Deep Learning the Landscape

YANG-HUI HE

何楊輝

Dept of Mathematics, **City, University of London**

Merton College, **University of Oxford**

School of Physics, **NanKai University**

幾何景觀之機器學習

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The Data Revolution

- C21st is driven by new science and new data, the **age of 'Big Data'** has already revolutionized biology, astronomy, experimental particle physics, . . .
- What about theoretical physics and pure mathematics?

  WELL-KNOWN string theory has revolutionized pure mathematics;

  LESS KNOWN also a bench-mark for computational mathematics in the last decade (e.g., collab w/ Macaulay2, Singular, GAP, SAGE)
- Plan: *use string theory & Calabi-Yau geometry as testing ground*
- A burgeoning enterprise (2017-8)
  - cf. Seong-Krefl, Ruehle, Carifio-Halverson-Krioukov-Nelson, Liu, Ballard-Mehta, Cohen-Freytsis-Ostdiek, Hashimoto, Wang-Zhang, et al.
  - *Data Science & Strings (Northeastern); String Data (Munich); Machine Learning CY (Sanya)*

Caveat Emptor

Big data is like teenage sex:

- *everyone talks about it,*
- *nobody really knows how to do it,*
- *everyone thinks everyone else is doing it,*
- *so everyone claims they are doing it.*

– Dan Ariely, James Duke Professor of Behavioural Economics

1984

- Heterotic string [Gross-Harvey-Martinec-Rohm]: $E_8 \times E_8$ or $SO(32)$, 1984 - 6
- String Phenomenology [Candelas-Horowitz-Strominger-Witten]: 1986
  - Het E_8 on $X \times \mathbb{R}^{1,3}$ for smooth, compact Calabi-Yau 3-fold Xs gives $\mathcal{N} = 1$ SUSY E_6-GUT theory on $\mathbb{R}^{1,3}$
  - E_6 commutant of $SU(3)$ in E_8 and $SU(3)$ is holonomy of T_X
  - $\mathcal{N} = 1$ follows from CY condition (existence of covariantly constant spinor)
  - Natural gauge unification
  - # net generations $= |h^{1,1}(X) - h^{2,1}(X)| = \frac{1}{2}|\chi(X)|$
- **Believed to be ToE**: het E_8 on CY3 gives GUT + gravity in 4-D (cf. Greene-Ross, Distler, et al)
- Need to construct CY3 with $\chi = \pm 6$ explicitly and to work out details

Complete Intersection Calabi-Yau (CICY) 3-folds

- immediately: (generic homog.) Quintic Q in $\mathbb{P}^4$ is CY3 (by Euler sequence $5 = 4 + 1 \rightsquigarrow c_1(T_X) = 0$), but $Q_\chi^{h^{1,1},h^{2,1}} = Q_{-200}^{1,101}$ so too may generations (even with quotient $-200 \notin 3\mathbb{Z}$)
- [Candelas-A. He-Hübsch-Lutken-Schimmrigk-Berglund] (1986-1990)
  - dim(Ambient space) - #(defining Eq.) $=$ 3 (complete intersection)

$$M = \left[\begin{array}{c|cccc} n_1 & q_1^1 & q_1^2 & \dots & q_1^K \\ n_2 & q_2^1 & q_2^2 & \dots & q_2^K \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ n_m & q_m^1 & q_m^2 & \dots & q_m^K \end{array}\right]_{m \times K}$$

– K eqns of multi-degree $q_j^i \in \mathbb{Z}_{\geq 0}$ embedded in $\mathbb{P}^{n_1} \times \ldots \times \mathbb{P}^{n_m}$

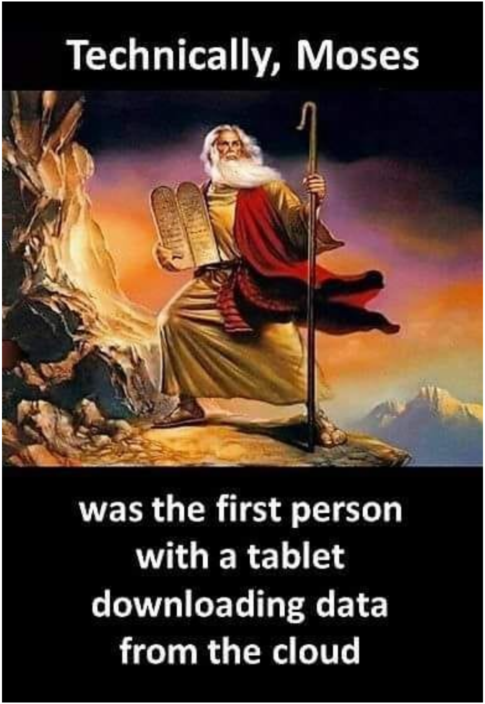
– $c_1(X) = 0 \rightsquigarrow \sum\limits_{j=1}^{K} q_r^j = n_r + 1$

– M^T also CICY

  - Famous Examples

The First Data-sets in Mathematical Physics/Geometry

- Problem: *classify all configuration matrices*; employed the best computers at the time (**CERN supercomputer**); q.v. magnetic tape and dot-matrix printout in Philip's office
  - 7890 matrices from 1×1 to max(row) $= 12$, max(col) $= 15$; with $q^i_j \in [0,5]$
  - 266 distinct Hodge pairs $(h^{1,1}, h^{2,1}) = (1,65), \ldots, (19,19)$
  - 70 distinct Euler $\chi \in [-200, 0]$ (all negative)
  - [V. Braun, 1003.3235] : 195 have freely-acting symmetries (quotients), 37 different finite groups (from $\mathbb{Z}_2$ to $\mathbb{Z}_8 \rtimes H_8$)
- [Candelas-Lynker-Schimmrigk, 1990] Hypersurfaces in Weighted P4
  - generic homog deg $= \sum_{i=0}^{4} w_i$ polynomial in $W\mathbb{P}^4_{[w_0:w_1:w_2:w_3:w_4]} \simeq (\mathbb{C}^5 - \{0\})/(x_0, x_1, x_2, x_3, x_4) \sim (\lambda^{w_0} x_0, \lambda^{w_1} x_1, \lambda^{w_2} x_2, \lambda^{w_3} x_3, \lambda^{w_4} x_4)$
  - 7555 inequivalent 5-vectors w_i, 2780 Hodge pairs, $\chi \in [-960, 960]$



The age of data science in mathematical physics/string theory not as recent as you might think

Ne Plus Ultra: The Kreuzer-Skarke Dataset

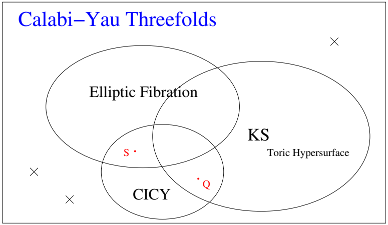
Generalize WP4, take reflexive polytope $\Delta_n \in \mathbb{R}^n$ (lattice convex polytope with a single interior point, take as origin); Need to classify up to $SL(n;\mathbb{Z})$;

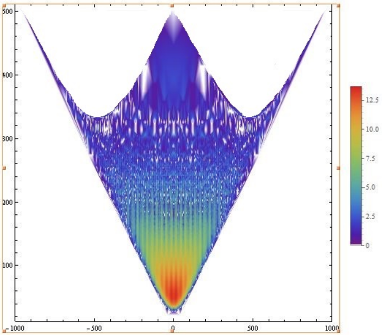
- THM [Batyrev-Borisov, 1994] hypersurface in toric variety $X(\Delta_n)$ is CY(n-1)
- Classically known: $n = 2$, there are 16 $\Delta_2 \rightsquigarrow$ elliptic curves
- Kreuzer†-Skarke 1997-2002: 4319 Δ_3 and 473,800,776 Δ_4, (last month: Δ_5, $> 185, 269, 499, 015$)
  - 30,108 distinct Hodge pairs, $\chi \in [-960, 960]$;
  - Dual polytope $\Delta \leftrightarrow \Delta^\circ$ = mirror symmetry
  - Vienna group (KS, Knapp,. . .), Oxford group (Candelas, Lukas, YHH, . . .), MIT group (Taylor,Johnson, Wang, . . .), Northeastern/Wits Collab (Nelson, Jejjala, YHH), Virginia Tech (Anderson, Gray, Lee, . . .)

Refined Structures in KS

The Compact CY3 Landscape

- 40 years of research by mathematicians and physicists
- 500 million data-points (and growing,. . .)
- Horizontal $\chi = 2(h^{1,1} - h^{2,1})$ vs. Vertical $h^{1,1} + h^{2,1}$: a Georgia O'Keefe Plot





CY3 Compactification: Recent Development

- E_6 GUTs less favourable, $SU(5)$ and $SO(10)$ GUTs: general embedding
  - Instead of TX, use (poly-)stable holomorphic vector bundle V
  - Gauge group$(V) = G = SU(n)$, $n = 3, 4, 5$, gives $H =$ Commutant(G, E_8):

$E_8 \to G \times H$	Breaking Pattern		
$SU(3) \times E_6$	248	$\to$	$(1, 78) \oplus (3, 27) \oplus (\overline{3}, \overline{27}) \oplus (8, 1)$
$SU(4) \times SO(10)$	248	$\to$	$(1, 45) \oplus (4, 16) \oplus (\overline{4}, \overline{16}) \oplus (6, 10) \oplus (15, 1)$
$SU(5) \times SU(5)$	248	$\to$	$(1, 24) \oplus (5, \overline{10}) \oplus (\overline{5}, 10) \oplus (10, 5) \oplus (\overline{10}, \overline{5}) \oplus (24, 1)$

- MSSM: $H \xrightarrow{\text{Wilson Line}} SU(3) \times SU(2) \times U(1)$
- Issues in low-energy physics $\sim$ Precise questions in Alg Geo of (X, V)
  - Particle Content $\sim$ (tensor powers) V Bundle Cohomology on X
  - LE SUSY $\sim$ Hermitian Yang-Mills connection $\sim$ Bundle Stability
  - Yukawa $\sim$ Trilinear (Yoneda) composition
  - Doublet-Triplet splitting $\sim$ representation of fundamental group of X

Algorithmic Compactification

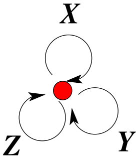
- Schön Quotients: **Penn Model** [Braun-YHH-Ovrut-Pantev 2005] exact MSSM
- Searching the MSSM, *Sui Generis?*
  - $\sim 10^7$ Spectral Cover bundles [Donagi, Friedman-Morgan-Witten, 1996-8] over elliptically fibered CY3 (2005-9), [Donagi-YHH-Ovrut-Pantev-Reinbacher, Gabella-YHH-Lukas,...]
  - $\sim 10^5$ (Monad) Bundles over all CICYs [Anderson-Gray-YHH-Lukas, 2007-9]
  - Monad Bundles over KS YHH-Kreuzer-Lee-Lukas 2010-11: ~ 200 in 10^5 3-gens
  - culminating in .. Stable Sum of Line Bundles over CICYs (Oxford-Penn-Virginia 2012-) Anderson-Gray-Lukas-Ovrut-Palti ~ 200 in 10^{10} MSSM
- meanwhile ... LANDSCAPE grew with D-branes Polchinski 1995, M-Theory/G_2 Witten, 1995, F-Theory/4-folds Katz-Morrison-Vafa, 1996, AdS/CFT Maldacena 1998, Flux-compactification Kachru-Kallosh-Linde-Trivedi, 2003, ...

A Geometer's AdS/CFT

- $\boxed{\text{Rep. Variety(Quiver)} \sim \text{VMS(SUSY QFT)} \sim \text{affine/singular variety}}$

  e.g $\mathcal{N} = 1$ Quiver variety = vacuum of F- & D-flatness = non-compact CY3

- $\mathcal{N} = 4$ $U(N)$ Yang-Mills
  - 3 adjoint fields X, Y, Z with superpotential $W = \text{Tr}(XYZ - XZY)$



  - Original AdS/CFT [Maldacena, '98]
  - N D3-branes (w.v. is $\mathcal{N} = 4$ in $\mathbb{R}^{3,1}$) $\perp \mathbb{R}^6$

    $\simeq \mathbb{C}^3$ = Vacuum Moduli Space

  - QUIVER = Finite graph (label = rk(gauge factor)) + relations from W
    - Matter Content: Nodes + arrows
    - Relations (F-Terms): $D_i W = 0 \rightsquigarrow [X, Y] = [Y, Z] = [X, Z] = 0$
  - Here $\mathbb{C}^3$ is real cone over S^5 (simplest Sasaki-Einstein 5-manifold), others?

Next Simplest Example, Orbifolds: $\mathcal{M} = \mathbb{C}^3/(\Gamma \subset SU(3))$

- Gimon-Polchinski, Douglas-Moore (1996); Greene-Morrison-Plesser, Johnson-Myers (1997);

  $\Gamma \subset SU(2)$, ADE (McKay Correspondence) $\Gamma \subset SU(3)$ (Hanany-YHH 9811183):

  Projection of parent $\mathbb{C}^3$ theory

- *Geometry of $\mathcal{M}$ and w.v. physics* $\sim Rep(\Gamma) = \{r_i\}$

  $\mathcal{R} \otimes \mathbf{r}_i = \bigoplus_j a_{ij}^{\mathcal{R}} \mathbf{r}_j$, $a_{ij}^{\mathcal{R}} \rightsquigarrow$ adjacency matrix of quiver

- Sasaki-Einstein base = Lens space of S^5

  Quiver Example

- Discrete finite subgroups of $SU(n)$ classified up to $n = 8$, usual pattern:

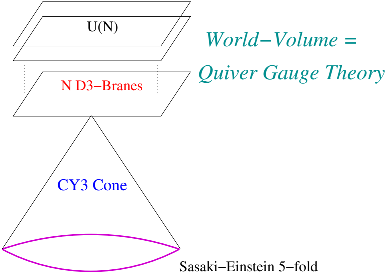
  1. $\mathbb{Z}_{m_1} \times \ldots \times \mathbb{Z}_{m_{n-1}}$;
  2. a few non-Abelian infinite families generalizing Dihedral group;
  3. a finite # Exceptional cases;

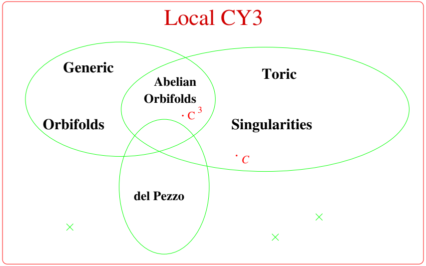
Non-Compact Toric CY3 $\mathcal{M} :=$ Spec$_{\text{Max}}\mathbb{C}[S_\sigma = x_i^{\text{gen}(\sigma^\vee)\cap\mathbb{Z}^d}]$

- Combinatorially: CYn $=$ convex lattice polytope in $\mathbb{R}^{n-1}$
- No known *compact* CY3 metrics, most known (non-compact) CY3 metrics are toric so far (SE Cone $U(1)^3$ isometry): infinite families $Y^{p,q}$, L^{abc} (conifold, special case); [Candelas-de la Ossa, Cvectic, Hanany, Pope, Sparks, Waldram 1990's-. . .]
- By far the largest class known and studied (Use Witten's GLSM Aspinwall, Beasley, Cachazo, Diaconescu, Douglas, Greene, Katz, Morrison, Plesser, Vafa et al., 1997-2000) Feng-Hanany-YHH 0003085: Inverse Algorithm toric diag $\rightsquigarrow$ gauge theory
- [Franco-Hanany-Kennaway-Sparks-Vegh 2005] gauge theory $=$ bipartite brane tiling T^2
  1. Mirror symmetry: Feng-Kennaway-YHH-Vafa 2005
  2. Recent generalizations to SUSY gauge theories in other dimensions Franco-Seong-Lee-Vafa 2016-7

The Landscape of non-Compact (affine, local, sing) CY3

a 15-year prog. joint with A. Hanany et al.





Classfications of Toric Gauge Theories

1. [Davey-Hanany-Pasukonis, Mekareeya, Torri, Sparks, Benishti, YHH] toric CY3,4: 2009-
2. [Franco-YHH-Sun-Yan] 1702.03958
3. Use reflexive polytope: Hanany-Seong, 2014, YHH-Seong-Yau, 2017

SUMMARY: Algorithms and Datasets in String Theory

- Growing databases and algorithms (many motivated by string theory): e.g., Singular, Macaulay2, GAP, SAGE, Bertini, grdb, etc; "Periodic table of shapes Project" classify Fanos
- Archetypical Problems
  - Classify configurations (typically integer matrices: polyotope, adjacency, . . .)
  - Compute geometrical quantity algorithmically
    - toric $\rightsquigarrow$ combinatorics;
    - quotient singularities $\rightsquigarrow$ rep. finite groups;
    - generically $\rightsquigarrow$ ideals in polynomial rings;
    - Numerical geometry (homotopy continuation);
    - Cohomolgy (spectral sequences, Adjunction, Euler sequences)
- Typical Problem in String Theory/Algebraic Geometry:

$$\begin{array}{ccc} INPUT & & OUTPUT \\ \boxed{\text{integer tensor}} & \longrightarrow & \boxed{\text{integer}} \end{array}$$

Where we stand . . .

The Good Last 10-15 years: several international groups have bitten the bullet Oxford, London, Vienna, Blacksburg, Boston, Johannesburg, Munich, . . . computed many geometrical/physical quantities and compiled them into various databases **Landscape Data** ($10^{9\sim10}$ entries typically)

The Bad Generic computation HARD: dual cone algorithm (exponential), triangulation (exponential), Gröbner basis (double-exponential) . . . e.g., how to construct stable bundles over the $\gg 473$ million KS CY3? Sifting through for MSSM not possible . . .

The ??? **Borrow new techniques from "Big Data" revolution**

A Wild Question

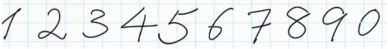
- Typical Problem in String Theory/Algebraic Geometry:

$$\overset{INPUT}{\boxed{\text{integer tensor}}} \longrightarrow \overset{OUTPUT}{\boxed{\text{integer}}}$$

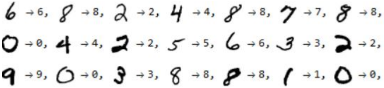
- Q: Can (classes of problems in computational) Algebraic Geometry be "learned" by AI ?
- 1706.02714 YHH: Experimentally, it seems to be the case for many situations

A Prototypical Question

- Hand-writing Recognition, e.g., my 0 to 9 is different from yours:



- How to set up a bijection that takes these to $\{1, 2, \ldots, 9, 0\}$? Find a clever Morse function? Compute persistent homology? Find topological invariants? <u>ALL are inefficient and too sensitive to variation.</u>
- What does your iPhone/tablet do? What does Google do?
  - Take large sample, take a few hundred thousand (e.g. NIST database)



. . .

  - Machine-Learn: (1) Data Acquisition; (2) Setup Neural Network (NN); (3) Train NN. generically, if the NN is sufficiently complex, called Deep Learning

A Single Neuron: The Perceptron

- began in 1957 (!!) in early AI experiments (using CdS photo-cells)
- DEF: Imitates a neuron: activates upon certain inputs, so define
  - Activation Function $f(z_i)$ for input tensor z_i for some multi-index i;
  - consider: $f(w_i z_i + b)$ with w_i weights and b bias/off-set;
  - typically, $f(z)$ is sigmoid, Tanh, etc.
- Given training data: $D = \{(x_i^{(j)}, d^{(j)}\}$ with input x_i and **known output** $d^{(j)}$, minimize

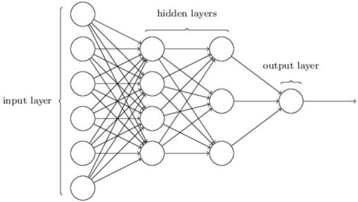
$$SD = \sum_j \left(f(\sum_i w_i x_i^{(j)} + b) - d^{(j)} \right)^2$$

  to find optimal w_i and $b \rightsquigarrow$ "learning"

- Essentially (non-linear) regression

The Neural Network: network of neurons $\rightsquigarrow$ the "brain"

- DEF: a connected graph, each node is a perceptron (Beta-version implemented on Mathematica 11.1 $+$)
  1. adjustable weights/bias;
  2. distinguished nodes: 1 set for input and 1 for output;
  3. iterated training rounds.



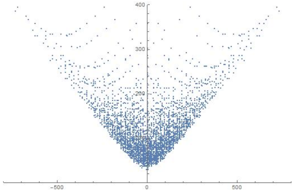
Simple case: forward directed only, called multilayer perceptron

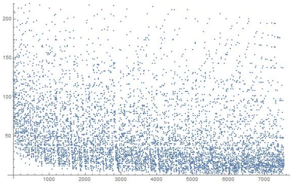
- use the simple MLP: e.g., Sigmoid $\to$ Linear $\to$ Tanh $\to$ Summation
- Essentially how brain learns complex tasks; **apply to our Landscape Data**

Hypersurfaces in $W\mathbb{P}^4$: Warmup I

Oftentimes, questions in pheno are **qualitative**, e.g.,

- large # complex structure how many have, say, $h^{2,1} > 50$?
  - [Candelas-Lynker-Schimmrigk] Landau-Ginzburg methods: many hours; using Euler sequence/Adjunction: many more hours

(a) 

(b) 

(a) Mirror plot of $(\chi, h^{1,1} + h^{2,1})$
(b) Distribution of $h^{2,1}$

  - With the MLP NN, 500 training rounds, under 1 min, learns $h^{2,1} > 50$ to 97% Cosine distance $D_C = 0.998$, Matthews $\phi = 0.84$.
  - consistency check (testing full set): cool and re-assuring but not useful

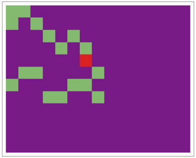
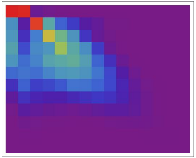
Hypersurfaces in $W\mathbb{P}^4$: Warmup II

- What if the data is not complete? Very often the case when computation powers are not yet capable (e.g., all triang for KS dataset: don't even know how many CY3 hypersurfaces in the 473 million toric varieties)
- Standard method: take partial training and validation data, s.t., $D = T \sqcup V$
  - train NN with random 2000/7555 inputs ($\sim 1/4$ only)
  - use the trained NN to predict value for the remaining UNSEEN 7555 - 2000
  - Get $\sim 91.8\%$ precision, $d_C = 0.91$, $\phi = 0.84$ in less than 20 sec on regular laptop! Learning Curves
- Another Question: How many have χ divisible by 3? (useful for # generations after Wilson line)

  2000 samples ~ 1 min: 80% precision, $d_C = 0.91$ when predicting 7555-2000
- **Endless possibilities of mathematical/physical queries...**

CICYs: a Colourful Example

- An image = a matrix (pixels) with entries denoting shade/colour; NN really good at images (e.g. hand-writing) [RMK: not using a convolutional NN here]
- CICY is a (padded) 12×15 matrix with 6 colours $\rightsquigarrow$ CICY is an image

(a)  (b) 

(a) typical CICY;

(b) average CICY

- Input more sophisticated, so greater accuracy expected: e.g. in learning large number of Kahler parametres $h^{1,1} > 5$:

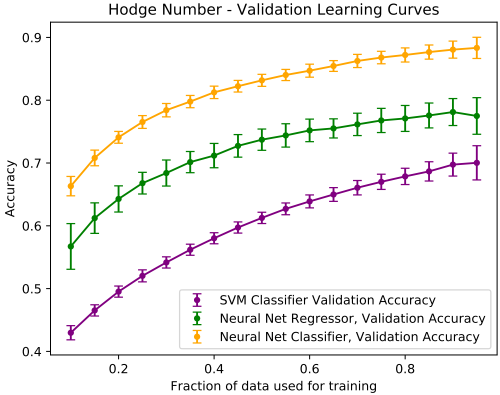
  learns 4000 samples ($< 50\%$) in ~ 5 min; validate against 7890-4000: 97% accuracy, $d_C = 0.98$, $\phi = 0.87$.

Learning Curves

CICYs: Detailed Analysis

Kieran Bull [Oxford] [Bull-YHH-Jejjala-Mishra: arXiv:1806.03121]

- **TensorFlow** Python's implementation of NNs and DL
- Compare NNs with Decision Trees, Support Vector Machines, etc

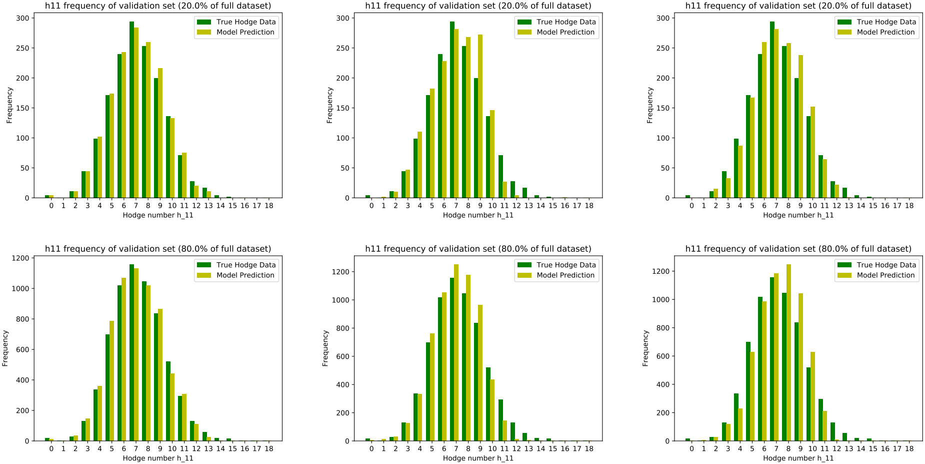


Can one learn the FULL information on Hodge numbers?

$h^{1,1} \in [0, 19]$ so can set up 20-channel NN classifer, regressor, as well as SVM

CICYs: Comparative Studies

$h^{1,1}$ for NN, Regressor, SVM at 20 and 80% training Sky's the Limit



Remarks and Sanity Checks

- Why does it work?
  - Short answer in the data-science community: nobody knows!!
  - Theorems still need to be proven about convergence, measure, etc., esp. for a large number of neurons; even a few neurons has many parametres
  - At the most basic level: problems in algebraic geometry boil down to finding kernels of integer matrices
  - NOT over-fitting training data $\cap$ validation data $= \{\}$
- A Reprobate: Try to predict the next prime; has to fail, otherwise crazy
  - Train our NN: gets a miserable 0.1% accuracy even on learning, forget about predicting, great! Better off just fitting $n \log(n)$ using PNT
  - expect other things like digits of π to utterly fail

Summary and Outlook

PHYSICS

- The string landscape now solidly resides in the age of Big Data
- Use Neural Networks as
  1. Classifier deep-learn and categorize landscape data
  2. Predictor estimate results beyond computational power

MATHS

- somehow bypassing the expensive steps of long sequence-chasing, Gröbner bases, dual cones/combinatorics and getting the right answer. **how is AI doing maths more efficiently without knowing any maths?**
- problems in geometry, combinatorics, etc, good; number theory, not so good.

Syntax and Semantics

- should prove useful to problems in fields ranging from string phenomenology to algebraic geometry and beyond
- many species of animals are capable of extremely sophisticated tasks (e.g., chimps with herbal medicine); we are such a species when confronted with the landscape; we can (deep-)learn by trial-error before we tackle the fundamental question of why in the future . . .
- Boris Zilber [Merton Professor of Logic, Oxford]: "you've managed syntax without semantics. . . "
- cf. *YHH: The CY Landscape, Springer 2019 (?)*

THAT YOU

Sophia (Hanson Robotics, HK)

First non-human citizen (2017, Saudi Arabia)

First non-human with UN title (2017)

Famous CICYs

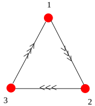
- The Quintic $Q = [4|5]^{1,101}_{-200}$ (or simply $[5]$);
- Yau Manifold: $TY = \begin{pmatrix} 1 & 3 & 0 \\ 1 & 0 & 3 \end{pmatrix}^{14,23}_{-18}$
  - no CICY has $\chi = \pm 6$
  - TY has freely-acting $\mathbb{Z}_3 \rightsquigarrow (TY/\mathbb{Z}_3)^{6,9}_{-6}$;
  - central to early string pheno
- Schön Manifold: $S = \begin{pmatrix} 1 & 1 \\ 3 & 0 \\ 0 & 3 \end{pmatrix}^{19,19}_{0}$ has $\mathbb{Z}_3 \times \mathbb{Z}_3$ freely acting symmetry
  - explored more recently;
  - The quotient is $M^0_{3,3}$.

Back to CICYs

Archetypal Quiver

- Archetypal example: $\mathbb{C}^3/\mathbb{Z}_3$ with action $(1,1,1) \rightsquigarrow U(1)^3$ quiver theory

$$W = \epsilon_{\alpha\beta\gamma} X_{12}^{(\alpha)} X_{23}^{(\beta)} X_{31}^{(\gamma)}, \qquad X_{12}^{(\alpha)}, X_{23}^{(\beta)}, X_{31}^{(\gamma)}, \alpha, \beta, \gamma = 1, 2, 3$$



Adjacency Matrix: $A = \begin{pmatrix} 0 & 3 & 0 \\ 0 & 0 & 3 \\ 3 & 0 & 0 \end{pmatrix}$

Incidence Matrix: $d = \begin{pmatrix} -1 & -1 & -1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & -1 & -1 & -1 \end{pmatrix}$

- F-terms non-trivial, counting GIO's complicated
- Moduli space: 27 quadrics in $\mathbb{C}^{10}$ Back to Quivers

Refined Structure in KS Data

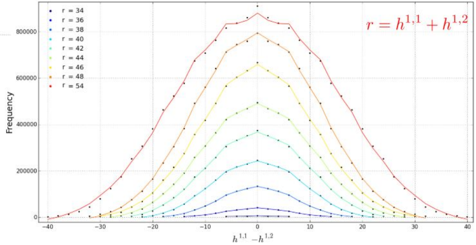
- DATABASES:

  http://hep.itp.tuwien.ac.at/~kreuzer/CY/

  http://www.rossealtman.com/

- Altman-Gray-YHH-Jejjala-Nelson 2014-17 triangulate Δ_4 (orders more than 1/2-billion): up to $h^{1,1} = 7$
- Candelas-Constantin-Davies-Mishra 2011-17 special small Hodge numbers
- Taylor, Johnson, Wang et al. 2012-17 elliptic fibrations
- YHH-Jejjala-Pontiggia 2016 distribution of Hodge, χ, Pseudo-Voigt

KS stats: YHH-Jejjala-Pontiggia arXiv:1512.01579 Return



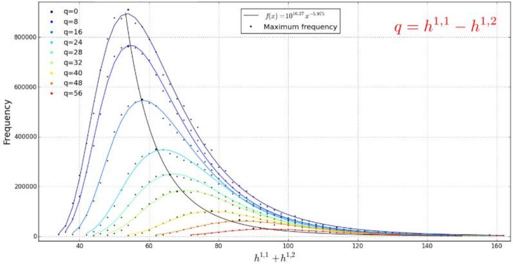
Pseudo-Voigt distribution

sum of Gaussian and Cauchy

$$(1-\alpha)\frac{A}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} + \alpha\frac{A}{\pi}\left[\frac{\sigma^2}{(x-\mu)^2+\sigma^2}\right]$$

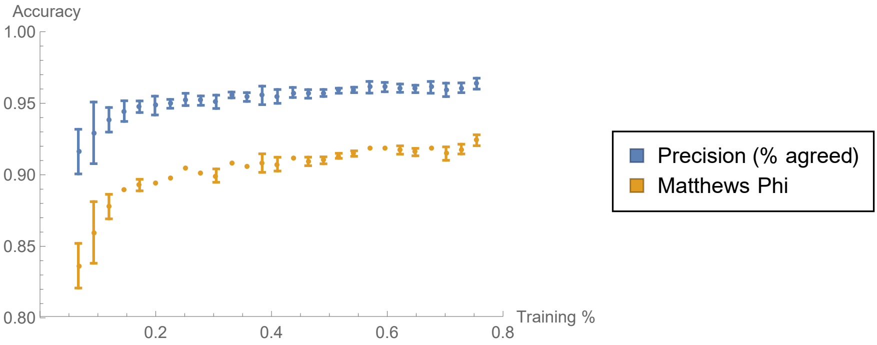
Planck distribution

$$\frac{A}{x^n}\frac{1}{e^{b/(x-c)}-1}$$



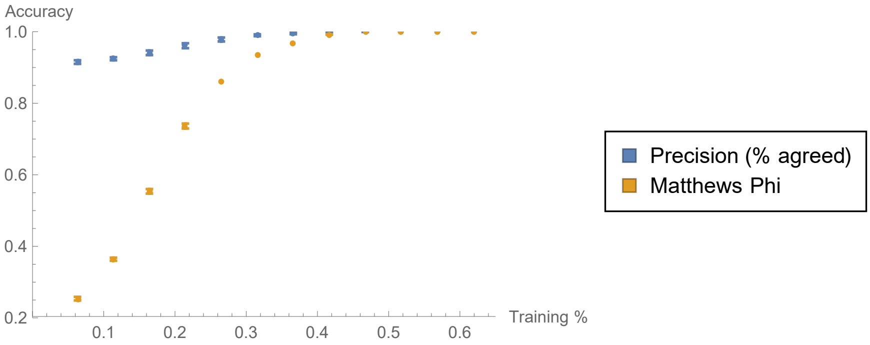
He, VJ, Pontiggia (2015)

Learning Curve: WP4



Return

Learning Curve: CICY

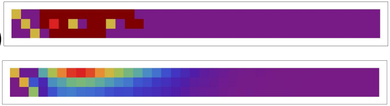


Return

KS Dataset: *Gradus ad Parnasum*

- 4319 reflexive Δ_3 correspond to compact K3 surfaces or non-compact CY3
- Each is an integer matrix (padded) 3×39 with entries in $[0, 28]$, pixelate with 28 shades of colour

(a) (b)



(a) typical Δ_3;

(b) average Δ_3

- Data size not so big for $n = 3$; training against for example, Sasaki-Einstein Volume or Picard Number achieves $\sim 60\%$ accuracy in a few minutes
- **GOAL:** to learn from geometrical quantities in a subset of $\sim 10^{5-6}$ (currently within computer power) to predict the full $\sim 10^{10}$ Δ_4 (currently beyond computer power) (to do ...)

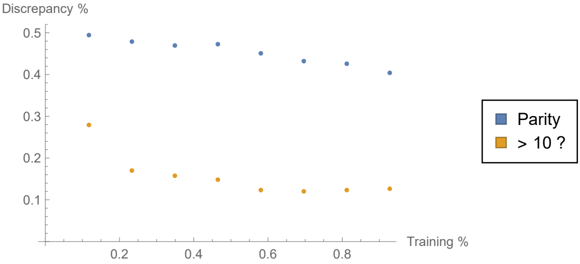
Toric Quiver Gauge Theories

- Infinite number of theories: any convex lattice polygon $\rightsquigarrow$ non-compact CY3 which D3-brane can probe; 2 databases so far:
  - Davey-Hanany-Pasukonis, 2009 (by terms in superpotential);
  - updated and expanded Chuang-Franco-YHH-Xiao, 2017 (by area of polygon)
- **computationally hard:** finding dual cone exponential-running; even with dimer/brane-tiling technology, Higgsing/perfect-matchings time-consuming
- Try on dataset1, (small) size = 375
  - INPUT: combined integer matrix Q_{DF}: incidence matrix from D-terms; exponent matrix from F-terms
  - OUTPUT: e.g., # gauge groups (train 100, predicts to $\sim 97\%$) Learning Curves
- TO DO: use this to **predict** unknown gauge theory given big toric diagrams

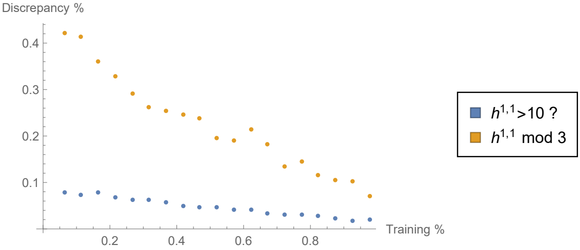
Back to CICYs

Learning Curves

Picard Numbers of K3 hyperfaces in toric Fano 3-folds from reflexive Δ_3



$h^{1,1}$ of CICYs:



Return