Dynamic Liquidity-Based Security Design

Emre Ozdenoren\textsuperscript{1} Kathy Yuan\textsuperscript{2} Shengxing Zhang\textsuperscript{3}

\textsuperscript{1}LBS and CEPR, \textsuperscript{2}LSE and CEPR, \textsuperscript{3}LSE
Motivation

- The determinants of liquidity in a dynamic economy
  - random productivity or endowment shocks
  - adverse selection
  - type of liquidity technology: optimal security design

- The amount of liquidity
  - repo contracts
  - haircuts, interest rates
Motivation

- The determinants of liquidity in a *dynamic* economy
  - random productivity or endowment shocks
  - adverse selection
  - type of liquidity technology: optimal security design

- The amount of liquidity
  - repo contracts
  - haircuts, interest rates
Key Mechanism

- **Adverse selection**
  - Without collateral borrowers cannot commit to paying back.
  - Productive assets provide liquidity because they can be used as collateral but are subject to adverse selection.

- **Inter-temporal feedback**
  - Collateral value depends on the re-sale value of the asset.
  - Re-sale value itself depends on the collateral value of the asset.
  - Leads to fragility and volatility in asset price and real output.
Key Mechanism

- **Adverse selection**
  - Without collateral borrowers cannot commit to paying back.
  - Productive assets provide liquidity because they can be used as collateral but are subject to adverse selection.

- **Inter-temporal feedback**
  - Collateral value depends on the re-sale value of the asset.
  - Re-sale value itself depends on the collateral value of the asset.
  - Leads to fragility and volatility in asset price and real output.
Main Results

- Equity contracts: Fragility and self-fulfilling
  - Pooling equilibrium: more liquidity and output
  - Separating equilibrium: less liquidity and output
  - Multiple equilibria

- Security design: liquid repo-debt contract (under monotone payoff constraints)
  - Unique equilibrium: both high and low types issue repo-debt and debt is liquid; low type issues equity
    - Eliminates fragility and improves liquidity
  - Improves social welfare relative to the separating equilibrium under equity contract
Main Results

- Equity contracts: Fragility and self-fulfilling
  - Pooling equilibrium: more liquidity and output
  - Separating equilibrium: less liquidity and output
  - Multiple equilibria

- Security design: liquid repo-debt contract (under monotone payoff constraints)
  - Unique equilibrium: both high and low types issue repo-debt and debt is liquid; low type issues equity
    - Eliminates fragility and improves liquidity
  - Improves social welfare relative to the separating equilibrium under equity contract
Agent $I$: investor or supplier; Agent $O$: entrepreneur

Agent $O$ has a CRS $z$-technology which produces $z > 1$ units of consumption goods with one intermediate good (capital, equipment) from Agent $I$.

Agent $I$ produces intermediate goods 1-to-1 from labor.

Both have a basic technology that produces consumption good 1-to-1 from labor.

Agent $O$ would like to borrow unlimited amount of intermediate goods from agent $I$.
  - because returns to scale of $z$-technology is $z > 1$.
  - ... but agent $O$'s promise to pay back is not enforceable.
Basic and z-Technology

- Agent $I$: investor or supplier; Agent $O$: entrepreneur
- Agent $O$ has a CRS z-technology which produces $z > 1$ units of consumption goods with one intermediate good (capital, equipment) from Agent $I$
- Agent $I$ produces intermediate goods 1-to-1 from labor
- Both have a basic technology that produces consumption good 1-to-1 from labor
- Agent $O$ would like to borrow unlimited amount of intermediate goods from agent $I$.
  - because returns to scale of z-technology is $z > 1$
- ... but agent $O$’s promise to pay back is not enforceable
Basic and $z$-Technology

- Agent $I$: investor or supplier; Agent $O$: entrepreneur

- Agent $O$ has a CRS $z$-technology which produces $z > 1$ units of consumption goods with one intermediate good (capital, equipment) from Agent $I$.

- Agent $I$ produces intermediate goods 1-to-1 from labor.

- Both have a basic technology that produces consumption good 1-to-1 from labor.

- Agent $O$ would like to borrow unlimited amount of intermediate goods from agent $I$.
  - because returns to scale of $z$-technology is $z > 1$.
  - ... but agent $O$’s promise to pay back is not enforceable.
Basic and z-Technology

- Agent I: investor or supplier; Agent O: entrepreneur

- Agent O has a CRS z-technology which produces $z > 1$ units of consumption goods with one intermediate good (capital, equipment) from Agent I

- Agent I produces intermediate goods 1-to-1 from labor

- Both have a basic technology that produces consumption good 1-to-1 from labor

- Agent O would like to borrow unlimited amount of intermediate goods from agent I.
  - because returns to scale of z-technology is $z > 1$

- ... but agent O’s promise to pay back is not enforceable
Utilities

- Utility in period $t$ is $U_t(x, l) = x - l$
- $x$ is the consumption good
- $l$ is labor
- Discount rate between periods $\beta$, with $0 < \beta < 1$. 
Three dates in each period.

- **Date 1:** Intermediate good is produced
  - perishes at the end of the period
  - no direct utility

- **Date 2:** Consumption good is produced
  - via the productive or basic technology.

- **Date 3:** Consumption takes place.
  - Consumption good perishes at the end of the period.
Timing

- Three dates in each period.
- **Date 1:** Intermediate good is produced
  - perishes at the end of the period
  - no direct utility
- **Date 2:** Consumption good is produced
  - via the productive or basic technology.
- **Date 3:** Consumption takes place.
  - Consumption good perishes at the end of the period.
Three dates in each period.

Date 1: Intermediate good is produced
    - perishes at the end of the period
    - no direct utility

Date 2: Consumption good is produced
    - via the productive or basic technology.

Date 3: Consumption takes place.
    - Consumption good perishes at the end of the period.
Timing

- Three dates in each period.
- **Date 1:** Intermediate good is produced
  - perishes at the end of the period
  - no direct utility
- **Date 2:** Consumption good is produced
  - via the productive or basic technology.
- **Date 3:** Consumption takes place.
  - Consumption good perishes at the end of the period.
Asset

- Long lived asset pays $s$ units of dividend as consumption good at date 3.

- Fixed supply of the asset is $A$.

- With prob. $\lambda$ dividend distribution is $F_L$ and $1-\lambda$ it is $F_H$.
  - $F_L, F_H \in \Delta[s_L, s_H], 0 \leq s_L < s_H$
  - $F_H$ first order stochastically dominates $F_L$
  - Quality $Q \in \{H, L\}$ is i.i.d. over time
  - $\tilde{F}_Q(s) = 1 - F_Q(s)$ for $Q \in \{L, H\}$
Asset

- Long lived asset pays $s$ units of dividend as consumption good at date 3.
- Fixed supply of the asset is $A$.
- With prob. $\lambda$ dividend distribution is $F_L$ and $1 - \lambda$ it is $F_H$.
  - $F_L, F_H \in \Delta[s_L, s_H], 0 \leq s_L < s_H$
  - $F_H$ first order stochastically dominates $F_L$
  - Quality $Q \in \{H, L\}$ is i.i.d. over time
  - $\tilde{F}_Q(s) = 1 - F_Q(s)$ for $Q \in \{L, H\}$
Asset

- Long lived asset pays $s$ units of dividend as consumption good at date 3.

- Fixed supply of the asset is $A$.

- With prob. $\lambda$ dividend distribution is $F_L$ and $1 - \lambda$ it is $F_H$.
  - $F_L, F_H \in \Delta[s_L, s_H]$, $0 \leq s_L < s_H$
  - $F_H$ first order stochastically dominates $F_L$
  - Quality $Q \in \{H, L\}$ is i.i.d. over time
  - $\tilde{F}_Q(s) = 1 - F_Q(s)$ for $Q \in \{L, H\}$
Agent $O$ uses the asset as collateral to borrow intermediate goods from agent $I$.

Agent $O$ privately observes asset quality $O$ at the beginning of each period.

- Adverse selection is within the period
- Shown later, agent $O$ purchases all collateral assets in equilibrium
- Privately informed about the quality because of
  - opportunity to temper with quality
  - incentive to acquire private information
Collateral Asset and Adverse Selection

- Agent $O$ uses the asset as collateral to borrow intermediate goods from agent $I$.

- Agent $O$ privately observes asset quality $O$ at the beginning of each period.
  - Adverse selection is within the period
  - Shown later, agent $O$ purchases all collateral assets in equilibrium
  - Privately informed about the quality because of
    - opportunity to temper with quality
    - incentive to acquire private information
Trading Environment: Two Markets

- Markets for intermediate goods at date 1
  - An agent $O$ randomly meets at least two agent $I$s
  - *in decentralized* market(s)
  - intermediate goods are traded for asset-based securities
  - i.e., borrowing against some forms of securities takes place

- Market for the collateral asset at date 3
  - After state is realized, asset price, $\phi_t$, is determined
  - in a *centralized* market
  - $\phi_t =$ present value of all the future cash flows of the asset.
Trading Environment: Two Markets

- Markets for intermediate goods at date 1
  - An agent $O$ randomly meets at least two agent $I$s
  - *in decentralized* market(s)
  - intermediate goods are traded for asset-based securities
  - i.e., borrowing against some forms of securities takes place

- Market for the collateral asset at date 3
  - After state is realized, asset price, $\phi_t$, is determined
  - in a *centralized* market
  - $\phi_t =$ present value of all the future cash flows of the asset.
Timeline

Period $t$         Period $t+1$

1          2          3          1          2          3

Production       Intermediate goods      Consumption occurs via z technology and basic technology

Markets          Securities traded in decentralized

$F_H$ or $F_L$ privately observed by O-agents

Asset traded in centralized

State is realized
Security Design

- Security Design is conducted ex ante before types are realised.
- An asset-backed security $y^j(s)$ is a state-contingent promise of consumption goods at date 3.

  - Two cases of interest:
    - Equity $y(s) = s + \phi_t, \forall s \in [s_L, s_H]$
    - Set of monotone securities
      \[ I_t(\phi_t) = \{ y : y(s) \text{ increasing in } s, y(s) \leq s + \phi_t, \forall s \in [s_L, s_H] \} \]
Security Design

- Security Design is conducted \textit{ex ante} before types are realised.
- An asset-backed security \( y^j(s) \) is a state-contingent promise of consumption goods at date 3.

- Two cases of interest:
  - Equity \( y(s) = s + \phi_t, \forall s \in [s_L, s_H] \)
  - Set of monotone securities
    \( \mathcal{I}_t(\phi_t) = \{ y : y(s) \text{ increasing in } s, y(s) \leq s + \phi_t, \forall s \in [s_L, s_H]\} \)
Security Design

- Security Design is conducted *ex ante* before types are realised.
- An asset-backed security $y^j(s)$ is a state-contingent promise of consumption goods at date 3.

Two cases of interest:
- Equity $y(s) = s + \phi_t, \forall s \in [s_L, s_H]$
- Set of monotone securities
  $\mathcal{I}_t(\phi_t) = \{ y : y(s) \text{ increasing in } s, y(s) \leq s + \phi_t, \forall s \in [s_L, s_H] \}$
Equilibrium in Security $j$’s Market

- Security trading occurs at date 1:
  - bilaterally between agent $O$ and multiple agent $I$s
  - in dedicated sub-markets for each available security.

- Suppose agent $I$ bids per-unit price $q_t^I$ for security $j$.

- If highest bid, agent $O$ offers him $a_t^Q(j)$ units of security $j$ for $q_t^I a_t^Q(j)$ intermediate goods.

- In equilibrium, winning bid $q_t^I$:
  - agent $I$: zero expected gain due to Bertrand Competition
  - IC for agent $O$: profitable for an informed $O$ agent type
Equilibrium in Security $j$’s Market

- Security trading occurs at date 1:
  - bilaterally between agent $O$ and multiple agent $I$s
  - in dedicated sub-markets for each available security.
- Suppose agent $I$ bids per-unit price $q^I_t$ for security $j$.
- If highest bid, agent $O$ offers him $a_t^Q(j)$ units of security $j$ for $q^I_t a_t^Q(j)$ intermediate goods.
- In equilibrium, winning bid $q^I_t$:
  - agent $I$: zero expected gain due to Bertrand Competition
  - IC for agent $O$: profitable for an informed $O$ agent type
Security trading occurs at date 1:
- bilaterally between agent $O$ and multiple agent $Is$
- in dedicated sub-markets for each available security.

Suppose agent $I$ bids per-unit price $q^I_t$ for security $j$.

If highest bid, agent $O$ offers him $a^Q_t(j)$ units of security $j$ for $q^I_t a^Q_t(j)$ intermediate goods.

In equilibrium, winning bid $q^I_t$
- agent $I$: zero expected gain due to Bertrand Competition
- IC for agent $O$: profitable for an informed $O$ agent type
Equilibrium in Security $j$’s Market

- Adverse selection index: higher $R^j_t$, lower adverse selection
  \[ R^j_t \equiv \frac{E_L y^j_t}{E_H y^j_t} \]

- Expected value of security $j$ when both $O$ types participate
  \[ \bar{q}^j = \lambda E_L y^j_t + (1 - \lambda) E_H y^j_t \]

- High $O$ type participates if adverse selection is low:
  \[ z \bar{q}^j - E_H y^j_t \geq 0 \iff R \geq \zeta \]

  where \( \zeta \equiv 1 - (z - 1)/\lambda z \)
If $R^j_t > \zeta$, both high and low $O$ types sell
- $q^j_t = \lambda E_L y^j_t + (1 - \lambda) E_H y^j_t$
- $a^L_t(q^j_t) = a^H_t(q^j_t) = a$.

If $R^j_t < \zeta$, only low type sells security
- $q^j_t = E_L y^j_t$
- $a^L_t(q^j_t) = a$ and $a^H_t(q^j_t) = 0$. 
Before learning asset quality, agent $O$ chooses security design $J_t(\phi_t) \subseteq I_t(\phi_t)$ to maximize

$$V_{o,t}(a) = \lambda \int \left( \sum_{j \in J_t(\phi_t)} a_t^L(j) \left[ z q^j_t - y^j_t(s) \right] \right) dF_L(s)$$

$$+ (1 - \lambda) \int \left( \sum_{j \in J_t(\phi_t)} a_t^H(j) \left[ z q^j_t - y^j_t(s) \right] \right) dF_H(s)$$

$$+ \int a(s + \phi_t) d \left[ \lambda F_L(s) + (1 - \lambda) F_H(s) \right]$$
Security Design: Constraints

- Each $O$ type optimally chooses how much to supply:
  - Low type $O$ agent always sells all since $E_L y^j_t(s) \leq E_H y^j_t(s)$

  $a^L_t(j) = a$ and $a^H_t(j) = \begin{cases} a & \text{if } R^j_t \geq \zeta \\ 0 & \text{if } R^j_t < \zeta \end{cases}$

- The security design must be overall feasible

  $\sum_{j \in J_t(\phi_t)} y^j_t(s) d\mu_{o,t} \leq s + \phi_t$

- Price is determined via Bertrand competition

  $q^i_t = \begin{cases} \lambda E_L y^j_t + (1 - \lambda) E_H y^j_t & \text{if } R^j_t \geq \zeta \\ E_L y^j_t & \text{if } R^j_t < \zeta \end{cases}$
Security Design: Constraints

Each $O$ type optimally chooses how much to supply:

- Low type $O$ agent always sells all since $E_L y_t^i(s) \leq E_H y_t^i(s)$

\[
a_L^t(j) = a \quad \text{and} \quad a_H^t(j) = \begin{cases} a & \text{if } R_t^j \geq \zeta \\ 0 & \text{if } R_t^j < \zeta \end{cases}
\]

- The security design must be overall feasible

\[
\sum_{j \in J_t(\phi_t)} y_t^i(s) d\mu_{o,t} \leq s + \phi_t
\]

- Price is determined via Bertrand competition

\[
q_t^i = \begin{cases} \lambda E_L y_t^i + (1 - \lambda) E_H y_t^i & \text{if } R_t^i \geq \zeta \\ E_L y_t^i & \text{if } R_t^i < \zeta \end{cases}
\]
Security Design: Constraints

- Each $O$ type optimally chooses how much to supply:
  - Low type $O$ agent always sells all since $E_L y_t^j(s) \leq E_H y_t^j(s)$

$$a_t^L(j) = a \text{ and } a_t^H(j) = \begin{cases} a & \text{if } R_t^j \geq \zeta \\ 0 & \text{if } R_t^j < \zeta \end{cases}$$

- The security design must be overall feasible

$$\sum_{j \in J_t(\phi_t)} y_t^j(s) d\mu_{o,t} \leq s + \phi_t$$

- Price is determined via Bertrand competition

$$q_t^j = \begin{cases} \lambda E_L y_t^j + (1 - \lambda) E_H y_t^j & \text{if } R_t^j \geq \zeta \\ E_L y_t^j & \text{if } R_t^j < \zeta \end{cases}$$
A stationary dynamic equilibrium consists of

- $\mathcal{J}_t(\phi_t)$ solves the security design problem
- security price $q^j_t$ satisfies the submarket Bertrand competition
- asset price $\phi_t$ solves the Euler equation given by:

$$\phi_t = \beta \left[ z \left( \sum_{j \in P_t} q^j_t + \lambda \sum_{j \in \mathcal{J}_t(\phi_t) \setminus P_t} q^j_t \right) + (1 - \lambda) \sum_{j \in \mathcal{J}_t(\phi_t) \setminus P_t} E_{Hy}^j \right]$$

where $j \in P_t$ iff $R^j_t \geq \zeta$. 
Collateral asset is the only security: No security design

security price depends on \( \frac{E_{LS} + \phi_t}{E_{HS} + \phi_t} \geq \zeta \):
- Pooling: \( q_t^P = \phi_t + \lambda E_{LS} + (1 - \lambda) E_{HS} \)
- Separating: \( q_t^S = \phi_t + E_{LS} \) otherwise.

asset price depends on \( \frac{E_{LS} + \phi_t}{E_{HS} + \phi_t} \leq \zeta \):
- Pooling: \( \phi^P = \beta z q_t^P = \frac{\beta z (\lambda E_{LS} + (1 - \lambda) E_{HS})}{1 - \beta z} \)
- Separating:
  \[
  \phi^S = \beta \left[ z \lambda q_t^S + (1 - \lambda) (\phi^S + E_{HS}) \right] = \frac{\beta [\lambda z E_{LS} + (1 - \lambda) E_{HS}]}{1 - \beta (\lambda z + 1 - \lambda)}
  \]
- \( \phi^P > \phi^S > PV = \frac{\beta [\lambda E_{LS} + (1 - \lambda) E_{HS}]}{1 - \beta} \)
**Benchmark: Dynamic Lemons Market**

- Collateral asset is the only security: No security design
- Security price depends on \( \frac{E_L s + \phi_t}{E_H s + \phi_t} \geq \zeta \):
  - Pooling: \( q_P^t = \phi_t + \lambda E_L s + (1 - \lambda) E_H s \)
  - Separating: \( q_S^t = \phi_t + E_L s \) otherwise.

- Asset price depends on \( \frac{E_L s + \phi_t}{E_H s + \phi_t} \leq \zeta \):
  - Pooling: \( \phi_P^t = \beta z q_P^t = \frac{\beta z (\lambda E_L s + (1 - \lambda) E_H s)}{1 - \beta z} \)
  - Separating: \( \phi_S^t = \beta [z \lambda q_S^t + (1 - \lambda) (\phi_S^t + E_H s)] = \frac{\beta [\lambda z E_L s + (1 - \lambda) E_H s]}{1 - \beta (\lambda z + 1 - \lambda)} \)
  - \( \phi_P^t > \phi_S^t > PV = \frac{\beta [\lambda E_L s + (1 - \lambda) E_H s]}{1 - \beta} \)
Benchmark: Dynamic Lemons Market

- Collateral asset is the only security: No security design

- security price depends on $\frac{E_L s + \phi_t}{E_H s + \phi_t} \geq \zeta$:
  - Pooling: $q_t^P = \phi_t + \lambda E_L s + (1 - \lambda) E_H s$
  - Separating: $q_t^S = \phi_t + E_L s$ otherwise.

- asset price depends on $\frac{E_L s + \phi_t}{E_H s + \phi_t} \geq \zeta$:
  - Pooling: $\phi^P = \beta z q_t^P = \frac{\beta z (\lambda E_L s + (1 - \lambda) E_H s)}{1 - \beta z}$
  - Separating:
    $\phi^S = \beta \left[ z \lambda q_t^S + (1 - \lambda) (\phi^S + E_H s) \right] = \frac{\beta [\lambda z E_L s + (1 - \lambda) E_H s]}{1 - \beta (\lambda z + 1 - \lambda)}$

- $\phi^P > \phi^S > PV = \frac{\beta [\lambda E_L s + (1 - \lambda) E_H s]}{1 - \beta}$
Collateral asset is the only security: No security design

- security price depends on $\frac{E_L s + \phi_t}{E_H s + \phi_t} \geq \zeta$:
  - Pooling: $q_P^t = \phi_t + \lambda E_L s + (1 - \lambda) E_H s$
  - Separating: $q_S^t = \phi_t + E_L s$ otherwise.

- asset price depends on $\frac{E_L s + \phi_t}{E_H s + \phi_t} \geq \zeta$:
  - Pooling: $\phi^P = \beta z q_P^t = \frac{\beta z (\lambda E_L s + (1 - \lambda) E_H s)}{1 - \beta z}$
  - Separating:
    \[
    \phi^S = \beta \left[ z \lambda q_t^S + (1 - \lambda) (\phi^S + E_H s) \right] = \frac{\beta [\lambda z E_L s + (1 - \lambda) E_H s]}{1 - \beta (\lambda z + 1 - \lambda)}
    \]
  - $\phi^P > \phi^S > PV = \frac{\beta [\lambda E_L s + (1 - \lambda) E_H s]}{1 - \beta}$
Fragility of Dynamic Lemons Market

- There can be multiple equilibria in a dynamic lemons market.

- Occurs when \( \frac{E_Ls + \phi^S}{E_Hs + \phi^S} < \zeta \leq \frac{E_Ls + \phi^P}{E_Hs + \phi^P} \).

- Plugging for \( \phi_S \) and \( \phi_P \) we obtain the condition for multiplicity as

\[
\frac{\zeta - \beta}{1 - \beta} < \frac{E_Ls}{E_Hs} < \frac{\zeta - \beta [1 - (1 - \lambda) (z - 1)]}{1 - \beta [1 - (1 - \lambda) (z - 1)]}
\]

- Easy to see that for intermediate values of \( E_{Ls}/E_{Hs} \) both equilibria exist.
Fragility of Dynamic Lemons Market

- There can be multiple equilibria in a dynamic lemons market.

- Occurs when \( \frac{E_{LS} + \phi^S}{E_{HS} + \phi^S} < \zeta \leq \frac{E_{LS} + \phi^P}{E_{HS} + \phi^P} \).

- Plugging for \( \phi^S \) and \( \phi^P \) we obtain the condition for multiplicity as

\[
\frac{\zeta - \beta}{1 - \beta} < \frac{E_{LS}}{E_{HS}} < \frac{\zeta - \beta}{1 - \beta} [1 - (1 - \lambda)(z - 1)]
\]

- Easy to see that for intermediate values of \( E_{LS}/E_{HS} \) both equilibria exist.
There can be multiple equilibria in a dynamic lemons market.

Occurs when \[ \frac{E_{Ls} + \phi^S}{E_{Hs} + \phi^S} < \zeta \leq \frac{E_{Ls} + \phi^P}{E_{Hs} + \phi^P}. \]

Plugging for \( \phi^S \) and \( \phi^P \) we obtain the condition for multiplicity as

\[ \frac{\zeta - \beta}{1 - \beta} < \frac{E_{Ls}}{E_{Hs}} < \frac{\zeta - \beta [1 - (1 - \lambda)(z - 1)]}{1 - \beta [1 - (1 - \lambda)(z - 1)]} \]

Easy to see that for intermediate values of \( E_{Ls}/E_{Hs} \) both equilibria exist.
There can be multiple equilibria in a dynamic lemons market.

Occurs when \( \frac{E_{LS} + \phi^S}{E_{HS} + \phi^S} < \zeta \leq \frac{E_{LS} + \phi^P}{E_{HS} + \phi^P} \).

Plugging for \( \phi_S \) and \( \phi_P \) we obtain the condition for multiplicity as

\[
\frac{\zeta - \beta}{1 - \beta} < \frac{E_{LS}}{E_{HS}} < \frac{\zeta - \beta [1 - (1 - \lambda) (z - 1)]}{1 - \beta [1 - (1 - \lambda) (z - 1)]}
\]

Easy to see that for intermediate values of \( E_{LS}/E_{HS} \) both equilibria exist.
There is a dynamic feedback loop.

If agents anticipate the asset to be traded in a pooling eqm in the decentralized market, then price is high.

In turn, when the price is high, the H-type O agent is willing to pool.

If agents anticipate the asset to be traded in a separating eqm in the decentralized market, price is low.

In turn, when the price is low, the H-type keeps the asset.
Intuition for Dynamic Multiplicity

- There is a dynamic feedback loop.
- If agents anticipate the asset to be traded in a pooling eqm in the decentralized market, then price is high.

- In turn, when the price is high, the H-type $O$ agent is willing to pool.

- If agents anticipate the asset to be traded in a separating eqm in the decentralized market, price is low.

- In turn, when the price is low, the H-type keeps the asset.
Intuition for Dynamic Multiplicity

- There is a dynamic feedback loop.
- If agents anticipate the asset to be traded in a pooling eqm in the decentralized market, then price is high.
- In turn, when the price is high, the H-type \(O\) agent is willing to pool.
- If agents anticipate the asset to be traded in a separating eqm in the decentralized market, price is low.
- In turn, when the price is low, the H-type keeps the asset.
Intuition for Dynamic Multiplicity

- There is a dynamic feedback loop.
- If agents anticipate the asset to be traded in a pooling eqm in the decentralized market, then price is high.
- In turn, when the price is high, the H-type $O$ agent is willing to pool.
- If agents anticipate the asset to be traded in a separating eqm in the decentralized market, price is low.
- In turn, when the price is low, the H-type keeps the asset.
We call a security traded in a pooling equilibrium in the decentralized market a liquid security.

First we show that Agent 0 is weakly better-off selling only one liquid security.

This is because if two securities are both liquid, combination is also liquid and generates at least as much value for Agent 0.

Also if security design is optimal, the feasibility constraint is binding.

W.l.o.g. can restrict attention to a liquid security $y(s)$ and an illiquid one $s + \phi - y(s)$. 
We call a security traded in a pooling equilibrium in the decentralized market a liquid security.

First we show that Agent $O$ is weakly better-off selling only one liquid security.

This is because if two securities are both liquid, combination is also liquid and generates at least as much value for Agent $O$.

Also if security design is optimal, the feasibility constraint is binding.

W.l.o.g. can restrict attention to a liquid security $y(s)$ and an illiquid one $s + \phi - y(s)$. 
We call a security traded in a pooling equilibrium in the decentralized market a liquid security.

First we show that Agent $O$ is weakly better-off selling only one liquid security.

This is because if two securities are both liquid, combination is also liquid and generates at least as much value for Agent $O$.

Also if security design is optimal, the feasibility constraint is binding.

W.l.o.g. can restrict attention to a liquid security $y(s)$ and an illiquid one $s + \phi - y(s)$. 
We call a security traded in a pooling equilibrium in the decentralized market a liquid security.

First we show that Agent $O$ is weakly better-off selling only one liquid security.

This is because if two securities are both liquid, combination is also liquid and generates at least as much value for Agent $O$.

Also if security design is optimal, the feasibility constraint is binding.

W.l.o.g. can restrict attention to a liquid security $y(s)$ and an illiquid one $s + \phi - y(s)$. 
Optimal Security Design

- We call a security traded in a pooling equilibrium in the decentralized market a liquid security.
- First we show that Agent $O$ is weakly better-off selling only one liquid security.
- This is because if two securities are both liquid, combination is also liquid and generates at least as much value for Agent $O$.
- Also if security design is optimal, the feasibility constraint is binding.
- W.l.o.g. can restrict attention to a liquid security $y(s)$ and an illiquid one $s + \phi - y(s)$.
Proposition

Assume that $\frac{f_L(s)}{f_H(s)}$ is decreasing in $s$. The optimal security is a unique standard debt contract $y_D$ such that

$$y_D(s) = \phi + \min(s, s^*),$$

for some $s^* \in (s_L, s_H)$. 

Optimality of Debt
Characterizing the Debt Contract

Proposition

Assume that \( \frac{f_L(s)}{f_H(s)} \) is decreasing in \( s \).

- If \( \frac{E_{LS}}{E_{HS}} < 1 - \frac{z-1}{z} \frac{1}{\lambda(1-\beta)} \),
  - ie, the separating region in the dynamic lemons market
  - a unique equilibrium and non-trivial tranching with
    \( D \in (s_L, s_H) \) and \( \phi \) solve:
    \[
    \phi = \frac{z}{z-1} \lambda \int_{s_L}^{D} \left[ \tilde{F}_H(s) - \tilde{F}_L(s) \right] ds - \int_{s_L}^{D} \tilde{F}_H(s) ds - s_L
    \]
    \[
    \phi = \frac{\beta}{1-\beta z} \left\{ z [\lambda E_{LS} + (1-\lambda) E_{HS}] - (1-\lambda)(z-1) \int_{D}^{s_H} \tilde{F}_H(s) ds \right\}
    \]
  - Otherwise, a “pass-through security” that promises the entire value of the asset and replicates the pooling equilibrium in dynamic lemons market
    \( D = s_H \) and \( \phi = \frac{\beta}{1-\beta z} z [\lambda E_{LS} + (1-\lambda) E_{HS}] \).
We show that security design equilibrium Pareto dominates all equilibria of the case in dynamic lemons market:

- more liquidity, more real output and less fragility
- even if only issue a “pass-through security” that mimics equity—replicate the pooling
Eliminates Low Liquidity Equilibrium

\[ F_0 = FL \]
\[ D = SL \]
Separating \( F_L \)
Pooling No

\[ \Phi_0 = \Phi_L \]
\[ D_1 = D(\Phi_L) \]
Separating \( \Phi_1 \)
Pooling No

\[ \Phi_1 \]
\[ D_2 = D(\Phi_1) \]
Separating \( \Phi_2 \)
Pooling No

\[ \Phi_2 \]
\[ D_3 = D(\Phi_2) \]
Separating \( \Phi_3 \)
Pooling No

\[ \Phi_3 \]
\[ D_n = D(\Phi_{n-1}) \]
Separating \( \Phi_n = \Phi^* \)
Pooling No

\[ \Phi_n = \Phi^* \]
\[ D = S_H \]
Separating \( \Phi_H \)
Pooling No

\[ F_n-1 = F^* \]
Discussions on Fragility and Robustness

- Unravelling results when security design option is introduced.
  - Suppose low asset price,
  - tranche a small senior liquid debt, asset price \( \uparrow \), which allows more liquid tranching \( D \uparrow \), which leads to asset price \( \uparrow \), ... converges to optimal.
Repo Features

- Face value: \( D + \phi \)
- Repo rate: \( \frac{D + \phi - q_D}{q_D} \)
- Haircut: equity tranche, \( q_E \)
Conclusion

Optimal security design in a dynamic lemons market

- Unique equilibrium: both high and low types issue repo-debt and debt is liquid; low type issues equity
- Eliminates fragility and improves liquidity
- Improves social welfare relative to the separating equilibrium under equity contract
- Endogenous amplification of shocks to asset quality and productivity