IMPRINTS
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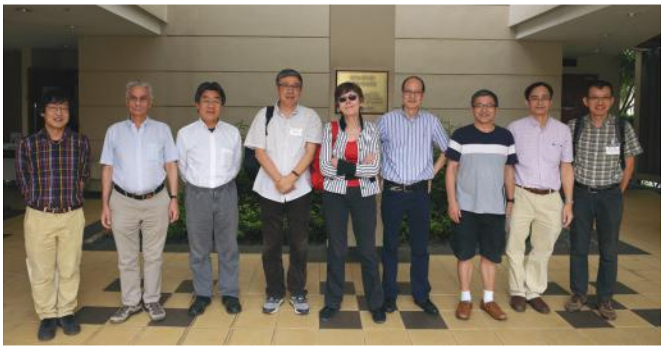
# Higher Dimensional Algebraic Geometry, Holomorphic Dynamics and Their Interactions >>>



From left: Keiji Oguiso, Nessim Sibony, Yujiro Kawamata, Gang TIAN, Hélène Esnault, De-Qi ZHANG, Shou-Wu ZHANG, Wing Keung TO and Tien Cuong Dinh

*[Editor's note: From 3 to 28 January 2017, the Institute hosted the program "Higher Dimensional Algebraic Geometry, Holomorphic Dynamics and Their Interactions". The program organizers and some of the program visitors contributed this invited article to* Imprints.*]*

From an algebro-geometric point of view, one likes to classify compact varieties according to their isomorphism classes or birational classes. The former classes are more rigid and the latter ones are more flexible in the following sense. If X is a variety, one can blow up a subvariety of X and get a new variety X′. Of course, X and X′ are not isomorphic to each other, but their relation is very clear -- they are the same modulo some proper nowhere dense subsets. Therefore, it makes more sense to determine the birational classes instead of isomorphism classes of varieties. In recent years there have been breakthroughs in the classification theory of higher dimensional compact algebraic varieties and complex manifolds. These results in algebraic geometry have profound influence on other areas of mathematics, including the study of higher dimensional dynamics and number theoretical dynamics.

The typical ways of classifying varieties are to look at the following three canonical fibrations on a variety X. The first fibration is the Iitaka (or Iitaka-Kodaira) fibration $I : X \to I(X)$, where its very general fibres F are varieties of Kodaira dimension zero and the base variety I(X) has dimension equal to the Kodaira dimension of X. This fibration reduces the classification of varieties of positive Kodaira dimension to those of Kodaira dimension zero or $-\infty$. The second fibration is the Albanese fibration $\mathrm{alb}X : X \to \mathrm{Alb}(X)$, where the codomain Alb(X) is a complex torus of dimension equal to the irregularity of X and hence has vanishing Kodaira dimension and Chern classes. Then to some extent, the classification of varieties is reduced to those of vanishing irregularity. The third fibration is the maximal rationally connected (MRC) fibration $r : X \to r(X)$ where the general fibres F are rationally connected varieties and the codomain is a non-uniruled variety. Recall that a variety F is rationally connected if every two general points of F are connected by a rational curve of F; we also recall that rational curves are the simplest curves, being of genus zero. From these three fibrations, the classification of varieties is reduced, to some extent, to the three types of building blocks: varieties of general type, varieties of vanishing Kodaira dimension and irregularity, and rationally connected varieties respectively.

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Next we turn our attention to the study of holomorphic dynamics, where we are interested in the symmetries f on a variety X. It is natural that one tries to apply the above algebro-geometric machineries to the study of automorphisms or endomorphisms f of X, or even birational or rational self-maps f of X. Such an approach provides an attractive immediate program to work on. At the same time, one will encounter some challenging obstacles in this dynamical setting, which still remain to be overcome. The IMS program provided a platform for discussion of some of these and other issues, and it assembled a group of algebraic geometers, complex geometers, arithmetic geometers and holomorphic dynamists. The main activity of the program was a 2-week workshop covering the major areas of algebraic geometry, arithmetic geometry and holomorphic dynamics with algebraic or number theoretic flavors. The program was highlighted by two lecture series by the following mathematicians:

1) Gang Tian of Peking University and Princeton University delivered two lectures elaborating the proof of Yau-Tian-Donaldson conjecture: the existence of a (geometric) Kähler-Einstein metric on a Fano manifold is equivalent to the (algebraic) K-stability condition. He also surveyed some of his related works and some open questions.
2) Shou-Wu Zhang of Princeton University gave two lectures and one colloquium talk covering the state-of-the-art development on several fundamental conjectures in arithmetic geometry and dynamics: the Manin-Mumford conjecture and its dynamical analogue, the Andre-Oort conjecture and Colmez' conjecture, and the ABC conjecture and BSD conjecture. He also illustrated the proof of Junyi Xie of Rennes University (also a program participant) on the Dynamic Manin-Mumford conjecture (proposed by Shou-Wu Zhang) when the endomorphism is a Frobenius lifting, using Peter Scholze's perfectoid space theory.

In addition to the numerous lectures delivered during the workshop fortnight, the program participants had plenty of research interactions and discussions throughout the entire duration of the program, and it is expected that these will lead to some fruitful research collaborations in the near future.

The Institute provided a comfortable and relaxed environment conducive to research, which benefitted the program tremendously. The superb service and untiring attention offered by the Institute's staff are highly appreciated by the program participants. The program organizers would like to take this opportunity to express their thanks to the Institute and its staff members for help in making the program a success.

Tien Cuong Dinh *(NUS)*
Hélène Esnault *(Freie Universität Berlin)*
Yujiro Kawamata *(The University of Tokyo)*
Keiji Oguiso *(University of Tokyo)*
Nessim Sibony *(Universite Paris-Sud)*
Gang Tian *(Peking University and Princeton University)*
Wing-Keung To *(NUS)*
De-Qi Zhang *(NUS)*
Shou-Wu Zhang *(Princeton University)*

## New Management Board Members >>>

The Institute is pleased to welcome two new members to the Management Board (MB) - Professor Frank Eisenhaber and Professor Andrew Lim.


Frank Eisenhaber

Professor Eisenhaber is currently the Executive Director of the Bioinformatics Institute, A*STAR. He received his PhD in molecular biology from the Engelhardt Institute of Molecular Biology, Russian Academy of Sciences in Russia. His main research interest lies in the discovery of new biomolecular mechanisms and uncharacterized genes and pathways. Frank Eisenhaber is one of the scientists credited with the discovery of the SET domain methyltransferases, ATGL, kleisins, many new protein domain functions and with the development of accurate prediction tools for posttranslational modifications and subcellular localizations.


Andrew LIM

Professor Andrew Lim is currently the Head of the Department of the Department of Industrial and Systems Engineering at the National University of Singapore (NUS). He received his PhD from the University of Minnesota. A prominent scientist and a technopreneur, he was recruited by NUS in 2016 under The National Research Foundation's Returning Singaporean Scientists Scheme. In 2013, he was recruited by the Nanjing University under China's Thousand Talents Program in the area of Management Science. His work draws from his expertise in computing, engineering and commerce. His current research interests include big data analytics, demand generation and supply management problems in the domains of healthcare, logistics and transportation.

The Institute would like to thank the outgoing MB members, Professors Paul Matsudaira, Alfred Huan and Kee Chaing Chua for their contribution in overseeing the Institute's operations and activities since they joined the Board in 2009, 2013 and 2014 respectively.

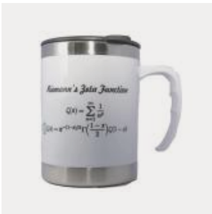
The Institute looks forward to strengthening its scientific programs further under the new and incumbent members of the MB.

Kwok Pui Choi
*National University of Singapore*

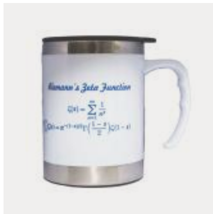
## News Highlights >>>

**New Souvenir**

IMS is happy to announce the launch of the new edition of the Riemann's Zeta Function mug! The new edition came in three designs.


White mug with black lettering


Blue mug with silver lettering


White mug with blue lettering

Visit our webpage to view more on our specially designed souvenirs, which are available in different designs, each with its distinctive mathematical theme. Purchases can only be made with cash at the IMS.

For more information, please visit ims.nus.edu.sg > Resources > Souvenir

## Programs & Activities >>>

### IMS Distinguished Visitor Lecture Series

Speakers invited to this new set of lecture series are prominent leaders in their fields. They are invited to participate in various activities of the Institute and will give a series of two lectures. The lectures are intended to highlight important developments in the field.

### Higher Dimensional Algebraic Geometry, Holomorphic Dynamics and Their Interactions (3 - 28 January 2017)

**10 January 2017**

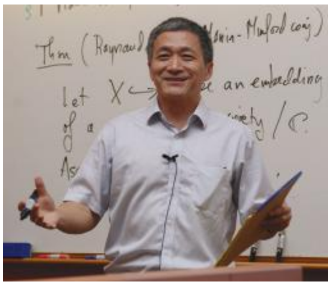
Professor Shou-Wu Zhang (Princeton University) delivered his first lecture "Torsion points and preperiodic points: the Manin-Mumford conjecture and its dynamical analogue".

**12 January 2017**

Professor Zhang delivered his second lecture "CM points and derivatives of L-functions: the Andre-Oort conjecture and Colmez' conjecture".

**17 and 19 January 2017**

Professor Gang Tian (Peking University and Princeton University) delivered his two-part lecture series on K-stability and Kähler metrics.



Shou-Wu ZHANG: Manin-Mumford conjecture and its dynamical analogue

Gang TIAN: K-stability and Kähler metrics

### Geometry, Topology and Dynamics of Moduli Spaces (1 - 19 August 2016)

Website: http://www2.ims.nus.edu.sg/Programs/016wgeo/index.php

**Co-Chairs**

Ser Peow Tan, *National University of Singapore*
Graeme Wilkin, *National University of Singapore*

This program focused on moduli spaces of geometric structures on Riemann surfaces and moduli spaces of Higgs bundles, for which the geometry, topology and dynamics give new information about geometric and topological problems in low dimensions.

In the first week of the program (1-5 August, 2016), the workshop "New perspectives on moduli spaces in gauge theory" aimed to focus on these moduli spaces and their connections to different areas of mathematics and physics. It consisted of a three-part mini-course by Rafe Mazzeo (Stanford University, USA) and 17 invited talks.

The second week was mostly devoted to research discussions and collaborative work. Two mini-courses by Ludmil Katzarkov (Universität Wien, Austria) and François Labourie (Université Nice Sophia Antipolis, France) were delivered, along with three other talks. For the third week from 15-19 August 2016, a second workshop on "Moduli spaces of geometric structures" aimed to focus more on the geometric aspects of moduli spaces. There were a total 19 invited talks in this week.

The program brought together two groups of researchers, one mostly working in gauge theory and moduli spaces of Higgs bundles and the other in the general area of geometric structures in low dimensions. There was ample time for the participants to interact and follow up on collaborative work initiated from previous programs held at IMS in 2010 and 2014. Interaction between the two groups of researchers has received a lot of recognition from the scientific community, with a large multi-year grant from the National Science Foundation (via the GEAR network) and a Luxembourg-Singapore bilateral grant supporting the funding of visitors in addition to the IMS program budget. During the program, 19 research papers/projects were initiated. Altogether 144 researchers and 24 graduate students participated in this program.

Takuro Mochizuki: Asymptotic behaviour of certain families of harmonic bundles on Riemann surfaces

Victoria Hoskins: The Voevodsky motive of the moduli stack of vector bundles

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Rafe Mazzeo elaborating on the asymptotic properties of the Hitchin moduli spaces


Jochen Heinloth and Oscar García-Prada: Mapping action of a subgroup on the moduli space representation


New Perspectives on Moduli Spaces in Gauge Theory

## Automata, Logic and Games (22 August - 25 September 2016)

Website: http://www2.ims.nus.edu.sg/Programs/016auto/index.php

**Co-Chairs**
Anthony Widjaja Lin, *Yale-NUS College and National University of Singapore*
Luke Ong, *University of Oxford*

This program was about computer-aided formal verification (especially model checking) of computing systems, and the mathematical and logical foundations underpinning it. The five workshops in this program revolved around six major themes: (1) Communicating, Distributed and Parameterized Systems; (2) Higher-Order Model Checking; (3) Probabilistic Model Checking; (4) Constraint-Solving; (5) Automatic Structures and (6) Parameterized Real-time Model Checking.

This program is concerned with the mathematical and algorithmic foundations of model checking, an approach to formal verification of computing systems that promises accurate analysis with push-button automation, and has been a truly successful application of logic to computer science. The field is one of the most important and influential research directions in computer science.

There were 18 tutorial speakers, and 83 invited talks in the five workshops. Results from a feedback survey showed that 19 research papers/projects were initiated during or after the program. There were a total of 139 participants which included 22 graduate students.


Luke ONG: Automata, logic and games for higher-type Bohm trees


Marta Kwiatkowska: Tutorial on PRISM


A semantics paradigm for effectful languages (From left: Nikos Tzevelekos, Andrzej Murawski, Steven Ramsay and Martin Sulzmann)


Probabilistic higher-order model-checking (From left: Mikolaj Bojanczyk, Denis Kuperberg and Charles Grellois)


Increasingly distributed and interactive consensus



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## Workshop on Mathematics of Information - Theoretic Cryptography (19 - 30 September 2016)

*— Jointly organized with Nanyang Technological University*

Website: http://www2.ims.nus.edu.sg/Programs/016wcrypto/index.php

**Organizing Committee:**
Ronald Cramer, *Centrum Wiskunde & Informatica and Leiden University*
Venkatesan Guruswami, *Carnegie Mellon University*
Yuval Ishai, *Technion - Israel Institute of Technology*
San Ling, *Nanyang Technological University*
Carles Padró, *Universitat Politècnica de Catalunya*
Chaoping Xing, *Nanyang Technological University*

In recent years, there has been a surge in the interaction between information-theoretic cryptography and several areas in mathematics -- algebraic geometry, algebraic number theory, coding theory, combinatorics and probability theory. The workshop aimed to enrich the understanding of the existing algorithms and techniques, expand the theoretical foundations and develop theories, methods, techniques and software tools that have direct applications to finance, defence, security and healthcare industries that require reliable and secure communication and storage.

The first week of the workshop consisted of the tutorials given by Yevgeniy Dodis (New York University, USA), Stefan Dziembowski (University of Warsaw, Poland), Antoine Joux (University Pierre and Marie Curie, France), Phong Nguyen (The University of Tokyo, Japan), Ignacio Cascudo Pueyo (Aarhus University, Denmark) and Luca Trevisan (The University of California at Berkeley, USA). The workshop in the second week (26-30 September 2016) had 31 invited talks. Results from a feedback survey showed that five research projects were initiated during or after the program. There were 115 participants including 17 PhD students.

Moni Naor: How to share a secret

Monogamy of entanglement and applications to quantum cryptography (From left: Kirill Morozov and Serge Fehr)

Mathematics of Information - Theoretic Cryptography

## Higher Dimensional Algebraic Geometry, Holomorphic Dynamics and Their Interactions (3 - 28 January 2017)

Website: http://www2.ims.nus.edu.sg/Programs/017hidim/index.php

**Co-chairs**
Tien Cuong Dinh, *National University of Singapore*
Keiji Oguiso, *University of Tokyo*
Wing Keung To, *National University of Singapore*

In recent years, there have been breakthroughs in the classification theory of higher dimensional compact algebraic varieties and complex manifolds. Seminal results include the proofs of finite generations of canonical rings of algebraic varieties by Caucher Birkar – Paolo Cascini – Christopher D. Hacon – James McKernan [BCHM06] and Yum-Tong Siu [Siu08]. These results have profound influence on many areas of mathematics – including the study of higher dimensional dynamics and number theoretical dynamics. The interactions of algebraic geometry and the study of these dynamics is exactly the main theme of this program.

The program aimed to discuss and update the progress towards the two conjectures (the existence of minimal model conjecture and the abundance conjecture) and the problem of making the minimal model program equivariant with respect to an automorphism.

This program ran a two-week workshop from 9 to 20 January 2017, covering the major areas of algebraic and arithmetic geometry, holomorphic dynamics with algebraic or number theoretic flavors, and their interactions. There were 46 invited talks and a two-part lecture series from each IMS

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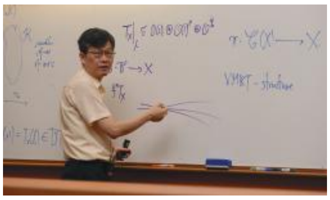
Distinguished Visitor, Zhang Shou-Wu (Princeton University) and Tian Gang (Peking University and Princeton University).

Results from a feedback survey showed that 15 research projects were initiated during or after the program. There were 104 participants including 14 graduate students.

Fabrizio Catanese: Rigid manifolds, Hirzebruch-Kummer coverings, projective classifying spaces

Hélène Esnault: Chern classes of automorphic bundles



Ngaiming MOK: Geometric substructures on uniruled projective manifolds

Idea-building from a variational approach (From left: De Qi ZHANG, Shigeharu Takayama and Sebastien Boucksom)

Dense set of periodic points under the action on algebraic geometry

## Public lectures:

Professor Hugo Parlier of the University of Fribourg, Switzerland delivered a public lecture on "From Puzzles to Moduli Spaces" in NUS on 17 August 2016. Beginning his lecture with the interesting question of defining the distance between two Rubik cube configurations, Professor Parlier gave many interesting examples of exploring geometry in a number of unexpected places. He then concluded the lecture with how puzzles illustrate more sophisticated mathematical objects such as moduli spaces.

Hugo Parlier: From Puzzles to Moduli Spaces

A total of 163 people attended the lecture.

Professor Moshe Y. Vardi of the Rice University, USA delivered a public lecture on "The Automated-Reasoning Revolution: From Theory to Practice and Back" on 1 September 2016. In his lecture, Professor Vardi described how SAT solving algorithms can be leveraged to accomplish other automated-reasoning tasks such as counting the number of satisfying truth assignments of a given Boolean formula or sampling such assignments uniformly at random. These problems have been thoroughly investigated since 1980s. However, approximation algorithms developed by the theoreticians do not scale up to industrial-sized instances. On the other hand, algorithms used by the industry offer better scalability but possibly at the expense of certain correctness guarantees. He concluded the lecture with a novel approach, based on universal hashing and Satisfiability Modulo Theory, capable of scaling to formulas with hundreds of thousands of variables without compromising correctness guarantees.

Moshe Vardi: The Automated-Reasoning Revolution: From Theory to Practice and Back

A total of 70 people attended the lecture.

## Current Program

### Oppenheim Lecture (15 February 2017)

*— Jointly organized with Department of Mathematics, NUS*

Website: http://ww1.math.nus.edu.sg/events.aspx?f=oppenheim-lecture2017

**On an Effective Proof of the Oppenheim Conjecture (joint work with G. A. Margulis)**

By Elon Lindenstrauss, *the Hebrew University of Jerusalem and Princeton University*

Activities Held in Conjunction with Oppenheim Lecture

- Workshop on Ergodic Theory & Dynamical Systems (14 – 16 February 2017)
- Conversation with Professor Lindenstrauss (16 February 2017)

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### 2nd NUS-USPC Workshop on New Challenges in Financial Risk Control (11-12 April 2017)

*— Jointly organized with the Centre for Quantitative Finance, NUS*

Website: http://cqf.nus.edu.sg/events.aspx?e=ws2017_2nd_NUS-USPC

**Organizing Committee:**
Jean-François Chassagneux, *University Paris Diderot*
Min Dai, *National University of Singapore*
Noufel Frikha, *University Paris Diderot*
Steven Kou, *National University of Singapore*
Huyên Pham, *University Paris Diderot*
Chao Zhou, *National University of Singapore*

## Programs & Activities in the Pipeline

### Complex Geometry, Dynamical Systems and Foliation Theory (1 – 26 May 2017)

Website: http://www2.ims.nus.edu.sg/Programs/017geo/index.php

**Organizing committee**
Tien Cuong Dinh, *National University of Singapore*
George Marinescu, *University of Cologne*
Xiaonan Ma, *Université Paris Diderot - Paris 7*
De-Qi Zhang, *National University of Singapore*

This program concerns the recent developments in complex analysis and its applications, and brings together experts working in these topics with an interest in pluripotential theory. Pluripotential Theory, a branch of Complex Analysis, is a very important tool with applications in many areas of mathematics: Complex Analysis, Complex Differential Geometry, Complex Algebraic Geometry, Dynamics, Foliations and Mathematical Physics.

#### Activities

- Informal Discussions and Seminars: 2 - 5 May 2017 and 22 - 26 May 2017
- Mini-workshop on Complex Analysis and Geometry: 3 - 4 May 2017
- Mini Courses: 8 - 12 May 2017
- Conference: 15 - 19 May 2017

### Geometric Structures and Representation Varieties (3 - 5 May 2017)

*— Jointly organized with Fonds National de la Recherche, University of Luxembourg and Department of Mathematics, NUS*

Website: http://math.uni.lu/schlenker/sling/

**Organizing committee**
Jean-Marc Schlenker, *University of Luxembourg*
Ser Peow Tan, *National University of Singapore*

The workshop will focus on the geometric structures on low-dimensional manifolds, representation varieties, and related questions.

### Data Sciences: Bridging Mathematics, Physics and Biology (29 May - 16 June 2017 and 30 November - 8 December 2017)

Website: http://www2.ims.nus.edu.sg/Programs/017data/index.php

**Co-chairs**
George Barbastathis, *Massachusetts Institute of Technology*
Hui Ji, *National University of Singapore*
Patrice Koehl, *University of California at Davis*

There are three sub-themes under the program: (1) Frame Theory and Sparse Representation for Complex Data, (2) Geometry and Shape Analysis in Biological Sciences, and (3) Computational Methods in Bio-imaging. This program will focus on the mathematical foundations and their applications. For examples, data-driven frame theory, frame theory for high-dimensional data and graphs, sparse representation of large data, efficient optimization methods; geometry and topology for representing, searching, simulating, analyzing, and visualizing biological data as well as the biological systems they represent; the theory underpinning quantitative phase imaging, and the opportunities in data sciences arising in biological imaging. Overall, this program will establish tangible links between theories and applications in data sciences, and provide a practical platform for interdisciplinary collaboration.

#### Activities

- Workshop on Frame Theory and Sparse Representation for Complex Data: 29 May - 2 June 2017
- Tutorial on Frame Theory and Sparse Approximation: 5 - 6 June 2017
- Tutorial on Geometry and Shape Analysis in Biological Sciences: 8 - 9 June 2017

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- Workshop on Geometry and Shape Analysis in Biological Sciences: 12 - 16 June 2017
- Tutorial on Bio-imaging: 30 November - 1 December 2017
- Workshop on Computational Methods in Bio-imaging Sciences: 4 - 8 December 2017

## IMS Graduate Summer School in Logic (19 June - 7 July 2017)

— *Jointly organized with Department of Mathematics, NUS*

Website: http://www2.ims.nus.edu.sg/Programs/017logicss/index.php

The Summer School bridges the gap between a general graduate education in mathematical logic and the specific preparation necessary to do research on problems of current interest in the subject.

### Activities

- Week 1: Lectures by Artem Chernikov (University of California, Los Angeles) and Steffen Lempp (University of Wisconsin)
- Week 2: Lectures by Theodore A. Slaman (The University of California, Berkeley) and Steffen Lempp (University of Wisconsin)
- Week 3: Lectures by Hugh Woodin (Harvard University)

## Quantitative Methods for Drug Discovery and Development (19 June - 14 July 2017)

Website: http://www2.ims.nus.edu.sg/Programs/017quan/index.php

**Co-chairs**

Wei-Yin Loh, *University of Wisconsin-Madison*
Weng Kee Wong, *University of California, Los Angeles*

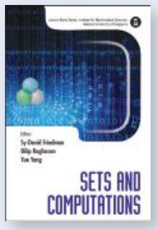
The program will bring together international experts from academia and the pharmaceutical industry to share their knowledge and discuss research ideas. There is already a U.S. industry working group called "Quantitative Sciences in the Pharmaceutical Industry" dedicated to sharing information on exploratory and confirmatory techniques for subgroup identification and analysis. The proposed program will further this goal as well as provide the opportunity for academic and industrial statistics professionals to learn from each other. It will also be of interest to health and medical professionals in the pharmaceutical and biotechnology community in Singapore, where more than thirty of the world's leading biomedical companies have offices.

### Activities

- Tutorial on Regression Tree Methods for Precision Medicine: 19 - 30 June 2017
- Tutorial on Medical Product Safety: Biological Models and Statistical Methods: 19 June 2017
- Workshop on Design of Healthcare Studies: 3-7 July 2017
- Workshop on Perspectives and Analysis Methods for Personalized Medicine: 10-14 July 2017

*For full list of upcoming events, visit our webpage at http://ims.nus.edu.sg/*

## Publications >>>

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**Sy David Friedman** *(University of Vienna)*,
**Yue Yang** *(National University of Singapore)*

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## Mathematical Conversations

# Cédric Villani: Ambassador of Mathematics *Extraordinaire* >>>

Cédric Villani

Interview of Cédric Villani by Y.K. Leong

Cédric Villani made fundamental contributions to the study of the change to Boltzmann equation [Ludwig Eduard Boltzmann (1844-1906)] in statistical mechanics, nonlinear Landau damping [Lev Davidovich Landau (1908–68)] in plasma physics and the theory of optimal transport. He is also well-known for his extraordinary efforts in promoting mathematics to the public.



In addition to major contributions to the theory of partial differential equations occurring in statistical mechanics, Villani provided a deep mathematical interpretation of the physical concept of entropy with unexpected ramifications in mathematics as well as physics. Jointly with Laurent Desvillettes and Giuseppe Toscani, he gave a rigorous answer to the problem of convergence of solutions not near to equilibrium for uniformly smooth solutions. With Clément Mouhot, he settled the problem of nonlinear Landau damping and vindicated the counter-intuitive conjecture of the Russian physicist Lev Landau (1908–68) about the behavior of plasmas. Another ground-breaking contribution is his joint work with Felix Otto which makes a surprising connection between gas diffusion and optimal transport theory, thus linking statistical physics and economics. An off-shoot of this connection is the modeling of the motion of a gas in an abstract landscape in which the theory of Ricci curvature can be applied. This interpretation enabled Villani, in joint work with John Lott, to arrive at a new and deeper level understanding of curvature.

He was awarded the Fields Medal in 2010 "for his proofs of nonlinear Landau damping and convergence to equilibrium for the Boltzmann equation." He has received numerous awards for his mathematical work: Louis Armand Prize, Peccot-Vimont Prize, Jacques Herbrand Prize, European Mathematical Society Prize, Henri Poincare Prize and Fermat Prize.

Villani was educated at the Lycée Louis-le-Grand and the École Normale Supérieure in Paris. He obtained his doctorate from Paris Dauphine University under the supervision of Pierre-Louis Lions (Fields Medal 1988). From 2000-2010 he was professor at the *École Normale Supérieure de Lyon*. In 2010 he moved to Lyons University (*Université Claude Bernard Lyon I*) and also serves as the director of *Institut Henri Poincaré,* which is the mathematical research part of University of Paris VI (*Université Pierre-et-Marie-Curie*). In 2011 he coordinated the setting up of a national centre of excellence *Laboratoire d'Excellence CARMIN* (*Centres d'Accueil et de Rencontres Mathématiques Internationales*) which is financed by *Investissements d'Avenir* and which comprises all the French national mathematical institutes *Institut Henri Poincaré (IHP), Institut des Hautes Études Scientifiques (IHÉS), Centre International de Rencontres Mathématiques (CIRM), Centre International de Mathématiques Pures et Appliquées (CIMPA)*.

Besides delivering invited lectures at many leading universities and major scientific meetings, he has made extraordinary efforts in promoting awareness in mathematics and reaching out to the public not only in France but around the world from the United States to Asia. His willingness and commitments in media presentations and participation clearly reflect a personal mission which is unmatched in the mathematical community. The fervor with which he spreads the message of the ubiquity and relevance of mathematics in daily life is so strong that the media has variously dubbed him as an ambassador, evangelist and proselytizer for mathematics. Members of the public who meet him for the first time will be invariably and pleasantly surprised at the colorful style of his dressing (a three-piece suit, velvet cravat, pocket watch and brooch) which one does not usually associate with a mathematician. In actual fact, this signature dressing of his has been adopted by him when he was a student at the age of 20. Together with his personable style, this has undoubtedly presented a friendlier and more approachable image of mathematics

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to the public. It is fair to say that no scientific personality in France has received as much national recognition as he has with the national awards of Chevalier in the National Order of Merit and of the Legion of Honor and medals from the National Assembly, *Conférence des Grandes Écoles* and the cities of Lyon, Brive and Toulouse.

In spite of the toll on his research schedule, he continues to be actively involved in both voluntary professional and obligatory administrative work. He serves on the editorial boards of major journals such as *Inventiones Mathematicae, Journal of Functional Analysis, Journal of Mathematical Physics* and *Journal of Statistical Physics*. He has also served in scientific bodies such as the pro-European Think-Tank EuropaNova and the Scientific Advisory Board of the Panafrican institute, African Institute for Mathematical Science (AIMS), of which he is currently the vice-chair. He has recently been appointed a member of the High-Level Group of the Scientific Advisory Mechanism of the European Commission (the Scientific Committee of the European Commission).

In May 2015 French President François Hollande alarmed and shocked the French scientific community with a proposed budget cut of 256 million euros in national spending for research and higher education. In a bid to reverse this unexpected policy move, five of France's illustrious Nobel laureates and one Fields Medalist met with the President. The Fields Medalist who actively gave his unwavering active support for the wider good was Villani. This unprecedented historic meeting resulted in the President agreeing to repay the whole cut, but only a part of it could be repaid that year because of budget timing issues. [The part which was immediately repaid was the critical one. - Villani]

In addition to numerous research papers, articles and monographs, he has written a popular book which was first published in French in 2012 as *Théorème Vivant* and had become an instant best-seller in France. This was translated into English as *Birth of a theorem: A mathematical adventure* by Malcolm DeBevoise and has received rave reviews. It gives a blow-by-blow account of a two-year research collaboration with his former student Clément Mouhot on a 60-year old mathematical problem (Landau dampening) in plasma physics. It attempts to portray the surreal realm of mathematical research with all its mental agony and joy set amidst the real world of human existence with all its passion and ambition.

Villani was on the organizing committee of the IMS (Institute for Mathematical Sciences, National University of Singapore) program *Hyperbolic Conservation Laws and Kinetic Equations: Theory, Computation, and Applications* (1 November - 19 December 2010). He was invited to the inaugural Global Young Scientists Summit (in Singapore) organized by the National Research Foundation of Singapore from 20 to 25 January 2013. Modelled after Germany's Lindau Nobel laureates meeting, it invited and brought together 280 post-doctoral fellows, PhD students and research scientists from Singapore and developed countries in a brain-storming atmosphere to pick the brains of 15 leading scientists who have won the Nobel Prize, Millennium Prize, Turing Award and Fields Medal. Villani was the Fields medallist invited. He gave a talk "On curvature, gas and human beings - From Monge to Boltzmann to Riemann" on 22 January 2013 at the Institute for Infocomm Research in Fusionopolis. During his short visit, Y.K. Leong took the opportunity to interview him on behalf of *Imprints* on 23 January 2013 at the Department of Mathematics, National University of Singapore. The following is an edited and vetted transcript of the interview in which he spoke with frankness and passion about his first love (Boltzmann equation) which led him to revolutionize mathematically the physical concept of entropy and his other love (popularization of mathematics) which has given him an international celebrity status. We also get a rare insight into the grand scientific tradition that has produced France's illustrious engineers, scientists and engineers, scientists and mathematicians.

**Acknowledgment.** Y.K. Leong would like to thank Eileen Tan, Senior Executive of the Institute for Mathematical Sciences, National University of Singapore for her help in preparing a raw draft of the transcript of the interview.

***Imprints (I):*** What made you get interested in questions in mathematical physics?

***Cédric Villani (V):*** Okay, first to tell the truth, if one looks back on my early training, it is a big surprise that I have been into mathematical physics. I was not very good in physics as a high school student. I mean, I had good grades but nothing spectacular. There was a big gap between my ease

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[ability] in physics and my ease [ability] in mathematics. After I entered L'Ecole Normale Supérieure, it was clearly the mathematics specialty [for me] although I did follow a few courses in physics and even one course in chemistry for a little while, done just out of curiosity. But later I went back to questions of physics. One of the main reasons was just accidental and the fact that my tutor of the time, Yann Brenier in École Normale Supérieure wanted me to work with Lions. And he suggested that the subject of Boltzmann equation was very good to put me as an equal to Laurent Desvillette, who was an assistant professor before I took the position. And before that, my goal was to do something like image processing, for instance. I did not think of doing something in mathematical physics. The second factor was that when I had to choose my PhD subject, not only was there this incentive of Brenier for me to work with Lions but also for some reason it was in the middle of a period of doubt for me whether mathematics was a good thing to do. And I was a bit depressed about the potential of mathematics activity and it was important for me at that time to work on a subject that would have real implications to, so to speak, real life, like the work of an engineer and so on. And so it's one of the questions that I asked Lions the first time we met when he presented me with the Boltzmann equation: "Is it really useful, etc?" And for me it was important. However, after some time spent with the darned equation, I literally fell in love with it and I found it too beautiful for many reasons – for reasons of history, for conceptual reasons and so on – that I really didn't care if you can have applications or not. At some point it was really the subject in itself that I liked and then I started to understand a lot of physics. I am one of those people who don't have physical intuition but that can be developed by looking at the mathematics. So I managed to understand some points about physics that physicists had missed because I came with a mathematical intuition. It's important that we work with different points of view to have a richer result. And I think one of my particularities is extreme curiosity. I read books about the Boltzmann equation, etc, discussed with people and so it grew until I became quite a decent mathematical physicist. It is one of my great pride to have won the Henri Poincaré Prize of Mathematical Physics in 2009. This really meant much to me especially given that I was not from the start gifted in physics.

***I:*** In some sense you are a self-made physicist.

***V:*** In some sense, yes. When you go in, you sometimes see that the distinction between mathematics and physics is very much blurred. Some of the great physicists of the past were, in fact, mathematicians as well, like [Isaac] Newton [Isaac Newton (1642-1726)]. The influence of Boltzmann on mathematics was considerable. Even in the 20th century, the work of the physicist Landau is very much mathematical. What changes for sure is that physicists and mathematicians don't always have the same appreciation of what are the important problems, the intuition and, of course, the appreciation of whether something is proven or not. One should not underestimate the role of the vocabulary. I think one of my strengths is that I am rather good at adapting my speech to the persons that they have in front of me. The physicists like to invite me because they know that I will adapt the vocabulary and concepts in such a way that they can understand.

***I:*** Did Pierre-Louis Lions have any influence on your philosophy of research?

***V:*** Obviously, yes. You know, the relation that you establish with your PhD advisor is usually a strong one. And you often define your identity with respect to this either in the same direction or by your own position. I like to think that there were four researchers that influenced me a lot during my early career. So, one was Lions and from him, in particular, I retain the importance of working on hard problems and unleashing whatever mathematical tools from analysis there are for a certain problem. And the second one was Yann Brenier (I mentioned earlier that he had an influence on me choosing Lions). He (Brenier) is a specialist in fluid mechanics. He was my tutor in École Normale Supérieure. He also was the head (director) of studies for the math department at that time. He pushed me to work with Lions. He also put me in contact with one of my first collaborators, Felix Otto, which was extremely important. And he [Otto] initiated me into the field of optimal transport of which he was a specialist. And I gained a lot of influence from him – this time very different from that of Lions. And, in particular, from Brenier I retained the importance to work on simple problems and to look for structure coincidences. Lions was more like let's do it with force and we want general things that are not related to a particular nature of the solution. With Brenier it was more of the inverse – let's try to find the special nature of things and their relation. And it's work on the simple

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things that Brenier was very excited about – discovering connections more than proving rigorous theorems. It had a lot of influence on me. Neither Lions nor Brenier gave me a lot of technical advice and Lions was very busy. I did not see him much and this was rather good for me because I wanted to be independent. But both of them always listened to my suggestions and what I had to say with a benevolent ear. Then there was Eric Carlen from Rutgers, who was one of the first to introduce tools from information theory into the field of the Boltzmann equation. Carlen had worked with Elliot Lieb and other people and had been raised in a spirit in which you are living in a beautiful world with nice inequalities, problems related to quantum physics, sharp inequalities and so on, and he had managed to move on to areas such as fluid mechanics and the Boltzmann equation which are much more "dirty" and hands-on and in which there is much more trouble. He arrived with new ideas and I admired very much his way of working. There was him and there was also Michel Ledoux, a well-known probabilist at the interface between analysis and probability, also working on topics related to Carlen's. Both Ledoux and Carlen were crazy about inequalities. Whenever there is a good problem there should be a nice inequality behind it and it is very clear from my PhD that you see the influence of these four people appearing in the various papers and sometimes combined. And at some point I diverged from some of them; for instance, I started with Lions in the field of generalized solutions, renormalized solutions of kinetic equations, and at some point I decided that, on the contrary, I would never again work on a generalized solution and only worked on classical solutions. While a lot of the work of Lions was his beautiful use of compactness theory, it was absolutely just his way to use compactness. At some point I decided – no more compactness – I would do things completely explicitly. So part of my style was inherited from him and part of my style was more like determined in contradiction with him.

***I:*** I counted twelve Fields Medalists who were originally of French nationality. This is quite a high proportion for a nation of moderate size (about seventy million) which is about between that of UK and Germany. UK and Germany has eight and one Fields Medalists respectively. Do you think that this could be the result of a long tradition of unfettered rational enquiry instilled since the time of Blaise Pascal[(1623-1662)] and René Descartes [(1596-1650)], perhaps reinforced by the spirit of the French Revolution.

***V:*** Okay. I'm not sure about the numbers but you can check it. I thought it was eleven, maybe twelve, of French nationality, maybe it's a bit less but the order of magnitude is this. It's true that it's a huge number and very high number among the promising internationally recognized young mathematicians (I mean in their thirties). Nowadays there are even a lot of French ones. I think that indeed, in part, this is a result of a long tradition. It is true that rational enquiry plays a role and the French take pride in this rational sense since the time of Pascal and Descartes. There is also a tradition of elegance and imagination, mostly simplified by the work of another of the great French mathematicians of the time, I mean Fermat [Pierre de Fermat (1607-1665)]. Extraordinarily inventive. Another, although a bit less important mathematician of the time was Girard Desargues [(1591-1661)] who was I think very influential and important, next to Pascal and Descartes, in his time. So indeed, the French people are known to like things that are rational and to like things a little abstract. It has often been rightly contrasted to a more hands-on practical spirit of the Anglo-Saxons and it is true also that the spirit of the French Revolution reinforced this at a time in which the political power was very much fond of revolution, very much fond of scientists. Let me comment more on the history of France. First, the cultural history of France is well-known for being one of the centers of the Enlightenment period. France is one of the more developed countries where a lot of people take pride in thinking and it was not only the philosophers but the mathematicians and the scientists. It all went together at the time. It was a matter of pride to be well versed in the sciences, at least for some of them like Voltaire [François-Marie Arouet (1694-1778)], Diderot [Denis Diderot (1713-1784)], Condorcet [Nicolas de Condorcet (1743-1794)], etc. And so the Enlightenment period is important. Then came the revolution. In both cases not only did it come with great people but also with great institutions. For instance, for engineers the *Arts et Métiers [ParisTech]* school was founded in the middle of the 18th century. Many of the schools that are most famous in France nowadays, were founded around the time of the revolution. And then came the Empire. Both the Empire of Napoleon I[Napoléon Bonaparte (1769-1821)] and Napoleon III [Louis-Napoléon Bonaparte (1808-1873)] were very much favorable to science, especially Napoleon I. It is known that he was able to sustain a scientific discussion with the best scientists of his time and he was not a scientist himself. It is very clear that if he had wanted, he could have become

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a top scientist. This is remarkable and the people in the government made sure that science was enforced.

Just for me to come here, in particular, one of these reasons is that you are one of the rare countries in the world in which at the highest level of the state you find scientists. And, by the way, we can see here that the state seems to be pretty efficient in setting up a national scientific program. So, in France you see from the history of the institutions, they are continued after the Empire. There was another big period which was favorable to the sciences, which was between the two world wars. At least from this, plans for INRIA (*Institut national de recherche en informatique et en automatique*) started and some important institutions were founded in France again. And also at that time they took seriously the problem of communicating to citizens. Now what can make a field successful is that some famous people act as role models and propagate the culture of the institutions that I spoke about. And another thing that is important is the idea of the formation of a community. And here also the organization of France was very helpful with the big cultural center Paris, very large and predates the wars. Paris has remained since the 18th century a mythical city for mathematics. No other city can compete with Paris in terms of the number of mathematicians. Of course, in some of the most prestigious American universities, you will find a high proportion of very famous researchers but in none of these famous universities you have the same density as in the Paris area. So this played an important role.

***I:*** And it seems to me that the cultural center of Paris has gone continuously, uninterrupted even by the wars, isn't it?

***V:*** In some sense this is true that Paris remained an important cultural center in spite of the wars. Its reputation remains to this day. It is still a place that makes people dream. However, things were difficult at times. After the First World War, the system was kind of bankrupt, and communications with the outside world were very difficult. My institute, *Institut Henri Poincaré* was born precisely with the goal to revive the changes by attracting foreign visitors and researchers. Nowadays we do have many structural problems in France about academia. The university system has become very complicated. People are grumpy and there is a lot of tension. The government has been a bit clumsy in trying to make things move. There are many problems, bureaucracy has increased – not only in Paris, by the way, but in many other developed countries we also see these tendencies. Let me add that there has been over the past decades (although it is right now reversing a bit) during the period 1980-2000 approximately, a big scientific and cultural development in regions outside Paris. I belong to this generation. I made my important discoveries in Lyon; I consider myself to be from Lyon. It was typical of many young people seeking better living conditions and, after being trained in Paris, to travel outside to develop themselves more freely than they would have in the Parisian environment.

***I:*** France has a long and continuous tradition of fundamental research at the interface of mathematics and physics, starting from Pascal, Laplace, Lagrange, Poisson, Fourier and others. Has this tradition been somehow woven into the fabric of the school system or at least into the university system of the main universities in France?

***V:*** In the school system the short answer is "No". And people getting out of the French school system have a very poor idea of the relation between mathematics and physics. Some systems do it much better. When you discuss, for instance, with French journalists they are often very surprised to hear of all the connections between mathematics and physics. They just do not realize how useful mathematics is for the study of physics; they think it's some form of abstract game. On the contrary, often I noticed journalists from Germany know this very well – that in many cases mathematics is a way to solve a physical problem. In the UK too, it's more practical. So, in the first approximation, we have failed to implement this link, and school teachers in mathematics in France are very uneasy with physics. These things tend to change because it has been understood and recognized and we are making progress on this. At the university level, it is not that well implemented either. And again you usually would develop mathematics or physics in France in a rather specialized way. There are countries in which the distinction is much more blurred. I see it in my colleagues who have been trained in Germany or in Italy – for them it's much more mixed. Some of them don't even bother to decide if they are mathematicians or physicists. In France we have this tendency rather to separate things into categories and define ourselves as in this category or that category, etc. I never wanted to belong to a particular category and always look for ambiguity in a way. All that being said, it is true we have a great tradition of fundamental research at the

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interface of mathematics and physics. As you know, as an example, I'm at the interface of mathematics and physics but I really had a mathematical training and it's only when I started my PhD that I really went into physics. Nowadays I regret that I don't have a better physics feeling in some stuff. This morning I was visiting the Quantum Computation Center here – beautiful center, all the experiments are nice and I had just a vague idea what they are doing and I wished I had a better intuition of all those things they were doing. Okay, I know that if I really want to, I can study and understand exactly what they are doing. But this will require some effort.

***I:*** I believe that the family name of Villani has Italian roots and that you speak Italian. Is your interest in Italian related to the roots of the Villani family?

***V:*** I think so, unconsciously, definitely. This is a part of my origin. Villani means "peasant" in Italian as opposed to "nobleman". Because I don't like categories I like the fact that my first name is a typical noble name from the Anglo-Saxon world and my family name is, on the contrary, a peasant name from the Mediterranean world. I also have a Greek origin as well as French both from the southwest and from Paris. I also have some origin from the Alsace region. And my Greek ancestors went to Corsica. They also were in Algeria. I am a heir of so many places. I like this too. Nowadays it is well perceived to be a mixture. I am a "mix" at a scientific level and also a "mix" at a personal level. Now, I did not learn Italian from my family. My father speaks some Italian but I did not learn with him. I learned by myself when I started during my first stay in Italy in 1997. So it was important for me to learn Italian and I did it in a serious way. I was a student at that time and so I was housed with many other students. I had asked everybody to speak to me only in Italian. I had this grammar that I would work on every night before going to sleep and so I learned seriously and very quickly. I am not very fluent in Italian because I did not practice since then. But after a few weeks I could sustain a conversation. If I had continued a bit, now I will be fluent for sure.

***I:*** The concept of entropy arises from both the Boltzmann and Vlasov [Anatoly Alexandrovich Vlasov (1908-1975)] equations which are both classical in nature. Does entropy manifest itself mainly at the macroscopic level? Does entropy have any significance at the subatomic level?

***V:*** It is true that the concept of entropy was formalized first by Boltzmann in classical mechanics and plays also an important role in the Vlasov equation, I would say by contradiction, in the fact that the Vlasov equation preserves the entropy. As a physical implication, it's completely different from the behavior of entropy for the Boltzmann equation. The concept of entropy, by definition, is a macroscopic one but it depends what you call macroscopic. The concept of entropy arises as soon as you have a discrepancy between several levels of description. There is a microscopic and a macroscopic level. The difference in the information that you gain at the macroscopic level from the information at the microscopic level will generate entropy. So whenever there are two different scales, two different degrees of accuracy in the description, you will have entropy coming in. At the subatomic level it is hard to know what it would be. I'm not aware of entropy defined at the subatomic level. I am aware of entropy which arises in a quantum context for gas of boson particles or something like this but, always, you need to have something which is macroscopic with respect to something else.

***I:*** In your work on optimal transport theory, the distribution of goods is likened to a configuration of gas particles and a finer configuration to an equilibrium state. This is a striking analogy between real life scenarios and inanimate physical systems. It seems that energies do play a role in bridging different disciplines. How much has energy played a role in your thinking?

***V:*** I think analogy plays an enormous role in my thinking, probably also for many other researchers. Analogies give you hindsight for finding relations and many of my works were about discovering, bumping into some unexpected relation, and then analogy helps you to think of proofs. You introduce an analogy; then it will naturally generate some link or direction where to pursue your reasoning. I like to offer it in my expository lectures and in my books. I like to put analogies. I put detailed proofs often but then I also explain in words what is the strategy of the proofs and so I often put analogies to help the reader form a mental image of the field.

***I:*** Some scientists believe that there is an unexplored potential in applying physical models and theories such as gauge theory to economics and quantitative finance. Have you ever thought of applying your ideas to mathematical finance?

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***V:*** I'm not too keen about mathematical finance and I have never considered working in that area myself. However, some people are very good at playing ideas from classical physics (statistical mechanics) to finance. In France, the two best known people in that spirit are Jean-Philippe Bouchaud and Cont (Iranian by origin). They come from statistical mechanics like the study of hydrodynamic limits of particle systems, things like this, and they applied their reasoning and intuition to problems in finance. Here again analogies play a very important role. And they obtain some very interesting results, some great new points of view. I think it is completely true that, in part, financial exchanges can be thought of as physical systems with some strange rules that often are not clearly known. All the more with the proliferation of everything in directions that came with high speed trading. I have heard, for instance, George Papanicolau advocating for taxing on financial exchanges based on the idea that dissipative activity will stabilize the system, thinking of these exchanges as a fluid mechanics problem. I think this is a very interesting analogy. I think also that one has to, by the way, mention this because, you know, after the 2008 financial crisis, many people blame mathematicians for devising bad formulas and so on, and they forget that in most cases the biggest problems came from the application of mathematical theories outside the conditions for which they had been devised. People knew theories like the one you mentioned without caring really if the assumptions were there.

***I:*** You have been very active in promoting mathematics to the general public. I think Wendelin Werner, your compatriot and Fields medalist in 2006 also believes that there is a need to improve communication between the public and mathematicians. But university academics are generally caught up in the process of publication papers for the purpose of tenure and so on. How do we reconcile one's personal need and the more professional responsibility at the community level?

***V:*** Well, this is tricky. First, yes, Wendelin also worked hard on improving communication between the public and the mathematicians, quite more than some of his predecessors. And I worked even quite harder than Wendelin on this. For me there was an opportunity that presented itself in the sense that the media response to my personality was extremely strong. There were factors which had nothing to do with mathematics such as my way of dressing, the fact that I am rather fluent on TV and radio – things like this. Recently I did, I think, a bit of a daring experiment since I published a popular book that goes completely at odds with the usual standards of communication from mathematicians, and this has been one of the library's successes of this year and has been a big boost again. Nowadays we get invitations for participating in public debates or broadcasting, either public lectures or radio interviews. So I think that one of the reasons why the response has been very strong is that precisely mathematicians usually are not so prepared to do it and there was a need and then I was ready to do this job. It is very much demanding and I understand very well why university academics don't have so much time to work with the media or they may be a bit shy. First they don't like to be exposed personally. They know that it is a whole community that should be exposed. They are afraid of the fact that the media work at a very fast pace. They write many mistakes; they don't care. All these make mathematicians uneasy. Nobel laureates, I think, are more prepared than we are for that. I could see here that the Nobel laureates invited in GYSS [Global Young Scientists Summit] were on the whole rather good in making a good show with the audience, making people laugh, talking with passion, etc. Mathematicians are less familiar with this. I guess it has to do with the fact that these Nobel laureates also had to fight for funding, they had to be team leaders; it's much more important than in mathematics. And so they developed all these communication skills I think more than the mathematicians. The question that you are asking (the way to reconcile personal needs and responsibilities at community level) is a really tricky one. We know that various activities can go into negative interaction together, becoming cumbersome for each other. On the other hand, things like teaching and research go well together. Often they reinforce each other, but administration, not so. It takes time and when you are in administration, first you need to handle the ego of people. You need to resolve some conflicts. You need to make decisions at a rather fast pace; you have to make a decision by a certain date whereas in research you can take your time to explore, check, investigate. When you go to a media presentation, it's even worse. Sometimes you have to decide by the hour. You have to give short interviews and then it's a question of minutes. Sometimes you are invited to some tea and you have thirty seconds to give an answer, or three minutes – then it's not bad – and so sometimes it's longer. So I don't have a general answer to your question. It is quite

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difficult to reconcile the personal needs of the researcher and professional responsibilities. And it's obvious that one has to protect some time. One has to make sure you are in an environment in which you don't lose your time. But apart from the general things I cannot really say. I have not managed myself really to resolve this because it is very clear that since the past year or two my presentation activity in particular has been a nuisance for my research activity. With the administrative activity it was okay. I managed rather well to do the two together in particular because I have a dedicated staff to help me at the *Institut Henri Poincaré*. But with the media activity I did not manage well. I will have to reduce the media activity if I want to increase research again.

***I:*** Would you agree that mathematicians are somehow, by nature, more introverted, so that exposure to the media is less attractive?

***V:*** On the whole, maybe mathematicians are a bit more introverted, and maybe they are more fussy about details. And then maybe they are less at ease with the media but this rule has important exceptions both within mathematicians and outside mathematicians. I think the way that the community looks at these activities has been changing and that people don't regard this nowadays as just a nuisance. The need to communicate is regarded as part of their duty as scientists to talk with society, also the duty of civil servants. Most of the researchers in France, at least, are state employed.

***I:*** You have once said that computer simulation gives important insight into understanding phenomena and formulation of theories. The computer technology and hardware have improved by leaps and bounds. Yet basically at the software level the corresponding advances have not been so dramatic and far-reaching. Do you think that in the near future there will be some fundamental breakthrough in the theoretical understanding of algorithms and computer science that will contribute to the solution of fundamental problems in mathematics and physics such as the Navier-Stokes equations?

***V:*** Progress in theoretical understanding of algorithms is stronger than one usually believes. There was a study, I think, by the AMS [American Mathematical Society] or maybe the EMS [European Mathematical Society] a few years ago about certain benchmarking algorithms that they use often in computer science and it was shown that over a certain period of time (I don't remember how much) the factor in time that has been gained from the progress in technology was comparable to the factor in time that would have been gained through the algorithmic. So, well-thought algorithms can really make a change and has really made changes on the efficiency. Now it is true that globally the software level lags behind the technology level. I am not sure whether this discrepancy will continue or whether, on the contrary, we will reach a limit in new technology and then know that progress would have to be made on the algorithms and their theoretical understanding. I think that we are making a lot of progress currently on the theoretical understanding of algorithms and there are all these fascinating works about the art of automatic verification and so on. However, I really don't see anything emerging for more complicated systems such as Navier-Stokes or Boltzmann equations or the problems I've been working on. I really don't see them occurring. I cannot imagine how this can occur. But this may be just a question of limitation of my imagination. Yes, it is clear that theoretical computer science has beautiful days ahead. As a sign of the times, in 2014, we at *Institut Henri Poincaré* shall house our first trimester devoted to theoretical computer science, its program centered precisely on these questions of logic of computing and formal proofs and automatic programs to check automatically the very details of the proof. I think this is one of the exciting areas of computer science nowadays. And we can ask ourselves if some day some computer programs will be useful as a help for finding proofs. For the moment, it's very far away but then at one time people believed that computers could never beat human beings in chess and now they do this routinely. So soon we may see computers able to check complicated proofs. Who knows in time to come, we will have the computer help us find complicated reasoning.

## Marie-France Vignéras: From Lycée to Langlands, *L'Express Femme* 100 >>>

Marie-France Vignéras

Interview of Marie-France Vigneras by Y.K. Leong

Marie-France Vigneras made important contributions to number theory, representation theory and the Langlands program, which provides a direction for a grand unifying theory for algebra, analysis and geometry.



Vigneras' early work in number theory resulted (in 1978) in the first examples of pairs of non-isometric, isospectral closed hyperbolic 2-manifolds and 3-manifolds, constructed using quaternion algebras associated with quadratic extensions of the rationals. From there she went on to modular representations, automorphic forms and problems arising from the Langlands program. This program springs from a web of conjectures, proposed by the Canadian mathematician Robert Langlands in a famous letter (1967) to André Weil, linking objects in algebra (Galois representations, congruences over finite fields) to objects in analysis (automorphic forms, infinite-dimensional representations). It has expanded into a vision of research with the goal of unifying algebra, analysis and geometry. One of her important contributions in this direction is the establishment (in 2000) of the mod-$l$ local Langlands correspondence for $GL(n)$. Currently, she is working on the recent exciting developments in the fast developing $p$-adic Langlands program.

Born in Senegal, Africa, Vigneras had her school education in the Lycée Van-Vollenhoven in Dakar. After her baccalaureate in Senegal, she moved to the University of Bordeaux where she obtained the *agrégation de mathématiques* in 1969 with the intention of becoming a school teacher. Her mathematical talent was quickly recognized and she was encouraged to go on to do research. In 1974, she obtained the *doctorat d'Etat* with a thesis on quaternion algebras, written under the supervision of Jacques Martinet. This interest in number theory led to her famous paper of 1978.

Vigneras started her research career with short stints in CNRS (*Centre national de la recherche scientifique*), and University of Paris XI (Orsay). In 1977 she was appointed director of the Centre of Mathematics of École Normale Supérieure Sèvres, a position which she held until 1983 while she rose through the ranks of *Université Paris Diderot* (University of Paris VII) from the position of *maitre de conférences* (lecturer) in 1977 to full professorship in 1985. She retired as emeritus professor in 2010 and continues to contribute to research developments.

She has been actively involved in the participation and organization of mathematical activities within France and without. She has expressed strong support for the cause of free access to research publications, even if it means giving up the prestige of service in a leading journal. The French newspaper *l'Express* came out with a list of the top 100 most influential women in France, only two of who are mathematicians. Vigneras is one of the two and the other (Stella Baruk) is a well-known mathematics educator.

Vigneras is one of 92 famous modern mathematicians featured in a book of photographic portraits by Mariana Cook, *Mathematicians: An Outer View of the Inner World*, in which she gave the following *raison d'etre* of being a mathematician. "Think of yourself in a forest. You enjoy the beauty of nature and it is not cold, but light becomes dim and it is time to leave the forest. You try a tiny path but it ends quickly. You walk back and try another one; they all look the same and it is darker. You stop and stay motionless. You wait and wait, with your senses alert to see the invisible, to feel the indescribable, to listen to the silence. And it happens suddenly: one direction becomes more dense, or more luminous. To experience this intense moment is the reason why I became a mathematician."

She has written more than 70 research papers and 4 books, of which two are with co-authors. She has been invited to major universities and research institutes throughout the world and, in particular, was an invited speaker at

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the European Congress of Mathematics (Barcelona, 2000) and the International Congress of Mathematics (Beijing, 2002). She was a fellow of the Harvard-Radcliffe Institute and Emmy Noether Professor at Göttingen University. She won the Albert Chatelet Medal, Silver Medal of CNRS, von Humboldt Prize and NSF Women Professorship Prize.

Vigneras was invited to two IMS (Institute for Mathematical Sciences) programs *Modular Representation Theory of Finite and p-adic Groups* and *New Developments in Representation Theory* for which she visited IMS during 1 - 26 April 2013 and 16 - 27 March 2016 respectively. In the first program, she gave a talk on "The Bernstein relations in the pro-*p*-Iwahori Hecke algebra of a general reductive *p*-adic group", and in the second program, a talk on "Pro-*p* Iwahori Hecke algebra, inverse Satake transform and change of weight in characteristic *p*". On 25 April 2013, Y.K. Leong interviewed her at IMS on behalf of *Imprints*. The following is an edited and vetted transcript of the interview in which she talked about the recognition and encouragement from her teachers that directed her career path towards mathematics and research, and some philosophical aspects of mathematics and mathematical research. We also get some insight into the French educational system and a glimpse of the Langlands program.

**Acknowledgment.** Y.K. Leong would like to thank Eileen Tan, Senior Executive of the Institute for Mathematical Sciences, National University of Singapore for preparing a raw draft of the transcript of the interview.

***Imprints (I):*** Were you already interested in mathematics when you were in school? Was there any person or event in your school days that had a decisive role in shaping your choice of career?

***Marie-France Vignéras (V):*** Well, I always liked mathematics when I was at school as I found it fun to do mathematics. During the last year of high school, there was a teacher who told my parents to send me to France to study mathematics. I was then living in Senegal, and going to the Lycée Van-Vollenhoven in Dakar, Africa.

***I:*** The teacher must have recognized your mathematical ability?

***V:*** Well, he thought it would be good if I could study more. I chose to go to the Lycée Massena in a place called Nice. In France, even now, the public schools are better than the private ones. But I stayed in a boarding school run by nuns, a place run by religious women only for girls.

***I:*** Which part of France was it?

***V:*** The south.

***I:*** But Nice is not near to Africa?

***V:*** No, but it is the closest to Africa that I could find.

***I:*** The topic of your doctoral thesis was number theory and was written in Bordeaux in the 1970s. How did you get interested in that field? Who was your thesis advisor?

***V:*** I got interested in this field because at the university the young professors were doing number theory. I liked algebra more than analysis and the young professors were more dynamic than the older professors doing analysis. It was thus natural for me to study number theory.

***I:*** Who was your advisor?

***V:*** Jacques Martinet. He was a professor in Bordeaux University.

***I:*** It must have been unusual for a female to pursue research in the 1970s and 1980s in number theory, which is predominantly male oriented, even up to now. Was there any affirmative action for women pursuing academic careers in France in those days?

***V:*** There was some affirmative action because there was École Normale Supérieure for girls and École Normale Supérieure for boys. But I did not want to go to École Normale Supérieure for girls, so there was no affirmative action for me. I went to the university in Bordeaux because I wanted to marry a student from Bordeaux. At that time, good students could pass an exam to get paid to study at the university. If you succeed, you sign that you will be teaching for ten years later after graduation. I got married and had a child immediately, and didn't think of research, I would be a wife. But my teachers at university suggested



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that I should do research. It was a difficult decision that I took because I thought that it was incompatible with being a doctor's wife.

***I:*** It's not very usual to start a family while doing doctoral studies.

***V:*** Indeed, people study and then start a family, but I started a family and simultaneously I studied.

***I:*** Did you face any gender related issues in your early career?

***V:*** I found that it was nice to be a woman because people notice if you prove something in mathematics, which is important. So in some sense it is positive. It's an advantage. There are disadvantages also.

***I:*** The complete classification of finite simple groups depended on the results of hundreds of research papers and it is fair to say that in the 1970s and 1980s no single person could possibly have read all of them and say with certainty that the proofs are all correct. Is this true for the current state of research in the Langlands program?

***V:*** I think it's totally different because in the Langlands program there are numerous concepts. It is possible to understand and to check what is written. If there are mistakes you can notice them – maybe not immediately but very quickly. You are developing mathematics and using a lot of mathematics. I am not a finite group theorist but it seems to me there are a lot of computations in it that you cannot re-check. The Langlands program is not like that. It is very theoretical and you can follow the proofs.

***I:*** I think in the case of the finite simple groups there is no sort of geometric objects at all.

***V:*** I don't know enough, but I think there is a lot of tricks to get something.

***I:*** I think even now they're trying to shorten all the stuff. Gorenstein, Ashbacher and some of the others are trying to shorten it into a few volumes. They are still doing it, or at least the details are already sorted out. I think Michael Atiyah once said that the finite group theorists should look for some geometric object for the group to act on.

***V:*** Yeah, yeah, to understand what they are doing.

***I:*** If I understand it correctly, Hichin fibrations play a crucial role in Ngô Bảo Châu's proof of the Fundamental Lemma and that Hitchin fibrations, which are geometric objects, have their origins in mathematical physics. I believe that Alain Connes [Fields Medalist 1982] has developed some ideas in "non-commutative geometry" which have found applications in string theory in physics. Has anyone tried to apply his geometric ideas in the Langlands Program?

***V:*** If you speak of these ideas of Alain Connes, I would say the answer is "no", but some months ago, Vincent Lafforgue - a younger brother of Laurent Lafforgue [Fields Medalist 2002] - proved the Langlands correspondence for reductive groups over function fields in the direction automorphic towards Galois. His beautiful geometric construction using shtukas is not related to non-commutative geometry although before this, V. Lafforgue proved the Baum-Connes conjecture for many groups, which was also considered as a *tour de force*. Still, I think that the answer is "no". If there is some link between non-commutative geometry and shtukas I don't see it.

***I:*** In the 1970s and the 1980s the classification problem of finite simple groups led to some kind of industry spearheaded by a group of strongly driven experts and which finally ironed out the details for a complete classification in the beginning of this century. Do you think the Langlands program might follow a parallel development within the next ten or twenty years?

***V:*** No, because this is totally different. You had this precise question, classification of finite simple groups and so it has an end. But the Langlands program is not a specific question. There is no bound to the Langlands program, which is still expanding.

***I:*** It is not a conjecture?

***V:*** There is no clear set of conjectures.

***I:*** But in the Langlands program there are some conjectures, isn't it?

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*V:* Yes, there are some conjectures as the Langlands local conjectures. I work now on the Langlands *p*-adic program, where there is no precise conjecture. By the Langlands philosophy, we think there are two categories with strong interaction. How to formulate this form of interaction we don't know.

*I:* Are you looking for structures or theories?

*V:* We are trying to understand Galois representations and Galois groups over Q. To do this we use automorphic representations and modular forms.

*I:* Is it the same as asking how to classify all the Galois groups?

*V:* Yes, you can say that. It is a kind of classification. Absolutely. We can also study analytic number theory using automorphic representations and try to classify the automorphic representations using Galois representations.

*I:* Do you need to use the results of the finite simple group people?

*V:* Oh, yes, we do, but not the classification. Proofs in Lie group theory use relatively more finite group theory than general group theory.

*I:* Is the problem of which group can be a Galois group solved?

*V:* It is not directly related to the Langlands program. The Langlands program may help to answer this question but I do not know.

*I:* Ngô Bảo Châu's proof of the fundamental lemma was listed as one of the top ten scientific discoveries by *Time* magazine in 2009. It may seem strange to classify a mathematical proof as a scientific discovery unless one holds the platonic view that the mathematical objects have an existence of their own. Do you think mathematics is created or discovered?

*V:* So that's a question that we always ask. I think that they are discovered. In fact, I don't know a single mathematician who doesn't think that mathematics is discovered. Well, generally, things are discoverable and they are not created. This is a complicated question, of course.

*I:* Do you believe that there are certain mathematical objects which sort of exist intrinsically on their own? We're not talking about the details but certain mathematical objects.

*V:* I don't know what it means "to exist on their own". For instance, when you speak about groups you assume that they exist and you see them acting on spaces. It's very convenient to define this notion of group to study all the symmetries and the symmetries exist. We all know what a group is … but not whether a group exists or not. Do we create it or we discover it? Maybe the group is the creation of the mind, I don't know.

*I:* For example, if there is some intelligent alien form in some other part of the universe would they have come to the same conclusion that say certain groups exist?

*V:* Yes, I think so.

*I:* So in that sense groups do really exist.

*V:* If you put it like that I would say "yes". But not knowing the answer doesn't prevent me from sleeping well. I work with them; there is something there. What do you think?

*I:* I think some of them may have existence on their own but you may have to create certain things to show that they exist.

*V:* Yes, in some sense.

*I:* You mentioned symmetries; symmetry is an intrinsic property.

*V:* Yes.

*I:* It's a very subjective point of view.

*V:* This is a philosophical question.

*I:* Roger Howe once mentioned that he is a geometer when it comes to mathematical thinking. What kind of intuition underlies your research work? Algebraic, geometric or something else?

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***V:*** For me, it is algebraic but sometimes I don't know if it is intuition. It is a kind of light. You are just walking, doing nothing, not sleeping, not dreaming and suddenly there is some light. You see something as if you are in the forest and suddenly you see a way out. I don't know the connection, it just happens and it gives you a pleasure that a mathematician has when he finally understands something. You see something, you don't really see it clearly but I don't draw pictures and don't know how to.

***I:*** So it's more of a formal manipulation of symbols?

***V:*** Because you have some idea that there is some structure somewhere and you have different objects with structure and that there should be some link with this structure … Very often it is the opposite way. You notice that something is probably false because there is contradiction and that there should be some coherent system.

***I:*** You mentioned you don't form pictures of the objects.

***V:*** You kind of link different things. If you have a map of a city and different parts are put together, it has to be coherent.

***I:*** I think there are some mathematicians who have some kind of geometric intuition.

***V:*** Indeed, I think it depends on what you do, I suppose – in which part of mathematics you are working. For people in topology, it's always visual.

***I:*** If I remember correctly, somebody said that [Mikhail] Gromov is visual. Most of the time, he sees the results before the proof.

***V:*** Yes, of course, it is always like that in mathematics. You know that you are going to prove something and you know what the result is before the proof. It is very exceptional that you are working and finding a theorem without knowing what you want to prove.

***I:*** French mathematicians have been awarded prestigious awards and prizes in a number that seems to be out of proportion with the population of France. Is there such a thing as a French school or tradition of mathematics?

***V:*** This is a very interesting question. The answer is yes. The French system of education and organization produces good mathematicians. Well, it is now disappearing but up till now we had a system where education was totally free and elitist. At École Normale Supérieure and École Polytechnique you get paid [to study]. At the university you pay some fees but it is comparatively nothing. Poor students have to work because they have to eat, but you don't pay fees. Back in the old days public schools were of very high quality. Well, they are no longer extremely good but in most cities there is a high quality high school where good students go to. After the baccalaureate at the end of high school, there is a special system called "*Classe Préparatoire*" where young, good students study very hard. There are some famous *Classes Préparatoires,* mostly in Paris, some in Toulouse, Bordeaux, Lyon, Nice. They are very selective in mathematics. After two or three years the students pass national competitive exams to enter a ``*Grande École*'', the two most famous ones are École Normale Supérieure and École Polytechnique. In this elitist and democratic way, they admit the best students - not because their parents are millionaires but because they succeed in the very difficult entrance exam. There are very few places at École Normale Supérieure, where they stay for four years. The very good students in mathematics are separated from the others and paid to study. It makes a big difference not having to worry about money for four years. For the PhD they go to the university.

***I:*** The top students, do they specialize early in their studies?

***V:*** After the baccalaureate, the students, who are between 17 and 19 and want to be admitted to a Grande École, work hard - they don't have much time to think about life and to mature – but they need to specialize: mathematics and physics, or biology, or social sciences, or literature and philosophy. There are some top students who cannot stand the system of *classes préparatoires* and go directly to the university. They also specialize, they are more open to life with its problems.

***I:*** But after they get in, do they specialize in any particular area?

***V:*** Yes. There are many possibilities but it depends where you are. In most Grande Écoles, you become an engineer.

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École Normale Supérieure and the universities are mostly for research and teaching professors.

***I:*** So this will force a student to make a decision early?

***V:*** More so for the students going to *classes préparatoires*.

***I:*** What if a student is interested in both physics and mathematics?

***V:*** Well, it is good. Mathematics and physics have bridges between them. The student may change his/her mind. Sometimes you start with mathematics and then go on to physics or vice versa.

***I:*** Doing research in pure mathematics seems to require the ability to think intuitively and naturally in a totally abstract language of its own. How does the beginning research student develop this kind of ability?

***V:*** I think the question is universal. I do not know the answer; it depends on the person. Some people have this natural talent and are not always aware of it. Professors are important to discover them. The interest and attitude of the professor who believes in a student helps him/her to trust himself/herself and to reach his/her goal. The student will enjoy entering the abstract mathematical world with his professor and friends. It is a "monkey" phenomenon.

***I:*** So you think a person either has it or doesn't have it?

***V:*** Some people find it easy and fascinating to do abstract mathematics. Others find it very difficult; their brains don't work that way.

***I:*** I think mathematics is itself a language. You have to master all the terms and concepts and then think using those concepts. It is not easy, isn't it?

***V:*** I don't know. Happily, your attraction to mathematics starts without knowing all the terms and concepts. For me, the concept never comes first. You are tackling a natural question and after a while you may feel the kind of concepts that could help you to approach a solution. It is easy to learn and to enjoy the power of abstract concepts when you see how they can be used on examples.

***I:*** What advice would you give to a student who wants to do pure mathematics?

***V:*** I will ask him what kind of life he/she wants to have. I say, "In many years when you get old, do you want to look like me, to have the life that I'm having? And look at other people around you. Do you want to be like them?" I would, of course, tell him that it's a wonderful life doing research, to have the pleasure to belong to the community of mathematicians, not to worry too much about money because you have enough to live and not too much. But I would tell him also that he will not become rich, that mathematics is a time-devouring passion and to forget about week-ends consisting of two days playing with children and drinking cocktails with friends by the swimming-pool.

***I:*** In pure mathematics do people collaborate with other people a lot?

***V:*** Collaboration in pure mathematics is a new phenomenon. People now start collaborating because they are pushed by politics which thinks that mathematics is like any other scientific field. Mathematicians today have to write applications to get grants to do research, and these grants imply collaborations and meetings. In biology, they work as a team, they cannot do anything by themselves. But in mathematics, you can work alone. I am collaborating since I retired from the university, I like to collaborate because it is more social, and the papers are better. We share ideas and write common papers but a lot of work is still done alone.

***I:*** Things are changing for mathematics.

***V:*** Yes, things are changing a lot. Maybe it's good that things should change.

## Call for Pre-Proposals >>>

The Institute for Mathematical Sciences (IMS) of the National University of Singapore (NUS) invites submissions of pre-proposals for April 2019 to March 2020 from researchers in the academia and industry in Singapore or overseas. The pre-proposals are for organizing thematic programs to be held wholly or partly at IMS with funding from the Institute. These programs, each lasting for one to six months, should have a well-defined theme or themes that are at the forefront of current research in an area of mathematical science or its applications. They should be of international interest and of interest or relevance to the local scientific community. Typically, a program should involve both local and international organizers. Pre-proposals on interdisciplinary programs in areas that interface with the mathematical sciences are welcome.

A soft copy of the pre-proposal should be sent to the Director of the Institute at imsdir@nus.edu.sg not later than 31 May 2017. The exposition of a pre-proposal should be aimed at the non-specialist and will be evaluated by a panel. Pre-proposals on interdisciplinary programs should indicate how the program would benefit the intended audience with diverse backgrounds and facilitate research collaboration.

Information on the Institute and its activities, as well as a detailed format for pre-proposals are available on the IMS website http://www2.ims.nus.edu.sg. Enquiries may be directed to imssec@nus.edu.sg.



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