Microwave Near-field Imaging in Real Time

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APPLICATIONS OF MICROWAVE NEAR-FIELD IMAGING

• penetration into optically obscured objects (clothing, walls, luggage, living tissue…)
  ➢ the lower the frequency the better the penetration
  ➢ frequency bands from 500 MHz into the mm-wave bands (≤300 GHz)

• compact relatively cheap electronics esp. in the low-GHz range

• diverse suite of image reconstruction methods

VARIOUS APPLICATIONS

whole body scanners  nondestructive testing  through-wall imaging

medical imaging  underground radar
APPLICATIONS: LUGGAGE INSPECTION, NDT


20 GHz to 30 GHz frequency range

Prof. Zoughi’s team at Missouri University of Science & Technology

[https://youtu.be/RE-PPXmtTeA]

Fig. 15. Example of video camera utility for imaging a box cutter and a pair of scissors inside a laptop bag. (a) Picture of laptop bag in front of the camera aperture with inset showing the objects inside the bag. (b) 3-D view. (c) 2-D image slice focused on the box cutter. (d) 2-D image slice focused on the pair of scissors.
APPLICATIONS: WHOLE BODY SCANNERS

[Sheen et al., “Near-field three-dimensional radar imaging techniques and applications,” Applied Optics 2010]

Pacific Northwest National Laboratory, Washington, USA

40 GHz to 60 GHz (U band) cylindrical scan

40 GHz to 60 GHz (U band) cylindrical scan

10 GHz to 20 GHz polarimetric cylindrical scan
APPLICATIONS: MEDICAL IMAGING


Prof. Kikkawa’s team at Hiroshima University, Japan

Figure 3. Dome antenna array design. (a) The top view of the antenna in x-y plane. (b) The side view of the antenna in x-z plane. (c) Top view photograph. (d) Bottom view photograph.
MICROWAVE NEAR-FIELD IMAGING: COMMERCIAL GROWTH

• mm-wave whole-body imagers for airport security inspection (> 30 GHz)

• through-wall and through-floor infrastructure inspection for contractors and home inspectors (UWB, 3 GHz to 8 GHz)

• numerous underground radar applications: detection of pipes, cables, tunnels, etc. (< 3 GHz)
OUTLINE

➢ specifics of near-field microwave imaging in
  • data acquisition
    • forward models
    • inversion strategies (real-time)

➢ examples and comparisons of methods
DATA ACQUISITION: SCANNING

Principle: N-D imaging (N=2,3) needs abundant and diverse N-D data sets

- **Spatial scans** – 1D (linear) or 2D (surface)
  - illuminate target from various angles
  - collect scattered signals at various angles/distances
  - acquisition surfaces – planar, cylindrical, spherical
  - mechanical vs. electronic scanning

- **Frequency/temporal sweeps**
- **Polarization diversity**

<table>
<thead>
<tr>
<th></th>
<th>Mechanical Scan</th>
<th>Electronic Switching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed</td>
<td>low</td>
<td>HIGH</td>
</tr>
<tr>
<td>Complexity</td>
<td>LOW</td>
<td>high</td>
</tr>
<tr>
<td>Flexibility in adjusting scan parameters</td>
<td>GREAT</td>
<td>limited</td>
</tr>
</tbody>
</table>
DATA ACQUISITION: SPATIAL SAMPLING

- spatial data diversity
  - each sample must add independent information – improves uniqueness
  - linearly dependent data may lead to ill-posed inversion problems
  - over-sampling: pros and cons
  - staying below but close to the maximum spatial sampling step ensures diversity

\[ \Delta \zeta \leq \Delta \zeta_{\text{max}} \approx \frac{\lambda_{\text{min}}}{4 \sin \alpha}, \quad \zeta \equiv x, y \]

- effective near-field wavelengths may be shorter than \( \lambda = \frac{2\pi}{k_b} \)

\[ \lambda_{\text{eff,min}} = \frac{2\pi}{k_{\text{max}}(x,y)} \]

\[ \tilde{S}(k_x, k_y) = \mathcal{F}_{2D}\{S(x, y)\} \]
DATA ACQUISITION: FREQUENCY SAMPLING

- frequency data diversity in frequency-sweep measurements
  - stay below but close to the **maximum frequency sampling step**
  - ensures that back-scattered signals (after IFFT) from all targets at distances \( \leq R_{\text{max}} \) do not overlap

\[
\Delta f \leq \Delta f_{\text{max}} = \frac{1}{2T_{\text{max}}} \approx \frac{v_b}{4R_{\text{max}}}
\]
DATA ACQUISITION: TEMPORAL SAMPLING

- temporal data diversity in time-domain measurements
  
  ➢ stay below but close to the maximum time sampling step
  ➢ ensures that all frequency components of the pulsed signals are fully used (Nyquist)

\[
\Delta t \leq \Delta t_{\text{max}} \approx \frac{T_{\text{min}}}{2} = \frac{1}{2f_{\text{max}}}
\]
FORWARD MODEL OF SCATTERING: S-PARAMETER DATA EQUATION

[Nikolova et al., APS-URSI 2016][Beaverstone et al., IEEE Trans. MTT, 2017]

- scattering from penetrable objects (isotropic scatterer)

\[
S_{ik}^{sc} = \frac{i\omega\varepsilon_0}{2a_i a_k} \iiint_{V_s} \Delta\varepsilon_r (r') E_i^{inc} (r') \cdot E_k^{tot} (r') \, dr' 
\]

data
complex permittivity contrast
Green’s vector function

\[ \Delta\varepsilon_r (r') = \varepsilon_r (r') - \varepsilon_{r,b} (r') \]

total internal field

\[ E_i^{inc} (r') : \text{incident internal field due to Rx antenna if it were to transmit} \]

\[ E_k^{tot} (r') : \text{total internal field due to Tx antenna} \]

\[ i, k = 1, \ldots, N_p \]

total number of experiments: \( N_p^2 \)

reciprocity: \( (N_p^2 + N_p) / 2 \)
**FORWARD MODEL:** BORN’S LINEARIZING APPROXIMATION

1. **Data Equation and Total Internal Field**
   - The total field $E_k^{\text{tot}}(r'; \Delta \varepsilon(r'))$ is generally unknown and depends on contrast: **data equation is nonlinear in the unknown contrast**.

2. **Born’s Approximation**
   - Born’s approximation of the total internal field linearizes the data equation:
     
     $$
     \frac{i \omega \varepsilon_0}{2 a_i a_k} \int \int \int_{V_s} \Delta \varepsilon_r(r') E_i^{\text{inc}}(r') \cdot E_k^{\text{tot}}(r'; \Delta \varepsilon_r(r')) \, dr'
     $$

3. **Main Challenge**
   - **main challenge of near-field imaging with linear inversion methods:** Incident-field distributions in antennas’ near zone are difficult to model.
any one of the conditions below implies near-field (short-range) imaging

- object is in the near-field region of antennas
  \[ r < D_{\text{far}} \approx \frac{2D_{A,\text{max}}^2}{\lambda} \]

- distance from object to antennas ≤ object’s size
  \[ r \leq D_{\text{OUT,max}} \]

- distance from object to antennas ≤ wavelength
  \[ r \leq \lambda \]

- implication: *resolvent kernel depends on incident fields which do not conform to analytical free-space far-zone propagation models*, e.g.,

\[
E^{\text{inc}}(r') \sim \hat{p}G(\theta, \varphi) \frac{e^{-ik_b r}}{r} \quad \text{not valid}
\]
LINEARIZED FORWARD MODEL: TIME DOMAIN

- Born’s linearizing approximation is applied in the same way

**frequency-domain (S-parameters)**

\[
S_{ik}^{sc} \approx \iiint_{V_s} \Delta \varepsilon_r(r') \hat{E}_{i}^{inc}(r') \cdot \hat{E}_{k}^{inc}(r') \, dr'
\]

**time-domain (pulsed-radar waveforms)**

\[
s_{ik}^{sc}(r_{Rx}, t; r_{Tx}) \approx \iiint_{V_s} \kappa(r') \left[ h_{Rx}^{inc} \ast (u_{Tx}^{inc})'' \right] \, dr'
\]

\[
\kappa(r') = \frac{\Delta \varepsilon_r}{v_b^2}
\]

[[Nikolova, Introduction to Microwave Imaging, 2017]]
LINEARIZED FORWARD MODEL: BORN vs. RYTOV DATA APPROXIMATION

• Born scattered-field data approximation

\[
S_{ik}^{sc}(\mathbf{r} \in S_a) \approx \left[ S_{ik}^{tot} - S_{ik}^{inc} \right]_{(\mathbf{r} \in S_a)} \approx \iiint_{V_s} \Delta \varepsilon_r(\mathbf{r'}) \bar{E}_i^{inc}(\mathbf{r'}) \cdot \bar{E}_k^{inc}(\mathbf{r'}) d\mathbf{r'}
\]

data calibration step

• Rytov scattered-field data approximation

\[
S_{ik}^{sc}(\mathbf{r} \in S_a) \approx S_{ik}^{inc} \ln \left( \frac{S_{ik}^{tot}}{S_{ik}^{inc}} \right)_{\mathbf{r} \in S_a} \approx \iiint_{V_s} \Delta \varepsilon_r(\mathbf{r'}) \bar{E}_i^{inc}(\mathbf{r'}) \cdot \bar{E}_k^{inc}(\mathbf{r'}) d\mathbf{r'}
\]

data calibration step

[Tajik et al., JPIER- B, 2017][Shumakov et al., Trans. MTT, 2018]
FORWARD MODEL: APPROXIMATIONS

- **total internal field approximation (Born)**
  - limitations on both size and contrast of the scatterer
  \[ a^2 \left| k_s^2(r) - k_b^2 \right| \ll 1, \ r \in V_s \]
  - if OUT violates limit: image contains artifacts which reflect differences between \( E_{\text{inc}}^{\text{Tx}}(r') \) and \( E_{\text{tot}}^{\text{Tx}}(r') \) rather than contrast
  - expect trouble in areas of strong multiple scattering & mutual coupling

- **data approximation**
  - Born – limitation on both size and contrast: \( 2a \left| k_s(r) - k_b \right| < \pi, \ r \in S_a \)
  - Born – neglects multiple scattering & mutual coupling between antennas and OUT
  - Rytov – limitation on contrast only: \( (k_s^2 - k_b^2) / k_b^2 < 1, \ r \in S_a \)
LINEAR INVERSION METHODS: THE ENGINES OF REAL-TIME IMAGING

**principle:** linearize forward model & solve resulting linear system of equations

\[
S_{ik}^{sc} \approx \iiint_{V_s} \Delta \varepsilon_r (r') \overline{E}_{i}^{\text{inc}} (r') \cdot \overline{E}_{k}^{\text{inc}} (r') \, dr'
\]

- contrast and compare with nonlinear inversion methods

**principle:** solve nonlinear forward-model equations for BOTH contrast and total field using nonlinear optimization and/or iterative methods

\[
S_{ik}^{sc} \approx \iiint_{V_s} \Delta \varepsilon_r (r') \overline{E}_{i}^{\text{inc}} (r') \cdot \overline{E}_{k}^{\text{tot}} (r', \Delta \varepsilon_r (r')) \, dr'
\]

Maxwell’s equations
**holography** refers to reconstruction methods that use both the magnitude and phase of the scattered waves recorded at a surface to produce a 3D image in a single inversion step.

- **reflected signals:** $S_{11}, S_{22}$
- **transmitted signals:** $S_{21}, S_{12}$

**type of response** | **number of values $S_{ik}(x',y')$**
--- | ---
co-pol X-X | $4N_\omega$
co-pol Y-Y | $4N_\omega$
cross-pol X-Y | $4N_\omega$
cross-pol Y-X | $4N_\omega$
**TOTAL** | $16N_\omega = N_T$

- number of responses acquired at each position.
\[ S_{\xi}^{\text{sc}}(x, y, \bar{z}; \omega) \approx \iiint_{V_s} \Delta \varepsilon_r(x', y', z') \left[ \mathbf{E}_{\xi, Rx}^{\text{inc}} \cdot \mathbf{E}_{\xi, Tx}^{\text{inc}} \right] (x', y', z'; x, y, \bar{z}; \omega) \, dx' \, dy' \, dz', \ \xi = 1, \ldots, N_T \]

approximate (Born) resolvent kernel \( K_{\xi}(r'; r; \omega) \)

- assume kernel is translationally invariant in \( x \) and \( y \) (background is uniform or layered)

Let \( K_{\xi}(x', y'; z'; \omega) \equiv K_{\xi}(x', y', z'; 0, 0, \bar{z}; \omega) \)

Then \( K_{\xi}(x', y', z'; x, y, \bar{z}; \omega) = K_{0, \xi}(x' - x, y' - y; z'; \omega) \)

\[ \Rightarrow S_{\xi}^{\text{sc}}(x, y, \bar{z}; \omega) \approx \iiint_{V_s} \Delta \varepsilon_r(x', y', z') K_{0, \xi}(x' - x, y' - y; z'; \omega) \, dx' \, dy' \, dz' \]
HOLOGRAPHY: RESOLVENT KERNEL, examples

- examples: analytical kernels used with far-zone reflection data \( \overline{E}_{\xi,Rx}^{\text{inc}} = \overline{E}_{\xi,Tx}^{\text{inc}} \)

  plane waves: \( \mathcal{K}_\xi(x', y', z'; x, y, z_{Rx}; \omega) = \overline{E}_{Rx}^{\text{inc}} \cdot \overline{E}_{Rx}^{\text{inc}} \sim e^{-i2k_b\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}} \)

  spherical waves: \( \mathcal{K}_\xi(x', y', z'; x, y, z_{Rx}; \omega) \sim \frac{e^{-i2k_b\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}}{(x-x')^2+(y-y')^2+(z-z')^2} \)

  cylindrical waves: \( H_0^{(2)}(2k\rho), \ \rho = \sqrt{(x-x')^2+(y-y')^2}, \ z = z' = \text{const} \)

- far-field analytical kernels do not work well with near-field data

- kernels computed from simulated incident-field distributions suffer from modeling errors [Amineh et al., Trans. AP, 2011]

- near-field kernels are best determined through measuring the system PSF [Savelyev&Yarovoy, EuRAD 2012][Amineh et al., IEEE Trans. Instr.&Meas., 2015]
• PSF-based kernels enable **quantitative imaging in real time**

• PSF is the system response to a point scatterer

• relating PSF to kernel

Let \( \text{PSF}_{0z',\xi}(x, y, \bar{z}; \omega) \equiv \text{PSF}_{\xi}(x, y, \bar{z}; 0, 0, z'; \omega) \)

Then \( K_{0,\xi}(x, y; z'; \omega) = \text{PSF}_{0z',\xi}(-x, -y, \bar{z}; \omega) / (\Delta \varepsilon_{r,sp} \Omega_{sp}) \)
DATA EQUATION OF HOLOGRAPHY IN TERMS OF PSF

• in real space

\[ S_{sc}^\xi (x, y, \bar{z}; \omega) \approx \frac{1}{\Delta \epsilon_r, sp \Omega_{sp}} \int \int \int \Delta \epsilon_r (x', y', z') \cdot \text{PSF}_{0z', \xi} (x - x', y - y'; \omega) \, dx' \, dy' \, dz' \]

2D convolution

• in Fourier (or \( k \)) space

\[ \tilde{S}_{sc}^\xi (k_x, k_y; \bar{z}; \omega) \approx \frac{\Delta x' \Delta y'}{\Delta \epsilon_r, sp \Omega_{sp}} \int \int \tilde{F}(k_x, k_y; \bar{z}') \cdot \text{PSF}_{0z', \xi} (k_x, k_y; \omega) \, dz' \]

\[ \text{FT}_{2D} \{ \Delta \epsilon_r (x', y', z') \} \]

• system of equations to solve at each spectral position \( \kappa = (k_x, k_y) \)

\[ \tilde{S}_{\xi}^{(m)} (\kappa) \approx \sum_{n=1}^{N_z} \tilde{f}(\kappa; z_n') \text{PSF}_{0z', \xi}^{(m)} (\kappa) \]

\[ m = 1, \ldots, N_\omega \]

\[ \xi = 1, \ldots, N_T \]

\[ \Omega_v \]

\[ \tilde{f}(\kappa; z_n') = \frac{\Delta x' \Delta y' \Delta z_n'}{\Delta \epsilon_r, sp \Omega_{sp}} \cdot \tilde{F}(\kappa; z_n') \]
ADVANTAGES OF SOLVING IN FOURIER SPACE: *DIVIDE AND CONQUER*

\[
\tilde{S}_{\xi}^{(m)}(\kappa_{ij}) \approx \sum_{n=1}^{N_z} \tilde{f}(\kappa_{ij} ; z_n') \text{PSF}_{0z',\xi}(\kappa_{ij})
\]

\[m = 1, \ldots, N_\omega \quad \xi = 1, \ldots, N_T\]

\[
\begin{align*}
A(\kappa_{ij})_{[N_TN_\omega \times N_z]} \cdot f(\kappa_{ij})_{[N_z \times 1]} &= d(\kappa_{ij})_{[N_TN_\omega \times 1]} \\
\kappa_{ij} &= (i\Delta k_x, j\Delta k_y) \\
i &= 1, \ldots, N_x; \quad j &= 1, \ldots, N_y
\end{align*}
\]

- we solve \((N_x \cdot N_y)\) such systems (on the order of \(10^4\) to \(10^5\))
- the size of each system is small: \(N_T N_\omega \times N_z\) (e.g. \(60 \times 5\))
- **typical execution times:** 2 to 3 seconds on a laptop using Matlab
- solution is orders of magnitude faster than solving in real space:

\[
N_D \times N_v \quad \text{where} \quad N_v = N_x N_y N_z \sim 10^6 \text{ to } 10^7
\]

\[
N_D = N_x N_y N_\omega N_T \sim 10^7 \text{ to } 10^8
\]
FINAL STEP: BACK TO REAL SPACE

- at each plane along range \((z' = \text{const})\)

\[
\Delta \varepsilon_r (x', y', z'_n) = \frac{\Delta \varepsilon_{r,sp} \Omega_{sp}}{\Delta x' \Delta y' \Delta z'_n} \mathcal{F}_{2D}^{-1} \left\{ \tilde{f} (\mathbf{k}; z'_n) \right\}, \; n = 1, \ldots, N_z
\]

\[
\varepsilon_r (x', y', z'_n) = \varepsilon_{r,b} + \Delta \varepsilon_r (x', y', z'_n)
\]
EXAMPLE: METALLIC TARGETS IN AIR

X-band (WR90) open-end waveguides ($f_c \approx 6.56$ GHz)

$\Delta x = \Delta y = 5$ mm

$\Delta f = 250$ MHz

<table>
<thead>
<tr>
<th>$f$ (GHz)</th>
<th>$\lambda$ (mm)</th>
<th>$D_{far}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>100</td>
<td>12.5</td>
</tr>
<tr>
<td>8.2</td>
<td>37</td>
<td>34</td>
</tr>
<tr>
<td>20</td>
<td>15</td>
<td>83</td>
</tr>
</tbody>
</table>

[photo credit: Justin McCombe]
METALLIC TARGETS IN AIR – RESULTS WITH SIMULATED KERNELS

[Amineh et al., Trans. Instr.&Meas., 2015]
METALLIC TARGETS IN AIR – RESULTS WITH MEASURED KERNELS (PSF)

[Amineh et al., Trans. Instr. & Meas., 2015]

expected spatial resolution

depth: \( \delta_z \approx 10 \text{ mm} \)

lateral: \( \delta_{x,y} \approx 4 \text{ mm} \)
EXAMPLE: IMAGING TISSUE

[photo credit: Daniel Tajik]

Tissue | Color Highlight | Relative Permittivity Averaged over 3 to 8 GHz
--- | --- | ---
Chicken Wing |  | NA
Bone |  | 21 ─ 10i
Skin |  | 13 ─ 6i
Muscle |  | 45 ─ 23i
Peanut Butter & Jam |  | 7 ─ 3i
Carbon Rubber |  | 10 ─ 3i

Δx = Δy = 3 mm
Δf = 100 MHz
f ∈ [3,8] GHz

a layer in the OUT
EXAMPLE: IMAGING TISSUE

[photo credit: Daniel Tajik]

<table>
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<td>Peanut Butter &amp; Jam</td>
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<td>$7 - 3i$</td>
</tr>
<tr>
<td>Carbon Rubber</td>
<td></td>
<td>$10 - 3i$</td>
</tr>
</tbody>
</table>

$\Delta x = \Delta y = 3 \text{ mm}$

$\Delta f = 100 \text{ MHz}$

$f \in [3, 8] \text{ GHz}$

Tajik et al., *JPIER-B* 2017

a layer in the OUT
EXAMPLE: ACQUIRING THE PSF

[photo credit: Daniel Tajik]

[photo credit: Daniel Tajik]

calibration object with small scattering probe at center: $\varepsilon_{r,\text{SP}} \approx 18 - i0$, radius 5 mm, height 10 mm
EXAMPLE: IMAGING TISSUE

[Tajik et al., JPIER-B 2017][ EuCAP 2018]
**QUALITATIVE IMAGING WITH SENSITIVITY MAPS**

- **reconstruction formula**

\[
D(r') = \sum_{\xi=1}^{N_T} \int_{\omega} \iint_{r \in S_a} \left[ S^{\text{inc}}_{\xi} - S^{\text{tot}}_{\xi} \right]_{(r,\omega)} \cdot \left[ \frac{\partial S^{\text{inc}}_{\xi} (r, \omega)}{\partial \varepsilon(r')} \right]^* d\mathbf{r} d\omega
\]

adjoint sensitivity formula using simulated incident fields

- **sensitivity map:** 3D image of Fréchet derivative of $\ell_2$ norm of the differences of all total and incident responses

\[
D(r') = \mathcal{J} \left\{ F[\varepsilon(r')] \right\} = 0.5 \sum_{\xi=1}^{N_T} \int_{\omega} \iint_{r \in S_a} \left\| S^{\text{tot}}_{\xi} (r, \omega) - S^{\text{inc}}_{\xi} [r, \omega; \varepsilon(r')] \right\|_2^2 d\mathbf{r} d\omega
\]

**Re** $\{ D^{(m)}(r') \} = \frac{dF^{(m)}}{d\varepsilon'(r')}$ → indicates where contrast in $\varepsilon'$ exists

**Im** $\{ D^{(m)}(r') \} = -\frac{dF^{(m)}}{d\varepsilon''(r')}$ → indicates where contrast in $\varepsilon''$ exists

[Tu et al., Inv. Problems, 2015]
from simulated incident fields to measured PSFs

\[
\frac{\partial S_{\xi}^{\text{inc}}(r, \omega)}{\partial \varepsilon(r')} \approx \frac{\Delta S_{\xi}^{\text{inc}}(r, \omega)}{\Delta \varepsilon(r')} \approx \frac{\bar{S}_{\xi, \text{sp}}^{\text{tot}}(r, \omega) - S_{\xi}^{\text{inc}}(r, \omega)}{\Delta \varepsilon_{\text{sp}}(r')} = \frac{\text{PSF}_{\xi}^{\text{sc}}(r, \omega; r')}{\Delta \varepsilon_{\text{sp}}}
\]

\[
\text{PSF}_{\xi}^{\text{sc}}(x, y, z, \omega; x', y', z') = \text{PSF}_{\xi, 0}^{\text{sc}}(x - x', y - y', z, \omega; z')
\]

uniform background along \(x\) and \(y\)

scattering probe at \(r'\)

scattering probe at center of \(z'\) plane
sensitivity reconstruction formula with PSFs: \textit{scattered-power maps (SPM)}

\[-\Delta \varepsilon_{\text{sp}} \cdot D(r') = M(r') = \sum_{\xi=1}^{N_T} \int_{\omega} \int_{r \in S_a} S_{\xi}^{\text{sc}}(r, \omega) \cdot \left[ \text{PSF}_{\xi,0}^{\text{sc}}(r, \omega; r') \right]^* \, dr \, d\omega\]

SPM reconstruction formula with planar scanning

\[M^{(m)}(x', y', z') = \sum_{\xi=1}^{N_T} \int_{\omega} \int_{S_a} \left[ S_{\xi}^{\text{sc}}(x, y, z, \omega) \right] \cdot \left[ \text{PSF}_{\xi}^{\text{sc}}(x-x', y-y', z, \omega; z') \right]^* \, dx \, dy \, d\omega\]

cross-correlation of response \(S_{\xi}^{\text{sc}}\) and PSF \(\text{PSF}_{\xi}^{\text{sc}}\) in \((x, y)\)

reconstruction is practically instantaneous – no systems of equations are solved

effort shifted to near-field system calibration – measuring PSFs or simulating incident fields to compute response sensitivity \(\partial S_{\xi}^{\text{inc}}(r, \omega) / \partial \varepsilon(r')\)
SCATTERED-POWER MAPS: SIMULATION EXAMPLE (Altair FEKO)

Simulation of PSF acquisition

Sample PSF: $S_{11}$ at 4 GHz MAG/PHASE

Simulation of data acquisition

$f_{\text{min}} = 3$ GHz
$f_{\text{max}} = 16$ GHz
$\Delta f = 1$ GHz
SCATTERED-POWER MAPS: SIMULATION EXAMPLE (F SHAPE)

- blurring typical for cross-correlation methods – diffraction limit, limited number of responses
QUANTITATIVE REAL-TIME IMAGING WITH SPM

• requires measured PSFs – they scale accurately with the scattering-probe contrast

• reconstruction solves the linear problem

\[ M(r') = \frac{1}{\Delta \varepsilon_{r,sp} \Omega_{sp}} \int \int \int_{V_s} \Delta \varepsilon_r(r'') M_{sp@r'}(r'')dr'' \]

qualitative image (SPM) of OUT
unknown contrast
qualitative image (SPM) of scattering probe

• real-time solution via inversion in Fourier space (similar to holography)

\[ M(x', y', z') = \frac{1}{\Delta \varepsilon_{r,sp} \Omega_{sp}} \int \int \int_{V_s} \Delta \varepsilon_r(x'', y'', z'') \cdot M_{sp@(0,0,z'')} (x' - x'', y' - y'', z') dx'' dy'' dz'' \]

correlation in \((x,y)\)
\[ \tilde{M}(k_x, k_y, z_p) = \frac{\Omega_v}{\Delta \varepsilon_{r,\text{sp}} \Omega_{\text{sp}}} \sum_{q=1}^{N_z} \tilde{f}(k_x, k_y, z_q) \cdot \tilde{M}_{\text{sp@}(0,0,z_q)}(k_x, k_y, z_p), \quad p = 1, \ldots, N_z \]

- small square system of equations to solve \textit{at each spectral position} \( \kappa = (k_x, k_y) \)

\[
\begin{bmatrix}
M(\kappa) x(\kappa) = m(\kappa)
\end{bmatrix}
\]

\[
x(\kappa) = \begin{bmatrix} \tilde{f}(\kappa, z_1) \cdots \tilde{f}(\kappa, z_{N_z}) \end{bmatrix}^T
\]

\[
m(\kappa) = \begin{bmatrix} \tilde{M}(\kappa, z_1) \cdots \tilde{M}(\kappa, z_{N_z}) \end{bmatrix}^T
\]

- final step: back to \((x, y)\) space

\[
\Delta \varepsilon_r(x', y', z_{n'}) = \frac{\Delta \varepsilon_{r,\text{sp}} \Omega_{\text{sp}}}{\Omega_v} \mathcal{F}_{2D}^{-1} \left\{ \tilde{f}(\kappa; z'_{n'}) \right\}, \quad n = 1, \ldots, N_z
\]
QUANTITATIVE SPM: SIMULATION EXAMPLE (F SHAPE)

[Re, Im ε_r]

[Nikolova, *Introduction to Microwave Imaging*, 2017]
5 cm thick carbon-rubber sample $\varepsilon_{r,b} \approx 10 - i5$

[Shumakov et al., IEEE Trans. MTT, 2018]
TIME DOMAIN FORWARD MODEL WITH PSF

[Nikolova, *Introduction to Microwave Imaging*, 2017]

- linearized time-domain resolvent kernel

\[
S^{sc}(\mathbf{r}, t) \approx \iint_{V_s} \kappa(\mathbf{r}') \left[ h_{Rx}^{inc} * h_{Tx}^{inc} \ast w''(t) \right] (\mathbf{r}', \mathbf{r}, t) \, dr'
\]

\Rightarrow \mathcal{K}(\mathbf{r}'; \mathbf{r}, t) = h_{Rx}^{inc}(\mathbf{r}'; \mathbf{r}_{Rx}, t) * h_{Tx}^{inc}(\mathbf{r}'; \mathbf{r}_{Tx}, t) * w''(t)

- resolvent kernel and PSF all over again:

measure response with each antenna pair and position of a point scatterer at \( \mathbf{r}' = 0 \)

\[
PSF_0(\mathbf{r}_{Rx}, \mathbf{r}_{Tx}, t) = \kappa_{sp} \Omega_{sp} \left[ h_{Rx}^{inc} * h_{Tx}^{inc} \ast w''(t) \right] (\mathbf{r}'=0; \mathbf{r}_{Rx}, \mathbf{r}_{Tx}, t)
\]

\[
= \kappa_{sp} \Omega_{sp} \mathcal{K}_0(\mathbf{r}_{Rx}, \mathbf{r}_{Tx}, t)
\]
• assuming uniform background: \( \mathcal{K}_r'(\mathbf{r}_{\text{Rx}}, \mathbf{r}_{\text{Tx}}; t) \approx \mathcal{K}_0(\mathbf{r}_{\text{Rx}}, \mathbf{r}_{\text{Tx}}; t - \Delta t(\mathbf{r}')) \)

• example: analytical far-zone kernel

\[
\mathcal{K}_r'(\mathbf{r}_{\text{Rx}}, \mathbf{r}_{\text{Tx}}; t) \sim \text{PSF}_0(\mathbf{r}_{\text{Rx}}, \mathbf{r}_{\text{Tx}}, t) \sim \delta\left(t - \frac{|\mathbf{r}' - \mathbf{r}_{\text{Tx}}|}{v_b} - \frac{|\mathbf{r}' - \mathbf{r}_{\text{Rx}}|}{v_b} - t_0\right) / r^2
\]

implied in \( r \)

some reference time
• cross-correlation – a measure of signal similarity

\[ X(t, r') = \sum_{\xi=1}^{N_T} \int_{r\in S_a} \text{PSF}_{\xi,0}(t, r, r') \otimes s_{\xi}(t, r) \, dr \]

steering filter for antenna pair at \( r \) toward voxel \( r' \)

\[ \sim \int \int \int_{r'' \in V_s} \kappa(r'') \cdot \sum_{\xi=1}^{N_T} \int_{r\in S_a} \text{PSF}_{\xi,0}(t, r, r') \otimes \text{PSF}_{\xi,0}(t, r, r'') \, dr \, dr'' \]

signal processing

• with large number of responses, \( X(r', t) \sim \kappa(r') \) as autocorrelation term dominates \( r'' \) integral

autocorrelation term: \( \kappa(r') \cdot \sum_{\xi=1}^{N_T} \int_{r\in S_a} \text{PSF}_{\xi,0}(t, r, r') \otimes \text{PSF}_{\xi,0}(t, r, r') \, dr \)

cross-correlation terms: \( \kappa(r'') \cdot \sum_{\xi=1}^{N_T} \int_{r\in S_a} \text{PSF}_{\xi,0}(t, r, r') \otimes \text{PSF}_{\xi,0}(t, r, r'') \, dr \)
- image generation: plot the “intensity” distribution

\[ I(r') = \int_{t=0}^{T_{\text{max}}} X^2(t, r') \, dt \]
DAS: CONCEPTUAL EXAMPLE
DAS: CONCEPTUAL EXAMPLE – 2

\[ I(r') = \int_t X^2(t, r') \, dt \]

\[ X(t, r') \]
CONCLUDING REMARKS

• we have just grazed the surface of an extensive subject
• real-time microwave imaging is rapidly growing and developing
  ➢ hardware – antennas & RF/radar electronics
  ➢ calibration methods
  ➢ inversion methods
• lateral (or cross-range) resolution

\[ \delta_{x,y} \geq \frac{\lambda_{\text{eff},\text{min}}}{4} = \frac{\pi}{2k_{x,y}^\max} \]

• depth (or range) resolution

\[ \delta_z \geq \frac{\lambda_{\text{eff},\text{min}}}{2} = \frac{\pi}{k_{z}^\max} \approx \frac{\nu_b}{2B} \]

• wide viewing angles are critically important: wide-beam antennas, large scanned apertures
Applications: Nondestructive Testing

[Sheen et al., “Near-field three-dimensional radar imaging techniques and applications,” *Applied Optics* 2010]

Pacific Northwest National Laboratory, Washington, USA

X-band (8 to 12 GHz) scanner with 16×16 electronically switched array: Absorber Inspection
APPLICATIONS: THROUGH-WALL IMAGING


Prof. Mostofi’s team at the University of California Santa Barbara

Area of interest – top view

3D binary ground-truth image of the unknown area to be imaged (2.96 m x 2.96 m x 0.4 m)

Our 3D image of the area, based on 3.84 % measurements

https://www.youtube.com/watch?v=THu3ZvAHI9A
LIMITATIONS OF BORN’S APPROXIMATION: TOTAL INTERNAL FIELD

- Limitations on both the size and the contrast of the scatterer
  \[ a^2 \left| k_s^2(r) - k_b^2 \right| \ll 1, \ r \in V_s \]

- If OUT violates the limits: images contain artifacts which reflect differences between \( E_{\text{Tx}}^{\text{tot}}(r') \) and \( E_{\text{Tx}}^{\text{inc}}(r') \) rather than contrast

**Example: Total vs. Incident Internal Field**

60 MM Thick Dielectric Slab in Air

\( f = 1 \text{ GHz} \)

[Nikolova, *Introduction to Microwave Imaging*, 2017]
NUMERICAL ASPECTS OF THE SOLUTION IN FOURIER SPACE

\[
\tilde{S}_\xi^{(m)}(\kappa_{ij}) \approx \sum_{n=1}^{N_z} \tilde{f}(\kappa_{ij} ; z'_n) \text{PSF}_{0z',\xi}^{(m)}(\kappa_{ij}) \\
m = 1, \ldots, N_\omega \\
\xi = 1, \ldots, N_T
\]

\[
\mathbf{A}(\kappa_{ij}) \cdot \mathbf{f}(\kappa_{ij}) = \mathbf{d}(\kappa_{ij})
\]

\[
\kappa_{ij} = (i \Delta k_x, j \Delta k_y) \\
i = 1, \ldots, N_x ; \ j = 1, \ldots, N_y
\]

solve \((N_x \cdot N_y)\) such systems

of size: \(N_T \cdot N_\omega \times N_z\)

the data vector:

\[
\mathbf{d}(\kappa_{ij}) = \begin{bmatrix} 
\mathbf{d}_1^T(\kappa_{ij}) \\
\vdots \\
\mathbf{d}_{N_T}^T(\kappa_{ij}) 
\end{bmatrix}_{N_T \cdot N_\omega \times 1} , \quad 
\mathbf{d}_\xi^T(\kappa_{ij}) = \begin{bmatrix} 
\tilde{S}_{\xi}^{(1)}(\kappa_{ij}) \\
\vdots \\
\tilde{S}_{\xi}^{(N_\omega)}(\kappa_{ij}) 
\end{bmatrix}_{N_\omega \times 1} , \quad \xi = 1, \ldots, N_T
\]

the contrast vector:

\[
\mathbf{f}(\kappa_{ij}) = \begin{bmatrix} 
\tilde{f}(\kappa_{ij} ; z'_1) \\
\vdots \\
\tilde{f}(\kappa_{ij} ; z'_{N_z}) 
\end{bmatrix}_{N_z \times 1}
\]

the system (PSF) matrix:

\[
\mathbf{A}(\kappa_{ij}) = \begin{bmatrix} 
\mathbf{A}_1(\kappa_{ij}) \\
\vdots \\
\mathbf{A}_{N_T}(\kappa_{ij}) 
\end{bmatrix}
\]

where \(\mathbf{A}_\xi(\kappa_{ij}) = \)

\[
\begin{bmatrix} 
\text{PSF}_{0z'_1,\xi}^{(1)}(\kappa_{ij}) \\
\vdots \\
\text{PSF}_{0z'_{N_z},\xi}^{(N_\omega)}(\kappa_{ij}) 
\end{bmatrix}
\]
THE FORWARD MODEL OF HOLOGRAPHY

• the $S$-parameter forward model

$$S_{\xi}^{\text{sc}}(x, y, z; \omega) \approx \iiint_{V_s} \Delta \varepsilon_r(x', y', z') \left[ \mathbf{E}_{\xi, \text{Rx}}^{\text{inc}} \cdot \mathbf{E}_{\xi, \text{Tx}}^{\text{inc}} \right]_{(x', y', z'; x, y, z; \omega)} \, dx' \, dy' \, dz', \quad \xi = 1, \ldots, N_T$$

• $\xi$ (response type) replaces $(i,k)$

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$(i, k)$</th>
<th>response</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1, 1)</td>
<td>$S_{11}$</td>
</tr>
<tr>
<td>2</td>
<td>(2, 2)</td>
<td>$S_{22}$</td>
</tr>
<tr>
<td>3</td>
<td>(1, 2) or (2, 1)</td>
<td>$S_{12} = S_{21}$</td>
</tr>
</tbody>
</table>

$N_T = 3$
THE FORWARD MODEL OF HOLOGRAPHY: PLANAR SCAN

- the $S$-parameter forward model

\[
S_{sc}^{\xi}(x, y, z, \omega) \approx \int \int \int_{V_s} \Delta \varepsilon_r(x', y', z') \left[ \mathbf{E}_{\xi, Rx}^{inc} \cdot \mathbf{E}_{\xi, Tx}^{inc} \right]_{(x', y', z'; x, y, z; \omega)} dx' dy' dz', \quad \xi = 1, \ldots, N_T
\]

- during scan, the Tx & Rx antennas in each $\xi$-th experiment are fixed wrt each other:
  - if $r_{Rx}$ is known then $r_{Tx}$ is known, e.g., $r_{Rx} \equiv (x, y, z)$ and $r_{Tx} \equiv (x, y, \bar{z} - D)$