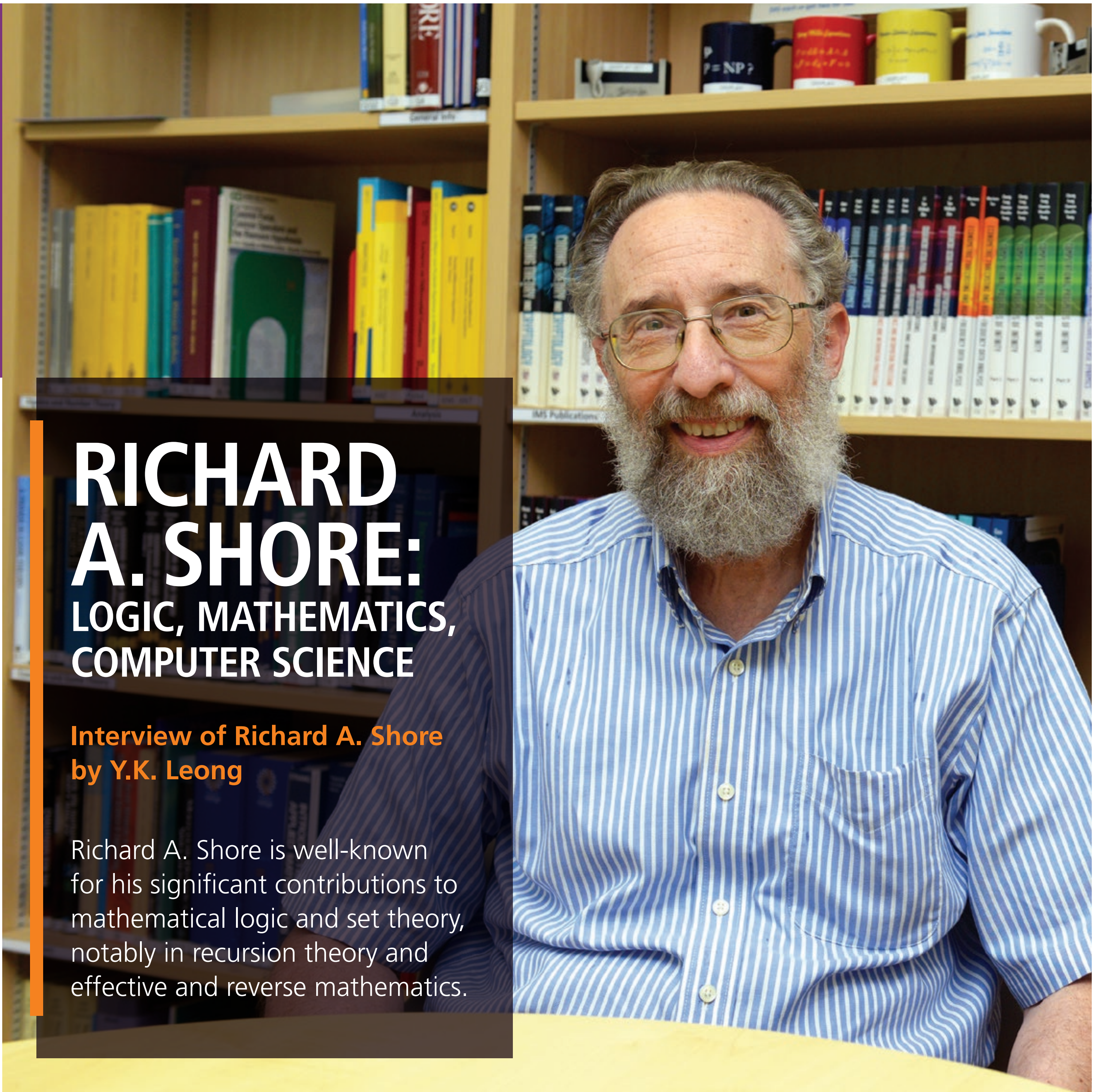
WORKSHOP CO-CHAIR:
Meng-Chwan Tan | National University of Singapore

On the Langlands Program: Endoscopy and Beyond

17 DECEMBER 2018 – 18 JANUARY 2019

CO-CHAIRS:
Dihua Jiang | University of Minnesota
Lei Zhang | National University of Singapore





RICHARD A. SHORE: LOGIC, MATHEMATICS, COMPUTER SCIENCE

Interview of Richard A. Shore by Y.K. Leong

Richard A. Shore is well-known for his significant contributions to mathematical logic and set theory, notably in recursion theory and effective and reverse mathematics.

Shore received an Artium Baccalaurens *summa cum laude* from Harvard University and a PhD from Massachusetts Institute of Technology (MIT). After graduate school at MIT and a two-year postdoctoral positon at the University of Chicago, he moved to Cornell University, where he rose through the ranks to full professor and is currently the Goldwin Smith Professor of Mathematics. However, throughout his career, he has been invited for short periods to collaborate and lecture at various universities such as University of Illinois, Chicago, University of Connecticut, Storrs, MIT; Hebrew University of Jerusalem, University of Chicago, University of Sienna, Italy, Mathematical Sciences Research Institute, Harvard University, Isaac Newton Institute for Mathematical Sciences, and National University of Singapore. He has actively supported research programs in developing countries in Latin America and in Greece, Israel, Italy and New Zealand. In 2007, a meeting in computability was held in his honour at MIT.

Early in his career, Shore made an impact on the development of computability theory with a short and elegant paper, which disproved the so-called "homogeneity conjecture" proposed by Hartley Rogers, Jr on the grounds that all the then known proofs in recursion theory could be "relativized"; that is, they would be true if one changed Turing reducible in any standard theorem about all sets to Turing reducible with access to an extra set C and restricted attention to all sets that compute C. More specifically, using earlier work on coding models of arithmetic in the structure of the Turing degrees $\mathbf{D}$ with the partial order of Turing reducibility, Shore showed that for any function f in which Kleene's $\mathbf{O}$ is computable, the ordering of Turing degrees (degrees of difficulty of computation of functions) is not isomorphic to the ordering of degrees of functions from which f is computable. A bit later, he improved the result to being not elementarily equivalent. Another significant result was his joint paper with Theodore Slaman, which showed that the Turing jump is definable in $\mathbf{D}$.

Shore's primary work has shed light on the extremely complicated nature, algebraically and logically, of the structures of relative complexity of computation of functions on the natural numbers. It has also led to connections with generalized recursion theory, complexity theory and set theory.

His work in effective model theory has developed methods for determining the effective content of standard mathematical theorems and, in particular, the complexity of proof of combinatorial theorems. He has helped develop a computational approach to the reverse mathematics associated with results in algebra, combinatorics and logic (see *Reverse Mathematics: The Playground of Logic, The Bulletin of Symbolic Logic*, Vol. 16 (2010) 378-402.).

From the beginning of his career, Shore has given a large number of invited talks at meetings, conferences and workshops throughout the world from the Americas, through Europe, to Asia. Among the distinguished lectures he gave was the Gödel Lecture of the Association of Symbolic Logic. His association with the National University of Singapore dates back to a month long visit as a Distinguished Visiting Professor in 1999-2000. It continued over the years with invited lectures at the Institute for Mathematical Sciences for the Institute's programs *Computational of Infinity Prospects* (20 June – 15 August 2005), *Computational of Infinity Prospects: All* (15 June – 13 July Aug 2005), another stint as Visiting Professor in 2011, and participation in *Sets and Computations* (30 March – 30 April 2015), and *New Challenges in Reverse Mathematics* (3 – 16 January 2016). He was also on the organizing committee of the latter program. One of his doctoral students (Yue Yang) is a Professor at NUS; another recently finished a stay as a postdoctoral fellow in NUS. During his latest visit to IMS, Y.K. Leong took the opportunity to interview him on behalf of *Imprints* on 12 January 2016.

The following is an enhanced version of the edited transcript of the interview in which he gives us an eye-opening account of the beginnings of logic and recursion theory (which led to computability theory and complexity theory), its connection and applications to computer science and a glimpse of a lesser known area in logic called "reverse mathematics".

ACKNOWLEDGMENT. *Y.K. Leong would like to thank Eileen Tan, Senior Executive of the Institute for Mathematical Sciences, National University of Singapore, for her help in preparing a raw draft of the transcript of the interview.*

IMPRINTS **I** You obtained Artium Baccalaurens *summa cum laude* in Mathematics at Harvard University, and then went to MIT to do a PhD in logic. Why did you not continue your graduate study at Harvard and why logic?

RICHARD SHORE **S** Those questions are related. First, the general phenomenon at US universities is that it is not expected that students stay at their undergraduate institutions for graduate work. That's unusual. The general advice is that you go somewhere else and learn from some new people. In my particular case, since I was interested in logic, even if Harvard had been interested in me, I would not have been interested in staying. There were many important logicians at Harvard, but they were all in the philosophy department. These included people like Quine [Willard Van Orman Quine (1908-2000)], Putnam [Hilary W. Putnam (1925-2016)], Dreben [Burton Spencer Dreben (1927 –1999)] and [Saul Aaron] Kripke. They also did mathematical logic. However, they were all in the philosophy department, and it would have meant studying philosophy instead of mathematics. I was interested in mathematics and mathematical logic. So it wouldn't have made sense for me to go to Harvard. MIT, on the other hand, also had a very good mathematics department and had important logicians in the mathematics department as well as in the philosophy department. So it was much more attractive. Geographically it was nearby and, on a personal level, although I was not yet engaged, the woman I married after my first year of graduate school was still finishing at Harvard. So it was attractive to stay in the area.

I Except for a short stint as instructor at University of Chicago after MIT, you have been on the faculty of Cornell University since 1974. What is it that makes you so attached to Cornell?

S I went to Cornell because it had a very strong tradition in logic and had very good people at that time and had a history of having good students. My own thesis advisor, Gerald Sacks, had been a student at Cornell and taught there before going to MIT. Rosser [John Barkley Rosser Sr. (1907-1989)] and [Anil] Nerode were very active logicians in the department for a long time even before I came. Michael Morley, one of the great model theorists, was there. It was attractive from that point of view. I was also happy to move back east for family reasons. That was why I went there originally. Over the years, I realized that it was a very pleasant department, unusually collegial and very democratic. They have, for example, a hiring committee, but it doesn't hire anyone. It does all the clerical and organizational work while the department as a whole makes all the decisions. Any decision of any importance really is handled by the faculty as a whole. The duty of the chairman is to do what the faculty wants rather than the other way around. It's a very friendly and pleasant place professionally. It also continued to get very good students in logic. I've been quite lucky in that I have had several very strong students. For example, there is a prize (the Sacks Prize) given annually for the past twenty three years to the best thesis in logic worldwide. Three of my students have won it, as well as another student in computer science for whom I was a co-advisor. Several of my students have won other prizes and fellowships as well. Another attractive aspect of the university outside of the mathematics department is its long history of interaction between logicians and other disciplines, including applied mathematics, philosophy and now primarily computer science. Students also go back and forth, and that's nice both for them and the faculty. Overall, it's a very pleasant place to be.

I Cornell is in the east?

S Yes, it's in upstate New York, that is, in the northern part of New York State.

I If I'm not mistaken, most of the logicians in the US are in the east. Is that true?

S Well, like many phenomena in the United States, it's bi-coastal, which means it's on both coasts, both sides. There are many logicians in California, in Berkeley, UCLA, Stanford and so on, as well as very strong groups in the east and in the middle such as Chicago, Madison and Urbana. As in many other things, it's in the east coast, west coast and central midwest.

I Your PhD thesis was on recursion theory. Could you briefly tell the layman what recursion theory is?

S Okay, I'll try. Thematically, I would say, recursion theory studies the notion of complexity in many different ways and in all kinds of mathematical structures. At one level you can talk about sets of numbers or functions from numbers to numbers and how hard it is to decide membership or compute them. You can talk about mathematical theorems and say how hard they are to prove in terms of the axioms needed. You can talk about mathematical constructions and say what you need to verify that they succeed or how complicated the objects they build are. You can talk about how hard it is to give the definition of some object. For example, you have some language in which you can describe or define an object of interest. Then you can measure the complexity of the description in terms of syntactic properties such as length or depth of quantification. This is a subject that tries to study these notions and see what the relationships are among the various notions of complexity. It also devotes much of its attention to the notion of relative complexity: What does it mean to say that one function is harder to compute than another, and then, what sort of structure is determined by this relation, and what are the connections between relative difficulty of computation and other measures of complexity.

I How old is the subject?

S I would say it goes back to the 1930s. The major figures include Church, Turing and Kleene [Alonzo Church (1903-1995), Alan Turing (1912-1954), Stephen Cole Kleene (1909-1994)]. The original motivating question, in some sense, was how do you even give a definition of what it means to be computable or what it means to be an algorithm? There were many classes of questions that mathematicians posed over the centuries that asked for an algorithm or effective procedure. Sometimes, someone would produce the desired procedure but sometimes not and not for centuries. Then how do you show that you can't find an answer that there is no effective or algorithmic procedure? It's harder to know what to do. What these people did was to give a mathematical definition of what it means to do something effectively and then used that definition to prove that there was no effective procedure that would solve the problem. So they solved problems that were very old in the sense that they proved that you can't do it the way you wanted to do it.

I I think that most people would associate logic with philosophy and that even most "mainstream" mathematicians believe that logic is more qualitative than quantitative. Do you think that there is some kind of communication or cultural gap between logic and "mainstream" mathematics and that some effort should be made to close this gap?

S I guess, in a word, "Yes". But, maybe I should add a little more. Historically, if one goes back to ancient times to the classical Greeks — Plato, Aristotle — all these things were somehow connected. Mathematics and philosophy were viewed, in some sense, as the same subject, and you could not do one without the other. It was reputed that the sign over Plato's school said; "Let no one ignorant of geometry enter." They expected you to be able to do mathematics in order to be able to do philosophy. That view prevailed through the Middle Ages in various ways. In more recent times, with the beginnings of modern logic in the 19th century, some of the motivation was philosophical and foundational. Some of the questions were: Does mathematics make sense? Is it consistent (so it can't prove both a sentence and its negation)? Can we do things in a way that we know we don't make a mistake? Some of these are also mathematical questions. After all, mathematicians, and not just logicians, don't like to make mistakes. What a mathematician wants, however, is not just a philosophical argument but a mathematical proof that shows no contradictions are derivable. Finding such proofs was the second of Hilbert's famous list in 1900 of problems for the 20th century. Gödel solved the problem negatively in the 1930s: No reasonably strong mathematical system can prove its own consistency.

So, at the beginning, there was a large interplay between logic and philosophy because logic was also concerned with representing thought and language and is traditionally a philosophical subject. It was Leibnitz's [Gottfried Wilhelm Leibniz (1646-1716)] dream that he would invent a mathematical procedure that would enable you to do all reasoning in every subject. Well, artificial intelligence has come a long way, but we are not there yet. There were certainly other connections and motivations and, as a result, there were many topics in logic that were slightly foreign to what was current in mathematics. Many prominent mathematicians were, however, very interested and involved in logic – people like Hilbert, Brouwer, von Neumann, Weyl [David Hilbert (1862-1943), Luitzen Egbertus Jan Brouwer (1881-1966), John von Neumann (1903-1957), Hermann Weyl (1885-1955)]. On the other hand, as you said, there is a view that working mathematician needn't think about it; they don't have to really know – "It's okay, someone will worry about it, and I'll just be safe." I think that has been changing quite a bit and at an accelerated pace more recently. In the past 10 to 15 years, there have been a growing number of applications of logic to a wide number of areas in mathematics, not just the things about algebraic structures that we talked about earlier on – all kinds of things in number theory, algebraic geometry, ergodic theory, operator theory, quantum physics and more. So there are more and more interactions. Several times over the past few years, logicians in my department while talking to someone in another field have heard things like "Oh yes, someone has just used (that part of logic) in my field to do x, y, or z." So it's beginning to change. I don't know how many decades it will take. One doesn't change the culture quickly.

I Recently logic is not in the philosophy department.

S Not primarily. It used to be the case but it is no longer true. As I said, Leibnitz wanted to formalize all of thought, all of argument. We're less ambitious. We like to do mathematics. Mathematical thought has its own rules that are at times different than the ones in everyday speech or in legal work. So the study of logic became very much the study of the logic of mathematics and became concentrated in mathematics departments. Now, maybe the largest number of people who might be called logicians are in computer science departments. Computer science has grown so enormously that, at my university, there are just as many logicians in the computer science department as there are in the mathematics department. Interestingly, many of the types of logic now studied and used in computer science are closely related to those that have traditionally been studied in philosophy. So there is a reintegration of sorts happening now.

I I think even in physics, logic has …

S Yes, logic has started to make contributions to mathematical models of physics. For example, there has been real mathematical work on the mathematical models for quantum physics using C^* algebras and methods from logic.

I One of the seven problems in the list of the Clay Mathematics Institute's seven millennium problems (each carrying a prize of one million US dollars) is the "$P = NP$" problem on computational complexity. Has recursion theory made any inroads into the solution of this problem?

S The question of $P = NP$ is roughly this. If you have an NP solution to a problem, it means that, if you guess an answer, you can easily check to see if it is correct. A P solution means that you can actually produce an answer easily. As I said, there are many measures of complexity. The one here is the running time, how long the machine takes to run the algorithm. Does it run in a polynomial time in the length of the input (this is what is called a function in P)? So I think the first thing to realize is that the entire question and the development of the subject out of which it grew is based on taking notions from recursion theory and transporting them to computer science.

In recursion theory, instead of asking "Is there an algorithm which quickly computes a solution to a problem?" you ask "Is there an algorithm that computes an answer at all?" I say to my students who come from the computer science department that the dividing line between here and the other end of the quad, where the computer science department is, is that here, when I show that there is an algorithm, I'm finished with the problem. I am primarily interested in the problems for which there are no algorithms. And at your end, if I prove that there's no algorithm, you're finished and you don't care. But the ideas are the same. From the recursion theoretic point of view, the question is: "Can I guess the answer, where 'guess' just means I produce a number?" and "Is there an algorithm that checks that it's right?" This is Turing's famous halting problem where one wants to know whether a given computer program will eventually halt (1) or not (0).

I That means you have to prove it will stop.

S Yes, that's the question. Your algorithm has to always give the right answer. So Turing famously proved that there is no such procedure. This then is an archetypical example of the kinds of problems we talked about before. Sometimes there is just no way to algorithmically solve a class of problems such as instances of the halting problem. So $P = NP$ appears to be a similar kind of question. You replace a computable algorithm by a computer algorithm that runs quickly. One then asks of various classes of problems if there is a fast algorithm to solve them. Then this seems like an analog of the halting problem, and almost everyone thinks the answer should be "no" (P does not equal NP). But no one has been able to prove it. So that's the setting of the problem in complexity theory. This area very often carries over recursion theoretic notions, and there are many more examples of this sort.

You asked whether the recursion theorists have helped make any progress in solving the $P=NP$ problem. Well, near the beginning of the study of this problem, there were proofs in a typical recursion theoretic style that certain methods can't suffice. You could use the methods of recursion theory to show that many general classes of arguments could not work. This was based on work that grew out of Turing's notion of relative computation. One says that B is computable from A if there is an algorithm that computes answers to membership questions about B as long as it is provided (by what Kleene called an "oracle') with answers to questions about A. In this kind of model of computation – if you can prove something, it really doesn't much matter if you add in this extra oracle, this extra free information. If the questions are sufficiently general, it doesn't make any difference. Most techniques that everyone knew had that property. This phenomenon lead to the Homogeneity Conjecture we discussed before. The techniques in computer science mostly had this property as well. People including logicians like Robert Solovay and others proved that you could use methods from recursion theory to show that such techniques in computer science would not work to solve various problems by showing that the answer depends on the oracle. The $P=NP$ question was a most striking example. For some oracles the answer is "yes, they are equal", but for others it's "no". So does that count as helping? I don't know. At least it says many well-known methods will not suffice to settle the problem.

I Do you think it will be solved in ten or twenty years?

S Oh, I don't know. I don't see any route to a solution. No one seems to have any likely idea. So it's very hard to predict. When there's something to work on, you can say "Aha, people will work on this and perhaps solve it the foreseeable future." But I think at the moment there isn't a good line of attack for a negative solution. On the other hand, proving that $P=NP$ would require someone to come up with a fantastic algorithm. So that could happen at any time. But until they do it, no one will know. If there is a really good algorithm for solving NP problems it will make a major impact on every industry, but that's probably not likely.

I There seems to be many paradoxes in logic and set theory. Have you found any of the results of your work to be surprising and counter-intuitive? Is it possible for you to explain it to the layman?

S Looking back now, more than once I realized at some point fairly far along in my own work in some area that what I had done developed into a paradigm shift. Many of the structures and notions that I have studied were originally thought to be simple in some way – "Oh, there are only five of these things; all of the things like these behave in a nice way." My results have almost always tended to say no, the situation is much more complex, the structures themselves are very, very complicated. There aren't just five ways of tackling this or that. There are dozens. This was surprising at the beginning. After a while, it seemed like the right thing to do all the time. I would say a common theme in much of my work is that the world is much more complicated than you thought.

I Can I ask you one question about reverse mathematics? This is an unfamiliar term to many of us. Could you tell us roughly what is it about?

S It's another example of measuring how complicated things are. How do you measure how complicated a theorem or mathematical construction is? Some are hundreds of pages long, some take years to prove. That's not a good mathematical measure. This subject tries to say, "Well, from one point of view, we know what a mathematical proof is. We know from high school what a proof in Euclidean geometry is. You have axioms, a proof and a theorem. So the question is: which axioms do you need to prove some theorem?" You want to make that precise in some way. One direction is easy. I write down the axioms and see whether I have a proof. High school students know how to check this. The other question is: how do you know you actually need those axioms? Maybe you need different ones. Maybe you don't need all of them. There is the famous question about the axioms of Euclid, whether you need them all, in particular, the parallel postulate. People over the centuries worked on this. The way this subject attempts to answer that question is that you take some very weak system of axioms, just about how arithmetic works on the numbers (with addition and multiplication), things you learn in grammar school, before high school even. Then you say, assume just that, and now suppose I assume the theorem instead of the other axioms. Could I then prove the axioms I used for my proof of the theorem instead? If so, we say the theorem is equivalent to (of the same strength as) this set of axioms. We call the proof of the axioms from the theorem a reversal (of the usual proof). Hence the name, reverse mathematics.

I Does it mean that you can replace all the other axioms by the theorem?

S Correct. So that's what the subject does. Some theorems are stronger than others because they prove more of the axioms. So you have an infinite array of possible axioms. One theorem may only require the first five axioms to prove it and it implies the first five but not the next five. Another theorem needs the next five as well and implies all ten. So it's stronger. That is the kind of argument one makes. Why do I have anything to do with it? It turns out that for almost all these types of complexity notions and results, there are connections between them. If the theorem proves that some function exists, it's almost always the case that the axiomatic complexity of the proof is connected with how complicated that function is in terms of computability. This may not be obvious at the beginning, but after you work on it, you say, "Oh, look, almost all the time the two approaches give the same answer."

I Do you take any theorem and apply your method?

S There is a sort of industry for going through the classical theorems and seeing how complicated they are. It provided an example of the phenomenon we discussed before. For the first couple of decades of the subject everybody said, "Oh, every theorem we look at turns out to be equivalent to one of these five axiom systems." There are, maybe, a few theorems that we don't know about yet, but there are only five systems. Then it started to be the case that other people found other examples that didn't fit any of the known five systems. Much of my work has been finding theorems that are strange in the sense that they don't correspond to any of the established systems. Now there's something on the web called the "reverse mathematics zoo", and you can go to the site and see a giant diagram of tens of different theorems, no two of which are equivalent in the sense of reverse mathematics. Most theorems still turn out to be equivalent to one of the major systems but I like to work on things that are different. I'm not interested in just working through things one by one. I want to find unusual examples and strange phenomena.

I While the usefulness of mathematics is widely recognized in physics, it is only with the increasingly important role of computers in science in everyday life that logic is beginning to catch people's attention. Even so, there is much ignorance among the general public about the role of logic in computer science. Could you tell us briefly how logic has contributed to the development of computer science?

S This is a big subject. Maybe I'll try and talk about a couple of examples. One is complexity theory that we have already talked about, $P = NP$. There, logic had a major influence. Others are perhaps less obvious. You know about programming languages. There are lots of programming languages, but how do you study them abstractly? How do you describe what kind of programming language a particular one is? What sort of things can programming languages do? You can think of a programming language as a syntax of strings of symbols with rules, with the signs and parentheses, and whatever you have has some meaning, otherwise you wouldn't be doing it. How you formalise the connection between the syntax and meaning is called semantics. Just as in linguistics, you have syntax and semantics; symbols and their meanings. You want to have some kind of connection between them. This is a topic in which logic has been interested from the beginning. Many of the models and methods of logic used in the analyses of these notions have turned out to be important for computer science. I mentioned Church. He had a computational system called "lambda calculus" which, at that time, seemed very strange. It was Turing who made the real impact in terms of convincing models of computation. But it turned out to be an important tool for giving an abstract representation and even semantics for programming languages.

“IN RECURSION THEORY, INSTEAD OF ASKING “IS THERE AN ALGORITHM WHICH QUICKLY COMPUTES A SOLUTION TO A PROBLEM?” YOU ASK “IS THERE AN ALGORITHM THAT COMPUTES AN ANSWER AT ALL?”

I SAY TO MY STUDENTS WHO COME FROM THE COMPUTER SCIENCE DEPARTMENT THAT THE DIVIDING LINE BETWEEN HERE AND THE OTHER END OF THE QUAD, WHERE THE COMPUTER SCIENCE DEPARTMENT IS, IS THAT HERE, WHEN I SHOW THAT THERE IS AN ALGORITHM, I'M FINISHED WITH THE PROBLEM.”

I But Turing was not very abstract.

S Correct, exactly. Turing's is a machine model of what a human could compute. It extracts the basic mechanical moves of a paper and pencil computation and considers simple machines that could implement them. Working on a “tape” it can read, write and erase any of a finite set of symbols and move from one space to the next. Moreover, it follows a finite list of instructions that tell it when to do which action. That's it and that's why everyone likes his formulation of computability – it captures that which we can clearly do. Church had a totally different formulation which turned out to be equivalent in the sense that each computed the same set of functions but the proof of equivalence was very complicated. Church's approach, however, turns out to provide a method for representing programming languages and prove theorems about them and even supply some sort of semantics. So that's a very important application of logic to computer science.

Another is program verification. This is a big item now. You have a program that is supposed to do something. You would like to know whether it actually does it as desired. You know the program is supposed to pilot your plane. You would like to know it won't crash. How do you know? You would like to prove that the program does what it is supposed to. Well, this is a natural application of logical methods. You have many different kinds of logic which fit different settings of programming languages and their analysis. But the ideas of syntax, proofs, semantics and the relationships among these concepts apply to all of them and are very important in terms of developing verified programs. There are even general theorems that say that if you can prove that a function exists in a certain nice kind of logic, then you actually can automatically produce an algorithm that works correctly. There are several large groups in computer science working on these ideas.

I What about debugging?

S Crudely, debugging is what you do when you don't have a proof of correctness. The situation here is you have a complicated program. Does it do something bad? How will you tell? Well, you may run it many times in different situations and see if all goes well. If not, you try to fix it and run it some more. If so, you have some assurance of correctness. After all, it didn't fail when you tried it. Then you sell it. Other people buy it. When it doesn't work, they complain and you fix it again. That is how debugging works.

Now there are actually industrial applications of the verification approach we just discussed. Even if they don't fully verify big programs, they verify parts or more basically that the chips underlying the workings of the computer perform correctly. There have been several real life examples where the basic chips made calculation mistakes in unusual situations or had other vulnerabilities. No one had formally checked them although they tried them out for a long time. Now there are formal verifications for the correctness of some of these basic building blocks of our computers.

I I believe that there are other types of logic like non-monotonic logic, database logic, probability logic and so on. Is it then meaningful to talk about “logics”, and if so, are there “hierarchies” of “logics”?

S There are indeed many kinds of logics. This is not unreasonable because the goal of a logic is to capture the methods of thinking and expression of a particular subject in a particular domain. They are not all the same. People discussing poetry and people discussing physics don't speak the same language in many ways. You can't expect it to be the same kind of analysis. Some logics might include others in terms of expressiveness and so be said to be stronger, but there is not a single hierarchy. There are different uses for various logics, and they are appropriate for different settings. This is an interesting phenomenon in the development of logic in both philosophy and computer science.

Many philosophers work on developing logics that are supposed to allow you to talk about knowledge or belief or time which mathematicians don't really usually talk about. I mean, we talk about truth, but we don't talk about whether it changes from yesterday to today. We don't worry about if “I believe this”. Either it is true or not. The logic for mathematics is not the same as the logic for belief or knowledge. You need different kinds of ways of talking about them. So instead of having symbols which just say “and” and “not” and “there is”, you have a symbol which says “I believe (or know) this”. But you can do the same kind of analysis as in mathematical logic. You can again describe syntax for the language, proofs systems, and semantics, just different ones, but with the same kinds of relationships among them.

INTERVIEW

“NON-MONOTONIC LOGIC, FOR EXAMPLE, IS ONE IN WHICH THINGS LIKE BELIEF OR KNOWLEDGE CHANGES WHEN YOU GET MORE INFORMATION. WHAT YOU THOUGHT YOU KNEW, OR WAS TRUE, YESTERDAY IS NO LONGER TRUE, OR BELIEVABLE, TODAY.”

Now you can ask what should be my proof rules and axioms based on what meaning you are trying to capture. You might think that if I know something, I believe it, or perhaps, if I believe it, I know it. Maybe you think that if you know something, you know that you know it, or maybe not. These are philosophical (or perhaps psychological) questions. Now you can design the logic and its axioms to capture the answers you want and then analyse the consequences of your choices. You can decide what you think represents the world (or the part of it of interest) and write down the axioms that you think are correct and see what is provable.

Non-monotonic logic, for example, is one in which things like belief or knowledge changes when you get more information. What you thought you knew, or was true, yesterday is no longer true, or believable, today. Many things in the world work that way. So if you like to model the kinds of performances or analyses that allow you to change your mind with new information, then you need a different kind of logic. If you want a logic that says, “Aha, right now the computer program is not doing anything wrong.” Suppose I let it run for a while, will it still be true that nothing is bad? Is there any way of describing that in our language and showing that it is correct? Logics that try to capture such time-bounded assertions are called temporal logics. For example, they may have symbols that apply to assertions and whose meaning is that the assertion will be true at some time in the future or will always be true.

You can also understand these symbols to mean the assertion about the state of the computer will be true after the next step, or at some point, or at every step of the procedure being analysed. So these logics were invented by philosophers to analyse philosophical questions, but now computer scientists are using them all and saying “Oh, what does a belief mean?” It is just a way of referring to the state of a processor at a given time. Well, we say of people Alice believes *A,* and Bob believes *B.* They communicate. Can they come to some common agreement? The referents for the proper nouns and pronouns in these statements may be people if you are studying philosophy, or they can be computer processors if you are doing computer science. The same languages and logics can be used to analyse both situations. So there are now ways of using many logics that were developed for philosophical reasons to analyse very natural problems in computer science.

I I believe that in computer science, they have some kind of measure of how reliable a proof is because the proof is very long and nobody can check every detail. How do you know there may not be errors somewhere?

S There are now several very large projects devoted to automatic theorem proving or verification. There are two questions and several approaches. One question is “Can the computer produce a proof?” Another, much easier one, is “Can the computer check a proof it is given?” There are projects that design computer languages and systems which, usually interacting with people, produce and then check proofs. You may provide the outline of a proof. Now, even if you know the proof in the sense that mathematicians usually mean, normally you can’t write out a fully formal proof. So there will be some sort of give and take. Perhaps you want to go from here to there, and the computer will say, “Aha, I can fill in the details.” Or it might say “I could try three possible approaches, or I don’t know what to do next. Can you help me now?” Then the person says, “Try this.” Eventually working together you can at times actually produce a formally verified proof of complicated theorems. Sometimes it take years.

Still these methods have been used for one or two important mathematical results that were so complicated that even the experts were not sure of their correctness. In one case, there were competing claims. One of the mathematicians spent years, developing a computer system that would produce a correct verified proof. This is not common, but on the other hand, these things can be used for practical applications in the sense that I mentioned before. If you can get a proof in a certain system, then you can automatically get an algorithm that will work correctly. This is a general kind of theorem. At Cornell we have a group working on this subject, and they have actually solved industrial problems. They did optimization problems for routings which were then provably optimal.

I Some students seem to view logic as a closed and self-contained area of knowledge in which they can readily get to do research in. What advice would you give to students who are keen to work in logic?

S I should say that they shouldn’t believe that view. There are many subjects you can do some work in without knowing much else, but it is not usual that you can do very good work without knowing other things. I think it is important for students in logic to get the same basic grounding in mathematics such as algebra and analysis that any mathematician would get. Some of those things will turn out to be useful and some not, but that’s the way mathematics works. Certainly, for me, algebra, combinatorics and analysis have all played roles in my work. Nowadays, I think it’s important for students of logic to study computer science as well, because many problems in logic are driven by computer science just the way physics used to drive mathematics. Even if you are not working on those problems in particular, it is important to understand how what you are doing is related to other subjects. In general, I think it’s important for many people, and certainly logicians, to have a basic education in mathematics and computer science.

VOLUME 35

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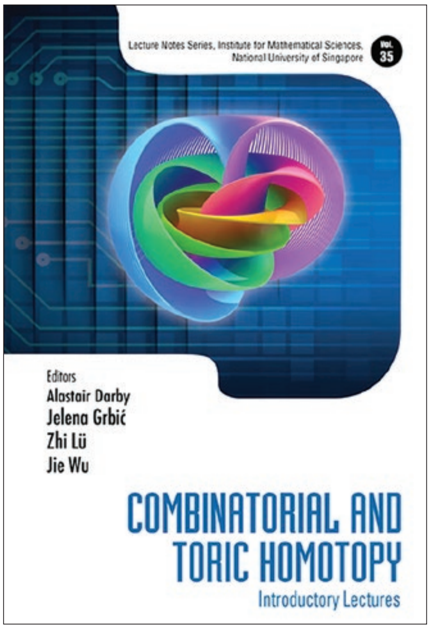

EDITED BY:

- **Alastair Darby** (Xi'an Jiaotong-Liverpool University)
- **Jelena Grbic** (University of Southampton)
- **Zhi Lu** (Fudan University)
- **Jie Wu** (National University of Singapore)

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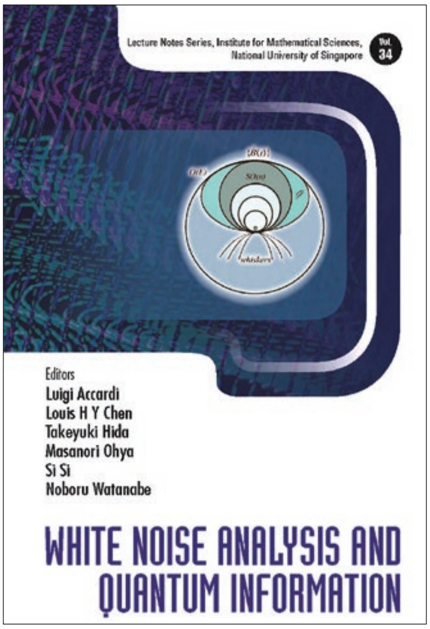

EDITED BY:

- **Luigi Accardi** (Università di Roma Tor Vergata)
- **Louis H Y Chen** (National University of Singapore)
- **Takeyuki Hida** (Nagoya University & Meijo University)
- **Masanori Ohya** (Tokyo University of Science)
- **Si Si** (Aichi Prefectural University & Yangon University)
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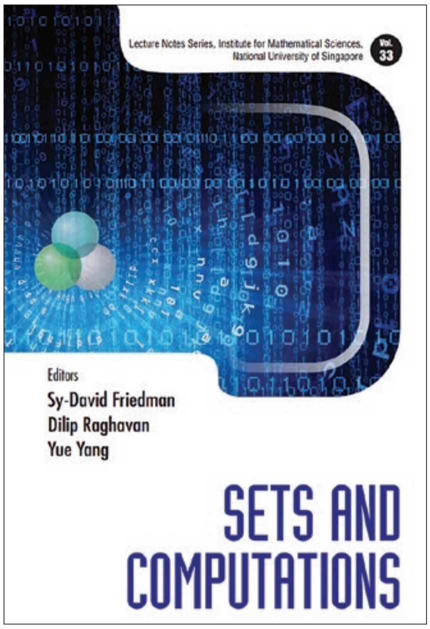

EDITED BY:

- **Raghavan Dilip** (National University of Singapore)
- **Sy David Friedman** (University of Vienna)
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PLEASE ADDRESS COMMENTS TO:
THE EDITOR, IMPRINTS
3 Prince George's Park Singapore 118402
PHONE: +65 6516-1897 | FAX: +65 6873-8292
EMAIL: ims@nus.edu.sg | WEBSITE: ims.nus.edu.sg

Editorial Team: Adrian Röllin (Deputy Director)
Kwok Pui Choi
Eileen Tan
Stephen Auyong San San
Wong Wai Yeng

Design: razorSHARK Design