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Tutorial on mathematical model, theory and simulation for Bose-Einstein condensation

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ABSTRACT

Since the first experiment of Bose-Einstein condensation (BEC) in 1995, numerous efforts have been devoted to the study of BEC. In this tutorial, we will briefly review the Gross-Pitaevskii (GP) theory for weakly interacting bosons modeling BEC, discuss the mathematical theory and numerical techniques. We will then extend the results to rotating BEC, BEC with dipole-dipole interactions, two component BEC with spin-orbit coupling, spinor BEC, etc.

Tutorial on multidimensional solitons

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ABSTRACT

It is commonly known that the interplay of linear and nonlinear effects gives rise to solitons, i.e., self-trapped localized structures, in a wide range of physical settings, including optics, Bose-Einstein condensates (BECs), hydrodynamics, plasmas, condensed-matter physics, etc. Nowadays, solitons are considered as an interdisciplinary class of modes, which feature diverse internal structures.

While most experimental realizations and theoretical models of solitons have been elaborated in one-dimensional (1D) settings, a challenging issue is prediction of stable solitons in 2D and 3D media. In particular, multidimensional solitons may carry an intrinsic topological structure in the form of vorticity. In addition to the "simple" vortex solitons, fascinating objects featuring complex structures, such as hopfions, i.e., vortex rings with internal twist, have been predicted too.

A fundamental problem is propensity of multidimensional solitons to being unstable (naturally, solitons with a more sophisticated structure, such as vortex solitons, are more vulnerable to instabilities). Recently, novel perspectives for the creation of stable 2D and 3D solitons were brought to the attention of researchers in optics and BEC. The present talk aims to provide an overview of the main results and ongoing developments in this vast field. An essential conclusion is the benefit offered by the exchange of concepts between different areas, such as optics, BEC, and hydrodynamics.

A new review article on the subject:

Y. Kartashov, G. Astrakharchik, B. Malomed, and L. Torner, Frontiers in multidimensional self-trapping of nonlinear fields and matter, Nature Reviews Physics, https://doi.org/10.1038/s42254-019-0025-7.

Tutorial on analytical techniques for multi-scale problems in materials

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ABSTRACT

In these lectures I review some (mostly) analytical techniques motivated from problems of passing from mesoscopic descriptions to continuum limits in contexts of problems in materials.

In the first part I survey the relative entropy method, which serves in comparing two different solutions of conservation laws, or two related thermomechanical theories. This is done first in a context of conservation laws following the early works of Dafermos and DiPerna motivated by thermodynamics.

Then a connection between variational calculus and the relative entropy method is developed and its ramifications to the system of quantum hydrodynamics will be indicated. A second application is the derivation of gradient flows from models of Hamiltonian flows with friction in the high-friction (or small mass limit).

Then I discuss applications of such ideas to the problem of passage from lattice systems to the equations of elasticity in the limit when the lattice size becomes small. Also, applications to variant models that are used for describing heavy ionic crystals.

In the last part, I focus on a class of models introduced by Doi describing suspensions of rod–like molecules in a solvent fluid. They couple a microscopic Fokker-Planck type equation for the probability distribution of rod orientations to a macroscopic Stokes flow. An objective is to compare such models with traditional models used in macoscopic viscoelasticity as the well known Oldroyd model. For the problem of sedimenting rods under the influence of gravity I will derive two such effective equations describing the emerging responses: (i) One ammounts to a classical diffusive limit and produces a Keller-Segel type of model. (ii) A second approach involves the derivation of a moment closure theory and the approximation of moments via a quasi-dynamic approximation. This produces a model that belongs to the class of flux-limited Keller-Segel systems.