FUND FLOWS AND PERFORMANCE UNDER UNOBSERVABLE DYNAMIC MANAGING ABILITY

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Motivation

Current literature, studying fund flows and performance, assumes unobservable constant manager ability to outperform passive benchmarks.

However, empirical evidence, supports dynamic abilities. E.g.,

- fund manager performance does not persist [e.g., Carhart (1997), Berk and Tonks (2007), Wang (2014)];
- fund family activities vary across time and affect managers’ ability to outperform benchmarks [e.g., Gaspar, Massa, and Matos (2006), Evans (2010)];
- dynamic macroeconomic factors affect fund managers’ abilities [e.g., Ferreira, Keshwani, Miguel, and Ramos (2012, 2013), Feldman, Saxena, and Xu (2019a, 2019b)].
This paper is doing the following:

- create a fund management model, with unobservable dynamic manager ability to beat a passive benchmark;
- explain and produce insights and implications on the fund flows and performance relation;
- improve empirical explanations of data.
Contribution

- Introduce a model of unobservable dynamic manager ability in studying fund flows and performance.

- Introduce a continuous-time framework of investors’ learning (unobservable) dynamic fund managers’ abilities.

- Provide empirical results that are better explained by dynamic fund managers’ abilities rather than constant such abilities.

- Offer policy implications.
Two strands of Literature

Apply the optimal nonlinear filtering techniques in studying the fund flow and performance relation.

Two strands of literature are relevant:
- optimal nonlinear filtering techniques and applications;
- models deriving the fund flow and performance relation.
Literature of Optimal Nonlinear Filtering

Optimal nonlinear filtering techniques [Liptser and Shiryayev (2001a, 2001b)]

Application of these techniques in asset pricing, e.g., an incomplete information framework with unobservable state variable [Detemple (1986), Dothan and Feldman (1986), and Feldman (1989)]

Application of these techniques in fund management, e.g., closed-end fund discount with unobservable constant manager ability [Berk and Stanton (2007)].
Current models deriving the fund flow and performance relation incorporate the following features:

- unobservable constant manager ability to outperform the passive benchmark,
- Bayesian update of manager ability by observing returns,
- decreasing returns to scale,
- investors’ risk preference.

E.g., Berk and Green (2004), Huang, Wei, and Yan (2007), and Lynch and Musto (2003).
An Intuitive Example

Consider a basket containing black balls and white balls. The ratio of black balls to white balls is unknown, but each time we can draw a ball from the basket, observe its color, and return it to the basket.

Case One: if the ratio is constant, then after sufficiently many observations, we can have a precise estimate of the ratio.
An Intuitive Example Cont.

**Case Two:** if the ratio is **dynamic** with a particular pattern, then we might find an optimal way to estimate the ratio, but

- the estimation precision can increase, decrease, or remain unchanged after each observation;
- after many observations, a new observation can be relevant;
- we face a forever tracking problem.
An Intuitive Example Cont.

We consider the unobservable manager ability as the ratio of black balls to white balls, and the observable fund returns generated by the manager as the results of the draws.

If the unobservable manager ability is constant, then we are in Case One, where we have a precise estimate of manager ability after a long history of observable fund returns.

If the unobservable manager ability is dynamic, then we are in Case Two, where we face a forever tracking problem. This is what our paper focuses on.
It is a Continuous-time model.

A representative active fund and a passive benchmark portfolio are available for investors to choose.

There are infinitely many small investors. Each investor’s actions do not affect the fund size. They are risk-neutral or risk-averse.

The fund manager is risk-neutral, i.e., a profit maximizer. The manager’s actions do not affect the benchmark portfolio’s return.

Investors and the manager are symmetrically informed.
Fund Returns and Manager Ability

\( \xi_t \): the active fund’s observable share (unit) price before fund fees and costs

\( d \xi_t / \xi_t \): the gross return from investing in the fund

\( \theta_t \): unobservable fund manager ability

We assume the fund has a beta loading of one on the benchmark, and we normalize benchmark return to zero. Thus, the fund’s gross alpha is \( d \xi_t / \xi_t - 0 = d \xi_t / \xi_t \).
Fund Returns and Manager Ability Cont.

The observable fund gross alpha and unobservable manager ability are associated as:

\[ d\theta_t = (a_0 + a_1 \theta_t)\ dt + b_1 \ dW_{1,t} + b_2 \ dW_{2,t}, \]
\[ d\xi_t / \xi_t = A_1 \theta_t \ dt + B dW_{2,t}. \]

- \( W_{1,t} \) and \( W_{2,t} \) are independent Wiener processes;
- initial conditions \( \theta_0 \) and \( \xi_0 \) are given;
- \( a_0, a_1, b_1, b_2, A_1, \) and \( B \) are known constants, and \( b_1 > 0, A_1 > 0, B > 0, \) and \( b_2 \geq 0 \).

Both the manager and investors learn \( \theta_t \) by observing the history of the fund gross alpha \( d\xi_t / \xi_t, \ t \geq 0 \).
Define the following:

$\mathcal{F}_t^\xi$: the $\sigma$-algebra of $\xi_s$, $0 \leq s \leq t$, with $\{\mathcal{F}_t^\xi\}_{t \geq 0}$ as the corresponding filtration.

$m_t \triangleq \mathbb{E}(\theta_t|\mathcal{F}_t^\xi)$: the conditional mean of $\theta_t$ given observations of $\xi_s$, $0 \leq s \leq t$, with initial value $m_0$;

$\gamma_t \triangleq \mathbb{E}[(\theta_t - m_t)^2|\mathcal{F}_t^\xi]$: the conditional variance of $\theta_t$ given observations of $\xi_s$, $0 \leq s \leq t$, with initial value $\gamma_0$;

We further assume the conditional distribution of $\theta_0$ given $\xi_0$, i.e., the prior distribution, is Gaussian $\mathcal{N}(m_0, \gamma_0)$. 
Posterior Estimation

Proposition 1 (Brief)

\( \overline{W}_t = \int_0^t \frac{d\xi_s}{\xi_s} \frac{A_1 m_s ds}{B} \) is a Wiener process w.r.t. filtration \( \mathcal{F}_t^{\xi} \) for all \( t \geq 0 \). Also, \( \mathcal{F}_t^{\xi} \) and \( \mathcal{F}_t^{\xi_0, \overline{W}} \) are equivalent.

The variables \( m_t, \xi_t, \) and \( \gamma_t \) are the unique, continuous, \( \mathcal{F}_t^{\xi} \)-measurable solutions of the system of equations

\[
\begin{align*}
    dm_t &= (a_0 + a_1 m_t) dt + \sigma_m(\gamma_t) d\overline{W}_t, \quad (3) \\
    d\xi_t / \xi_t &= A_1 m_t dt + Bd\overline{W}_t, \quad (4) \\
    d\gamma_t &= \left[ b_1^2 + b_2^2 + 2a_1 \gamma_t - \sigma_m^2(\gamma_t) \right] dt, \quad (5)
\end{align*}
\]

where \( \sigma_m(\gamma_t) \triangleq \frac{b_2 B + A_1 \gamma_t}{B} \) is the sensitivity of expected manager ability to shocks to fund gross alpha.
Remarks of Proposition 1

- The “estimation error” $\gamma_t$ can increase, decrease, or be unchanged, dependent on parameter values. This is different from the Berk and Green (2004) model, where the “estimation error” of the constant manager ability decreases as more observations of returns arrive.

- The sensitivity of expected manager ability to shocks to fund gross alpha $\sigma_m(\gamma_t)$ can increase, decrease, or be unchanged, dependent on parameter values. This is different from the Berk and Green (2004) model, where this sensitivity decreases across time as the estimation converges to the true value across time.
Investors’ Problem

Our baseline model studies risk-neutral investors. Define the following terms:

$q_t$: the active fund’s size;

$q_t^a$: the amount of the fund that is under active management;

$C(q_t^a)$: fund costs, and we assume $C(q_t^a) = cq_t^a^2$;

$f$: constant percentage management fee

$S_t$: the per share price of the active fund’s asset under management that is net of fund costs and management fee;

$dS_t/S_t$: net return of fund share, and as the benchmark return is zero, it is also the fund net alpha.
Investors’ Problem Cont.

Then, the fund net alpha is

\[
\frac{dS_t}{S_t} = \frac{q_t^a \ d\xi_t}{q_t \ \xi_t} - \frac{C(q_t^a)}{q_t} dt - f dt
\]

Similar to Berk and Green (2004), we assume that risk-neutral investors supply capital with infinite elasticity to funds that have positive expected net alphas. Then, in equilibrium

\[
E\left( \frac{dS_t}{S_t} \mid \mathcal{F}_t^\xi \right) = 0, \ \forall t.
\]

Consequently, we have \( fq_t = A_1 m_t q_t^a - cq_t^{a2} \).
The Fund Manager’s Problem

The fund manager solve the following

$$
\max_{q_t^a} f q_t = A_1 m_t q_t^a - c q_t^{a^2},
$$

$$
s.t. \ 0 \leq q_t^a \leq q_t.
$$

The optimal amount under active management is

$$
q_t^{a^*} = \frac{A_1 m_t}{2c}.
$$

Then, in equilibrium, the fund size is

$$
q_t^* = \frac{(A_1 m_t)^2}{4cf}.
$$
Fund Flow and Performance Relation

Apply Itô’s Lemma, and we have

$$\frac{dq^*_t}{q^*_t} = \frac{A_1 \sigma_m(\gamma_t)}{fB} \left( \frac{dS_t}{S_t} \right) + \frac{A_1^2 \sigma_m^2(\gamma_t)}{4f^2 B^2} \left( \frac{dS_t}{S_t} \right)^2 + \left( \frac{2a_0}{m_t} + 2a_1 \right) dt.$$ 

We define the lowest conditional expected manager ability that make the fund survive, \( \underline{m}_t, \underline{m}_t \geq 0. \) Then,

- if \( m_t > \underline{m}_t, \) then in equilibrium, we have fund size as \( q^*_t \) and fund flow as \( dq^*_t/q^*_t \) as shown above;
- if \( m_t \leq \underline{m}_t, \) then in equilibrium, we have fund size and fund flow as zero.

We focus on the case where \( m_t > \underline{m}_t. \)
Proposition 2 (Brief)

Where $m_t > m_t$, the fund flow and performance relation has the following characteristics:

- fund flow increases with and is convex in fund net alpha;
- a higher volatility of fund gross alphas, $B$, induces a lower sensitivity of fund flow to fund net alpha;
- a higher sensitivity of expected manager ability to shocks, $\sigma_m(\gamma_t)$, induces a higher sensitivity of flow to net alpha;
- a higher management fee $f$ induces a lower sensitivity of fund flow to fund net alpha;
- a higher sensitivity of gross alpha to manager ability, $A_1$ induces a higher sensitivity of flow to net alpha.
Relation to Berk and Green (2004)

We can also show that our model degenerates to a continuous-time analog of Berk and Green (2004)’s model if the unobservable manager ability becomes constant. In particular, if $a_0 = a_1 = b_1 = b_2 = 0$ and $A_1 = 1$, then we have

$$\sigma_m(\gamma_t) = \frac{\gamma_0 B}{B^2 + \gamma_0 t},$$

$$\frac{dq_t^{*}}{q_t^{*}} = \left(\frac{dS_t}{S_t}\right) \left(\frac{\gamma_0}{B^2 + \gamma_0 t}\right) + \left(\frac{dS_t}{S_t}\right)^2 \left(\frac{\gamma_0}{B^2 + \gamma_0 t}\right)^2.$$

One of the key differences in the equilibrium fund flow and performance relation is that, their fund flow sensitivity to net alpha decreases across time, whereas ours can increase, decrease, or be unchanged, depending on the parameter values.

This is because, their sensitivity of expected manager ability to shocks, $\sigma_m(\gamma_t)$, decreases across time, making fund flow being less and less sensitive to net alpha. However, in our model, $\sigma_m(\gamma_t)$ can increase, decrease, or be unchanged, resulting in an increase, decrease, or unchanged fund flow sensitivity to net alpha across time.
Simulation Results

We set the parameter values as follows.
\[ f = 1.12\%, \quad m_0 = 1.17\%, \quad \gamma_0 = (2.09\%)^2, \quad B = 5.04\%. \]

For other parameters, we analyze three cases:

- **Case One**: \( a_0 = 0, \quad a_1 = 0, \quad A_1 = 1, \quad b_1 = 0, \quad b_2 = 0; \)
- **Case Two**: \( a_0 = 0.01, \quad a_1 = -0.1, \quad A_1 = 0.1, \quad b_1 = 0.02, \quad b_2 = 0.01; \)
- **Case Three**: \( a_0 = 0.005, \quad a_1 = 0.001, \quad A_1 = 0.01, \quad b_1 = 0.06, \quad b_2 = 0.01. \)
Simulation Results: Case One

The fund flow sensitivity to fund net alpha decreases over time, as the sensitivity of expected manager ability to shocks decreases over time.
The fund flow sensitivity to fund net alpha stays unchanged over time, as the sensitivity of expected manager ability to shocks stays unchanged over time.
Simulation Results: Case Three

The fund flow sensitivity to fund net alpha increases over time, as the sensitivity of expected manager ability to shocks increases over time.
Remarks of Simulation Results

The fund flow sensitivity to fund net alpha changes with the sensitivity of expected manager ability to shocks to gross alphas. In particular, if the latter increases, decreases, or does not change, then the former increases, decreases, or does not change, respectively.

Case One is the case of Berk and Green (2004), whereas our model allows difference cases.
To study how risk-aversion affects our equilibrium, we assume that investors are mean-variance risk-averse who maximize instantaneous Sharpe ratios of their portfolios.

We find that the results of the fund flow and performance relation in Proposition 2 still hold.

The intuition is that investors’ risk-aversion affects amount of the investment allocated to the risky active fund, $q_t^*$, so it affects $dq_t^*$. However, when the fund flow is calculated as percentage flow $dq_t^*/q_t^*$, the effects of risk-aversion cancel out.
Empirical Study

The literature has studied the empirical fund flow and performance relation under different context [e.g., Lynch and Musto (2003), Bollen (2007), Huang, Wei, and Yan (2007), Chen, Goldstein, and Jiang (2010), Spiegel and Zhang (2013)].

Our empirical hypothesis is as follows:

- if managers’ abilities are constant, the fund flow sensitivities to performances decrease (fast) monotonically over time.
- if managers’ abilities are dynamic, the fund flow sensitivities to performances can have different time patterns.
Empirical Methodology: Fund Net Alpha Estimation

We measure alpha in comparison to the "closest set" of index funds using a Style-Matching Model (Sharpe (1992), Berk and Binsbergen (2015)):

\[ R_{i,t} = \alpha_{i,t} + b_{i,t}^1 F_1^t + b_{i,t}^2 F_2^t + \ldots + b_{i,t}^n F_n^t \]

- \( R_{i,t} \): fund net of fee return
- \( F_n^t \): factor \( n \)'s net of fee return
- \( b_{i,t}^n \): factor loading to \( n^{th} \) factor
  - \( 0 \leq b_{i,t}^n \leq 1 \) (no short-sales of factors)
  - \( \sum_n b_{i,t}^n = 1 \) (weights to \( n \) factors replicate the \( R_{i,t} \))
- \( \alpha_{i,t} \): fund net alpha

We use observations from \( t - 1 \) to \( t - 60 \) (i.e., previous 60 months) to estimate out of sample \( \alpha_{i,t} \) (i.e., next month).
Empirical Methodology: Time Pattern of Fund Flow Sensitivity to Fund Net Alpha

We divide a fund’s time-series observations of fund flows and net alphas into three parts: the first five years of observations in the sample (i.e., Period 0), the second five years (i.e., Period 1), and the observations in the remaining period (i.e., Period 2). Then, we develop our 3-Period Model:

\[
\text{Flow}_{i,t} = \beta_0 + \beta_1 \alpha_{i,t-1} + \beta_2 \alpha_{i,t-1} \times L1_{i,t} + \beta_3 \alpha_{i,t-1} \times L2_{i,t} + \beta_4 L1_{i,t} + \beta_5 L2_{i,t} + \text{Controls}_{i,t} + \epsilon_{i,t}
\]

Where \(L1_{i,t}\) (\(L2_{i,t}\)) is 1 if the time is in Period 1 (Period 2) and 0 otherwise. In this model, Period 0 is the base group.

Control Variables include the following

- $Fee_{i,t}$: fund management fee, approximated by annual expense ratio;
- $\ln TNA_{i,t}$: the logarithm of the fund’s total net asset under management;
- $Flow_{i,t-1}$: the lagged fund flow;
- $Age_{i,t}$: fund age, calculated as the number of month from the fund’s inception date;
- $Vol_{i,t}$: fund volatility, calculated as the standard deviation of the fund’s net returns in the prior 12 months;
- Fund dummies and Year dummies.
Data

Morningstar US mutual funds (open-end funds), from Jan 1979 to Dec 2014

All returns are net of administrative and management fees and other costs taken out of fund assets, and fund share class data is aggregated to fund data, using funds’ net asset under management as weights

Obtain active equity funds using filters similar to Feldman, Saxena, and Xu (2019 a)

We further require active equity funds to have at least 15 years’ monthly fund net alpha and fund flow observations

Use index funds’ net returns and Fama-French risk-free rate as factors in the Style-Matching Model, requiring index funds to have non-missing observations in the sample period
the R-squared of the style-matching model is high, with an average of 85%, implying that missing many relevant factors in the Style-Matching model is unlikely.

In addition, the total fund size in our sample occupies around 10% of the equity market capitalization.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs.</th>
<th>Mean</th>
<th>Std.</th>
<th>1st</th>
<th>25th</th>
<th>50th</th>
<th>75th</th>
<th>99th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Fund-Level Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fund Flow (Decimal)</td>
<td>139511</td>
<td>0.0112</td>
<td>2.8754</td>
<td>-0.1166</td>
<td>-0.0129</td>
<td>-0.0035</td>
<td>0.0088</td>
<td>0.1753</td>
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<td>Fund Net Return (Decimal)</td>
<td>139511</td>
<td>0.0086</td>
<td>0.0504</td>
<td>-0.1413</td>
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<td>0.0127</td>
<td>0.0384</td>
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<td>Fund Net Alpha (Decimal)</td>
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<td>0.0005</td>
<td>0.0209</td>
<td>-0.0584</td>
<td>-0.0089</td>
<td>0.0003</td>
<td>0.0098</td>
<td>0.0610</td>
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<td>Style-Matching Model R-sqr (Decimal)</td>
<td>139511</td>
<td>0.8473</td>
<td>0.1222</td>
<td>0.4027</td>
<td>0.8002</td>
<td>0.8818</td>
<td>0.9315</td>
<td>0.9861</td>
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<td>Fund TNA (in 100 Million Dollar)</td>
<td>139511</td>
<td>26.9061</td>
<td>80.0711</td>
<td>0.0902</td>
<td>1.5122</td>
<td>5.9664</td>
<td>19.4942</td>
<td>375.5497</td>
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<td>Fee (Decimal)</td>
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<td>0.0112</td>
<td>0.0042</td>
<td>0.0022</td>
<td>0.0088</td>
<td>0.0107</td>
<td>0.0131</td>
<td>0.0222</td>
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<tr>
<td>Age (Month)</td>
<td>139511</td>
<td>273.9791</td>
<td>189.8273</td>
<td>64</td>
<td>144</td>
<td>221</td>
<td>328</td>
<td>918</td>
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<tr>
<td>Fund Net Return Volatility (Decimal)</td>
<td>139511</td>
<td>4.5796</td>
<td>2.1164</td>
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<td>2.9968</td>
<td>4.1661</td>
<td>5.6711</td>
<td>11.3820</td>
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<td>Panel B: Industry-Level Data</td>
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<td>Industry Size to Equity Market (Decimal)</td>
<td>371</td>
<td>0.0868</td>
<td>0.0360</td>
<td>0.0238</td>
<td>0.0469</td>
<td>0.1052</td>
<td>0.1152</td>
<td>0.1293</td>
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Empirical Results: 3-Period Model on Panel Data

**Panel A**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
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<tbody>
<tr>
<td>Lag_Net_Alpha</td>
<td>0.2603***</td>
<td>0.2781***</td>
<td>0.2569***</td>
</tr>
<tr>
<td></td>
<td>(0.0295)</td>
<td>(0.0337)</td>
<td>(0.0437)</td>
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<tr>
<td>Lag_Net_Alpha * L1</td>
<td>0.9313</td>
<td>0.9221</td>
<td>0.9076</td>
</tr>
<tr>
<td></td>
<td>(1.0154)</td>
<td>(1.0092)</td>
<td>(1.0196)</td>
</tr>
<tr>
<td>Lag_Net_Alpha * L2</td>
<td>-0.0832**</td>
<td>-0.0779**</td>
<td>-0.0795*</td>
</tr>
<tr>
<td></td>
<td>(0.0361)</td>
<td>(0.0386)</td>
<td>(0.0477)</td>
</tr>
<tr>
<td>L1</td>
<td>0.0249</td>
<td>0.0179</td>
<td>0.0237</td>
</tr>
<tr>
<td></td>
<td>(0.0332)</td>
<td>(0.0269)</td>
<td>(0.0337)</td>
</tr>
<tr>
<td>L2</td>
<td>-0.0142***</td>
<td>-0.0428</td>
<td>0.0102</td>
</tr>
<tr>
<td></td>
<td>(0.0036)</td>
<td>(0.0279)</td>
<td>(0.0236)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.0130***</td>
<td>-0.0483</td>
<td>0.1104</td>
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<tr>
<td></td>
<td>(0.0035)</td>
<td>(0.0600)</td>
<td>(0.0966)</td>
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</table>

Controls: No, Year Dummies: No, Fund Dummies: No

**Panel B**

<table>
<thead>
<tr>
<th>Lag_Net_Alpha * L2 - Lag_Net_Alpha * L1</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
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<tr>
<td>Coefficient</td>
<td>-1.0145</td>
<td>-0.9999</td>
<td>-0.9870</td>
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<tr>
<td>Standard Error</td>
<td>1.0151</td>
<td>0.9997</td>
<td>0.9899</td>
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<tr>
<td>Student-t Statistics</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
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<tr>
<td>Two-Tailed P-Value</td>
<td>0.32</td>
<td>0.32</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Observations: 139,551, 138,993, 138,993
R-squared: 0.00005, 0.00012, 0.00054
Empirical Results: 3-Period Model on Panel Data Cont.

We find that $\beta_2$ and $\beta_3 - \beta_2$ are positive and negative respectively, but both are not significant, showing that the change of the fund flow sensitivity to fund net alpha from Period 0 to Period 1, and that from Period 1 to Period 2, are not statistically significant.

$\beta_3$ is negative and significant in different model specifications, showing a significant decrease in fund flow sensitivity to fund net alpha from Period 0 to Period 2.

Thus, fund flow sensitivity to fund net alpha decreases in the long term, but does not change significantly in short term.
Empirical Results: 3-Period Model on Individual Funds

We also re-run our 3-Period Model for each individual fund, and report the numbers of funds (out of 527 funds) whose relevant coefficients are significant.

<table>
<thead>
<tr>
<th>Significance</th>
<th>*</th>
<th>**</th>
<th>***</th>
<th>Total</th>
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<tbody>
<tr>
<td>$\beta_2 &gt; 0$</td>
<td>20</td>
<td>15</td>
<td>11</td>
<td>16</td>
</tr>
<tr>
<td>$\beta_2 &lt; 0$</td>
<td>27</td>
<td>28</td>
<td>9</td>
<td>64</td>
</tr>
<tr>
<td>$\beta_3 &gt; 0$</td>
<td>19</td>
<td>12</td>
<td>8</td>
<td>39</td>
</tr>
<tr>
<td>$\beta_3 &lt; 0$</td>
<td>29</td>
<td>24</td>
<td>15</td>
<td>68</td>
</tr>
<tr>
<td>$\beta_3 - \beta_2 &gt; 0$</td>
<td>14</td>
<td>15</td>
<td>2</td>
<td>31</td>
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<tr>
<td>$\beta_3 - \beta_2 &lt; 0$</td>
<td>26</td>
<td>25</td>
<td>6</td>
<td>57</td>
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Empirical Results: 3-Period Model on Individual Funds Cont.

Results show that from a particular period to another, around 20% of funds experience a significant increase or decrease in fund flow sensitivity to fund net alpha.

The majority of funds have insignificant change in fund flow sensitivity to fund net alpha.
Robustness Check: 4-Period Model

We develop a 4-Period Model, and divide a fund’s time-series observations of fund flows and net alphas into four parts: the first four years of observations (i.e., Period 0), the second four years (i.e., Period 1), the third four years (i.e., Period 2), and observations in the remaining period (i.e., Period 3).

We find consistent results:

- in the panel regression, the fund flow sensitivity to fund net alpha decreases in the long term (from Period 0 to Period 3), but does not change significantly in shorter terms;
- individual funds’ results also show that around 20% of funds experience an significant increase or decrease in fund flow sensitivity to fund net alpha from one period to another, but the majority does not have significant change.
Remarks on Empirical Results

The empirical results are not well explained by the framework of constant manager ability. If manager ability is constant, we should observe that, both in the whole sample and in individual funds, the fund flow sensitivity to fund net alpha decreases quickly and monotonically over time. However, this is not the case.

The results are better explained by the framework of dynamic manager ability, where these dynamic manager ability processes vary across funds.
Policy Implications

When we evaluate whether investments flow into (out of) funds with better (worse) performance, we need to take the dynamics of the unobservable manager ability into consideration. Failure to do so could result in a wrong evaluation of the market efficiency of the active fund management industry.

Policy makers not only need to consider how their policy affects the level of fund manager ability in producing returns, but also need to take care how it influences the progress of fund manager ability over time.
We develop a continuous-time model to derive the fund flow and performance relation, where we assume that the unobservable fund manager ability is dynamic.

We show that empirical time patterns of the fund flow and performance relation can be explained better by the framework of the dynamic fund manager ability than by the framework of the constant fund manager ability.

Our results show that, the dynamics of manager ability has relevant effects on the fund flow and performance relation, so this dynamics should be taken care of by policy makers.