Computational imaging of micro-structured media at small scale — from one-shot first-order solutions to full-wave iterative ones

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w. thanks to
H. Tu, now Scientist, North Automatic Control Technology Institute, Taiyuan,
Changyou Li, now Associate-Professor, NPU Xi’an
[C. Li is still engaged with us closely]
• **Today’s thread**
How to **locate w. enhancement of resolution or, even better, super-resolution**, vs. wavelength of operation of (simple) defects/radiators **within micro-structured systems** (circular cylindrical dielectric or metal rods gridded in regular fashion, 2D and 3D)

• **Main example in a 2D scattering situation**
*Imaging missing fibers in periodic planar laminates*: *one-shot* MUSIC and *iterative* sparsity-constrained imaging

• **Complementary in 2D scattering situation**
*Imaging missing rods or sources within finitely numbered set of rods in free space*: *one-shot* time-reversal and *iterative* sparsity-constrained imaging

• **Going 3D full wave supported by lab experiments**
Short monopole antennas inserted in finite set of regularly distributed, shorter thin metal wires: *the monopole that radiates to be located from far field patterns* (so far, TR, soon ... learning?)

• **Time permitting/work in progress**
How to shift to **supervised learning** involving feature analysis and regression
Electromagnetic inspection of disorganized fiber-based composite laminate
Small-scale EM model of fiber-reinforced layering

Direct modeling

- Material: glass, graphite, metal fibers; epoxy, polyester matrix
- Topology: planar layers w. circular cylindrical fibers
- Defects: missing, displaced, shrunk/expanded fibers, bubbles
- Source: planar & line
- Frequency: MHz to THz

Inversion

- Main goal: locating/imaging damages inside
- Methods: MUltiple SIgnal Classification & Joint-Sparsity-Based
Equivalence theory (1)

Expand field into CWEs with coefficients $A_m, B_m, C_m, Q_m$

Fiber boundary conditions lead to

$$B_m = R_m A_m + T_m Q_m$$

$R_m, T_m$ reflection & transmission coefficients.

missing fiber $\rightarrow$ no scattered field $\rightarrow B_m = 0$

$$Q_m = (-R_m / T_m) A_m$$

EM equivalent line source inside sound fiber

**Condition:** linear relation between $Q_m$ and $A_m$ satisfied.
Equivalence theory (2)

According to scattering linearity

\[ V = V^{inc} + \sum_l V^\text{equ}_l \]

\( V^{inc} \) due to incident source, \( V^\text{equ}_l \) to equivalent source inside \( l \)-th fiber.

**Decomposition of \( V^\text{equ}_l \):** \( V^\text{equ}_l (\sum_m Q^l_m) = \sum_m Q^l_m V^\text{equ}_l (\delta_m) \)

**Solution to \( Q^l_m \) or \( A^l_m \):**

\[
A^l_m = A^{inc}_{l,m} + \sum_j \sum_{n \in \mathbb{Z}} Q^j_n A^\text{equ}_{l,m} (\delta_n)
\]

\[
Q^l_m = (-R_m/T_m) A^l_m
\]

\[
A = [A^l_m], \quad I \text{ identity matrix}, \quad D = [A^\text{equ}_{l,m}(\delta_n)], \quad \Gamma = \text{diag}\{R_m/T_m\}, \quad A^{inc} = [A^{inc}_{l,m}]
\]

Apply similar idea to fiber displacement, shrinkage/expansion & bubbles.
4-layer graphite-fibered laminate (anomalous E)

\[ d/4 \text{-radius}, d\text{-thick graphite-fibered epoxy layers,} \]
\[ \text{missing fibers } (l, -2) \& (l, 2), l = 1, \cdots 4 \]
\[ \text{TM, } \lambda^{inc} = d, \text{ source at } (0, d), d = 0.1 \text{ mm} \]
4-layer glass-fibered laminate (anomalous E)

\[ \frac{d}{4}\text{-radius, } d\text{-thick glass-fibered epoxy layers, missing fibers } (l,-2) \text{ & } (l,2), l = 1, \cdots 4 \]

TM, \( \lambda^{inc} = d \), source at \((0,d)\), \(d = 0.1\) mm
Inverse source problem

Denote $g = \tilde{E}_y(r) - E_y(r)$, domain integral formulation leads to $g = Wq$

$l$-th element of $q$ is non-zero if $l$-th fiber removed; otherwise zero

**Multiple sources**

For each $\nu$, one has $g_\nu = Wq_\nu$

Set $Y = [g_\nu], F = [q_\nu], Y = WF$

**Zero-mode approximation**

\[ \sum_{m \in \mathbb{Z}} \Rightarrow \sum_{m = -M}^{M}, M \to 0 \text{ for "low frequency"} \]

\[ Y = WF \xrightarrow{M = 0} Y = AX + N \]

$A = W^0, X = F^0, N$ approximation error + data noise
Joint-sparsity-based imaging (1)

\[ Y = AX + N \]

**Character of desired \( X \)**

- Number of damaged fibers \( \ll L \)
- \( L \) number of fibers in detection
- A few nonzero elements in each column of \( X \), i.e., it is sparse
- Position of defects independent from sources
- Different columns of \( X \) share same sparsity, i.e., joint sparsity

**Measurement of joint sparsity**

\[
\| x \|_p^p = \sum_{l=1}^{L} x^l \| x^l \|_p^p , \text{ where } x^l = \| X^l \|_2
\]

\( X^l, x^l \) being \( l \)-th row of \( X \) and \( x \), respectively.

**Solution from minimization problem with standard optimization**

\[
\min J (X) = \| Y - AX \|_F^2 + \tau \| x \|_p^p
\]

\( \tau \) regularization parameter, \( 0 < p \leq 1 \).
Joint-sparsity-based imaging (2)

\[
\min J (X) = \|Y - AX\|_F^2 + \tau \|x\|_p^p
\]

Minimization conducted with factored-gradient approach:

\[
\nabla J (X) = 0 \implies X = (A^H A + \lambda \Pi (X))^{-1} A^H Y
\]

leads to iterative formula

\[
X^{(k+1)} = (A^H A + \lambda^{(k)} \Pi (X^{(k)}))^{-1} A^H Y
\]

where \( \lambda = p \tau / 2 \), \( \Pi (X) = \text{diag}\{(||X^l||_2^2 + \epsilon)^{p/2-1}\} \), \( \epsilon = 10^{-3} \max \{||X^l||_2^2\} \).

1. Initialization: \( X^{(0)} = 0 \), \( p = 0.8 \), \( k = 0 \);
2. Update \( \Pi (X^{(k)}) \) and \( \lambda^{(k)} \) based on L-curve method, then \( X^{(k+1)} = (A^H A + \lambda^{(k)} \Pi (X^{(k)}))^{-1} A^H Y \) and \( k = k + 1 \);
3. If \( \|X^{(k)} - X^{(k-1)}\|_F < 10^{-3} \), stop; otherwise, repeat step 2.

0 matrix full of zeros, \( ^H \) conjugate transpose
Truncation of CWE in imaging procedure

\[ \sum_{m \in \mathbb{Z}} \Rightarrow \sum_{m = -M}^{M} \text{ in modeling, but } \uparrow M \text{ means } \uparrow \text{ ill-posedness} \]

1 fiber corresponds to \(2M + 1\) coefficients.

So, \(\sum_{m = -N}^{N}\) is used in imaging, \(N \leq M\). As a result,

\[ Y = WF \Rightarrow Y = W^{(N)}F^{(N)} + N^{(N)} \]

where \(N^{(N)}\) is truncation error. Identify damaged fibers by nonzeros of

\[ f_l = \sum_{m = -N}^{N} \left| F^{(N)}_{l,m} \right|, l \text{ indicating } l\text{-th fiber} \]

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Imaging with different \(N\)

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Data collected w. 100 line sources \((\lambda^{inc} = d, \text{TM})\), and receivers uniformly set along \(-10d \leq x \leq 10d, z = d, d = 0.1\text{mm}, c_l = c = d/4, o_l = (ld,0)\).
Anomalous fields with graphite-fibered laminate

\[ \mathbf{r}_s = (-d, d) \]

\[ \mathbf{r}_s = (0, d) \]

\[ \mathbf{r}_s = (d, d) \]

\[ \mathbf{r}_s = (2d, d) \]

\[ \mathbf{r}_s = (3d, d) \]
Anomalous fields with glass-fibered laminate

\[ \mathbf{r}_s = (-d, d) \]

\[ \mathbf{r}_s = (0, d) \]

\[ \mathbf{r}_s = (d, d) \]

\[ \mathbf{r}_s = (2d, d) \]

\[ \mathbf{r}_s = (3d, d) \]
Imaging graphite-fibered laminates with joint sparsity

\[ r_s = (0, d), M = 14 \]

\[ \epsilon(N) = \frac{||N^{(N)}||_2}{||Y||_2} \]

\[ SNR = 30 \text{ dB} \]
Imaging glass-fibered laminate with joint sparsity

\[ \mathbf{r}_s = (0, d), M = 14 \]

\[ \mathbf{e}(N) = \frac{||N^{(N)}||_2}{||Y||_2} \]

SNR = 30 dB
Imaging with data sampled at half wavelength

Data collected w. 41 line sources ($\lambda^{inc} = d$, TM) and receivers uniformly set along $-10d \leq x \leq 10d$, $z = d$, with interval $\lambda^{inc}/2$. SNR = 30 dB

Graphite

Glass

MUSIC

Sparsity
Direct modeling


Imaging

2-D inverse source and inverse scattering in micro-structure
• Frequency-diverse or transient (*localization unknown*) ideal line source – elec., TM, magn., TE – in finitely-numbered periodically-distributed (dielectric) circular cylindrical rods ⇒ Inverse source pb.

• Dual w. (*localization unknown*) one/several rods missing, sources and receivers outside ⇒ Inverse scattering pb.

• All scalar 2-D, yet subwavelength (radii << λ, spacing << λ) w. far (20λ) field data, λ as median wavelength

• Multiple Scattering Theory (MST) w. cylindrical wave field expansions (*modes*) truncated to some pertinent index, yields workhorse for all situations
(TE) TIME-REVERSAL WITHIN MICRO-STRUCTURE

**Left** Reference (TE, centered source, relative rod permittivity 10, 64 of them, spaced $d = \lambda/5$, radius $\lambda/50$, frequency 3 GHz, 30 TRM at 100$d$)

**Right** (left to right) shifted sources by 3$d$ & 10$d$

No enhancement in effect (1.029 effective permittivity)

\[
\tilde{\epsilon}_r = \frac{S_1 + S_2}{S_1/\tilde{\epsilon}^{(1)}_r + S_2/\tilde{\epsilon}^{(2)}_r}
\]
(TM) TIME-REVERSAL WITHIN MICRO-STRUCTURE

**Left** Reference (TM, centered source, relative rod permittivity 10, 64 of them, spaced $d = \lambda/5$, radius $\lambda/50$, frequency 3 GHz, 30 TRM at $100d$)

**Right** (left to right) shifted source by $3d$ & $10d$

Enhancement: like 1.28 effective permittivity space

$$\bar{\varepsilon}_r = \frac{\varepsilon_r^{(1)} S_1 + \varepsilon_r^{(2)} S_2}{S_1 + S_2}$$
(TE) JOINT-SPARSITY-BASED IMAGING

Like before, permittivity $10, 64$ of them, spaced $d = \lambda/5$, radius $\lambda/50$ but now $1, 2$ or $3$ rods missing, $30$ sources & receivers (data SNR $30$ dB) on the $100d$ circle being used.

About same results with various $M$ as number of modes (here $M = 0$)
Like before, permittivity 10, 64 of them, spaced $d = \lambda/5$, radius $\lambda/50$ but now 1, 2 or 3 rods missing, 30 sources & receivers (data SNR 30 dB) on the $100d$ circle being used. About same results with various $M$ as number of modes (here $M = 0$)
REFERENCES


On the Diagnosis of a Micro-Structured Wire Antenna System
• **P1 to P3 wire antennas** (*monopoles*) 256 mm-long (*288 MHz resonant*) 26 mm apart, in grid of 5 x 13 regularly distributed, 205 mm-long 3 mm-Ø metal (soldered-to-ground) (*passive*) rods (*358 MHz resonant*) 13 mm apart
• **Monopoles** fed via coaxial connectors, those not excited 50 Ω loaded
• ≈ 0.2 - 1 GHz band of operation (median wavelength as of 500 mm)
Measurement in anechoic chamber yielding S-parameters as well as radiation patterns (e.g., 360° in horizontal plane) via standard E-probe. See, when no micro-structure:
Measured (reflection) S-parameters (P1 off-center, P2 centered) compared with those computed by CST MS

Results fitting together, save extremely sharp resonances

That ground plane not finite in practice changes little, nor true field probe not modeled, whereas if no micro-structure, simulation & measurement fit extremely well, no difference.
Simulated (CST) & measured electric fields vs frequency (in MHz) at 1 m in front of P2, with either P1 or P2 emitting

Encircled zones show the narrow frequency band (around 0.34 GHz), wherein super-localization may occur with time-reversal
SIMULATED FAR-FIELD PATTERNS (at 340 MHz)

Without micro-structure

With micro-structure

(center) P2 emits (off-center) P3 emits
MORE ON PATTERNS (300-340 MHz) w. MICRO-STRUCTURE

Radiation patterns at 340 MHz evidence effects of micro-structure, linked w. resonances in quite narrow band (compare to 300 MHz behaviors)

300 MHz

340 MHz
MORE ON PATTERNS (300-340 MHz) wo. MICRO-STRUCTURE

No evidence of changes vs. frequency when no micro-structure around emitting monopole antennas.

300 MHz

340 MHz
SWE (300 MHz = NO RESONANCE) w. & wo. MICRO-STRUCTURE

[To remind, power-normalized Spherical vector Wave Expansions of the field outside the minimal sphere enclosing the micro-structure are]

\[ \vec{E} = \frac{k}{\sqrt{\eta}} \sum_{j=1}^{J_{\text{max}}} Q_j \vec{F}_j^{(3)}(\vec{r}) \]

\[ J_{\text{max}} = 2(N_{tr}(N_{tr} + 2)) \]

\[ N_{tr} = [kr_{\text{min}}] + 10 \]
SWE (340 MHz = RESONANCE) w. & wo. MICRO-STRUCTURE

WITHOUT (P1 off-center)

WITH (P1 off-center)

WITHOUT (P2 center)

WITH (P2 center)
TIME-REVERSAL MIRRORING (TRM)?

TRM operated from field data, P2 emitting

Normalized signals collected by loaded P1 to P3 displayed

Monopole associated to port of excitation P2 shows up clearly: “super-localized”

Results depend upon
- number & distribution of rods
- position (from simulations) of source within the micro-structure

Far-field in horizontal plane, yet to discuss if collected on hemisphere or via SWE (previous slides)

LEARNING???

Database generation
N (known) rods, Nr receivers, 1 source (imposed) at center

— 1 missing rod
X = [X₁, X₂, X₃ ... Xₙ], Xᵢ = [x₁ᵢ, x₂ᵢ, x₃ᵢ ... xᵣᵢ], xᵢj field at each receiver
Y = [Y₁, Y₂, Y₃ ... Yₙ], Yᵢ = [y₁ᵢ, y₂ᵢ, y₃ᵢ ... yᵣᵢ], yᵢj = 1 rod missing, 0 otherwise, here only 1 value = 1

— 2 missing rods
X = [X₁, X₂, X₃ ... X₂ₙ], N₂ = Cᴺ², Xᵢ = [x₁ᵢ, x₂ᵢ, x₃ᵢ ... xᵣᵢ], xᵢj field at each receiver
Y = [Y₁, Y₂, Y₃ ... Yₙ], Yᵢ = [y₁ᵢ, y₂ᵢ, y₃ᵢ ... yᵣᵢ], yᵢj = 1 rod missing, 0 otherwise, only two values = 1

— and so on ...

— with many variants of configurations
Feature extraction

In each case, the input $X$ has $N$ elements, there is redundancy

$\rightarrow \text{reducing high dimensional feature space to low dimensional one}$

Application of \textbf{Partial Least Squares algorithm}

PLS attempts to maximize covariance between $X$ and $Y$. It requires addition of weights $W$ to maintain orthogonal scores. Factors are calculated sequentially by projecting $Y$ through $X$. One has something like \textit{(in a box)}:

\textbf{Principal Component Analysis} $X = T_k P_k^T + E$

\textbf{Pseudo-inverse} $X^+ = W_k (P_k^T W_k)^{-1} (T_k^T T_k)^{-1} T_k^T$, $k$ number of factors to be kept
Inverse Model Generation – Training - Test

Support Vector Regression algorithm is now used to find a non-linear function to generate the model. The math core of SVR is just about:

\[ f(x) = \sum_i \alpha_i K(x_i, x_j) + b \]

e.g., \( K(x_i, x_j) = \exp(-\|x_i - x_j\|^2/\delta^2) \)

(Gaussian radial basis function)

A lot of subtle(r) stuff, vast literature, many available tools (Python!), ... and the need to show efficiency ... on our cases.

Flow chart of inversion
TENTATIVE CONCLUSION

• Success relies on accurate computations to an extent only. 3D hard (CST!); CDA not proven, yet in 2D even some rather loose CWE yields great inversion

• Resolution enhancement in 2D easy from first-order, yet no theoretical tool in 3D to get there, and even multi-layered laminates beyond grasp (save brute-force homogenization “layer-per-layer”)

• Super-resolution associates w. resonances, those tracked to singular behavior of Green dyads, to excite the micro-structure properly. Age-old Singularity Expansion Method rejuvenated somehow?

• All depends from modus operandi, caring about feeds & realizations, model & data uncertainties, w. lack of theory, beyond infinitely many rods and ideal sensors in/nearby

• Learning requires training (w. feature selection, building up feature space, choice of predictor) & test (entailing regression, e.g., Support Vector R.). From many examples (beyond ours!) 2D em OK, 3D full-wave less sure; here spherical wave expansions compactly yielding radiation patterns as “the” way?