Two extremely active areas of modern mathematical logic are Computability Theory and Set Theory. These fields are intensively researched in many parts of the USA, Europe and Asia, including Singapore. An impetus for the launching of the Sets and Computations Program at the IMS was to exploit the recent dramatic growth in interactions between these two fields, resulting in exciting new collaborations between two distinct groups of mathematical logicians.

Set Theory dates back to Cantor’s discovery of transfinite counting. Let \( C \) denote a closed set of real numbers. The Cantor derivative of \( C \), denoted by \( C' \), is the set of limit points of \( C \). One can iterate the Cantor derivative infinitely many times to obtain \( C, C', C'', \ldots \) with intersection \( C(\text{infinity}) \). Cantor came to the surprising realisation that this process may continue even further to \( C(\text{infinity})', C(\text{infinity})'', \ldots \), a “transfinite iteration”.

Set Theory was born out of Cantor’s work, which provided this “counting into the transfinite” with a meaningful analysis, ultimately leading to modern-day axiomatic set theory. An important added benefit to the development of set theory is that it provides an excellent foundation for mathematics. Indeed, with rare exceptions, the concepts of mathematics can be expressed in the language of set theory and the theorems of mathematics then become theorems of the standard system ZFC of axioms for set theory.

ZFC is however not a complete system, and in set theory we in fact expect quite the opposite, which is that most interesting questions are not decidable in ZFC. This has led to a huge body of work, combining the use of large cardinal axioms (which assert the existence of large infinities) with the forcing method (invented by Cohen to show that the continuum hypothesis CH is not provable in ZFC) to produce a wide array of distinct interpretations (models) of the ZFC axioms. Producing these models is essential for understanding the status of set-theoretic assertions relative to ZFC and to each other.

[Editor’s note: In March/April 2015, the Institute hosted the program “Sets and Computations”. One of the members of the organizing committee, Sy-David Friedman (University of Vienna), contributed this invited article to Imprints.]
The rare exceptions to the completeness of ZFC for questions outside of set theory include important problems such as the Borel Conjecture (measure theory), Whitehead Problem (group theory) and Kaplansky Conjecture (functional analysis). Such problems can be solved using the axiom $V = L$ (Goedel’s axiom of constructibility), but the common view is that such an axiom reflects a “restricted” view of the concept of set.

Recent work however shows that such problems can also be resolved using a relatively new breed of axioms called “forcing axioms”, which not only have great mathematical power but which also do not restrict the concept of set in the way that $V = L$ does. This is certainly not the end of the story, as other recent work on “cardinal characteristics of the continuum” yields yet another compelling perspective on set theory whereby the size of the continuum (the set of real numbers) is even larger than what forcing axioms imply.

This is all very exciting and rapidly-developing work, uncovering the rich combinatorial complexity of the notion of set through the tools of large cardinals and forcing.

Computability Theory was born out of Goedel’s work on the incompleteness of axiomatic systems, which as a consequence yielded a notion of “recursive function”. Subsequent work revealed the robustness of this concept, as it was shown that a wide spectrum of different models for the theory of computation resulted in the very same class of computable functions. The most attractive of these models was presented by Turing in terms of his Turing Machine. A key observation, closely related to Goedel’s work on incompleteness, was the fact that there are sets which can be “computably enumerated” but which are not computable. Turing also introduced computability “relative to an oracle”, and the subsequent study of this notion of Turing-reducibility became and still is a vast and intricate area of study.

Recent work in computability theory exploits the fact that the notion has natural connections to numerous other areas of mathematical logic, such as computable model theory (where one studies structures that are presented in a computable way) and computable randomness (which adds an algorithmic feature to the theory of randomness). Further, much work on metamathematics, where one studies the axiom systems needed to prove mathematical statements, is closely related to computable mathematics, where one studies computable analogues of such statements.

Recently, important and surprising connections between Set Theory and Computability Theory have emerged and one of the key successes of the Sets and Computations program was to reveal and further develop these connections. These include: Computational Complexity in Set Theory (including a notion of polynomial-time computability on arbitrary sets), Computable Polish Group Actions (injecting computability theory into a central topic of descriptive set theory), Computable analogues of Cardinal Characteristics of the Continuum (mentioned above) and Computability on Uncountable Structures (placing Computability Theory on the ordinal numbers into a much wider context). Our timing was just right, as these connections have only become apparent in the last few years. And of course we look forward to seeing them develop further in the near and long-term future.

Sy-David Friedman
University of Vienna
**Professor Louis Chen Conferred the Emeritus Professorship Award**
Professor Louis Chen Hsiao Yun, the founding Director of IMS, was presented the Emeritus Professorship award by Provost Professor Tan Eng Chye at an appreciation dinner organised by the Department of Mathematics on 8 April at Hotel Miramar. He served as Director of the Institute for 12 ½ years.

![Louis CHEN receiving the Emeritus Professorship award scroll and appreciation gift from Provost Professor Eng Chye TAN. Photo Credit: Department of Mathematics, NUS](image)

**Visitors to IMS**
In January 2015, the Institute hosted Professor Yoshiaki Maeda and his colleagues from the Tohoku Forum for Creativity. Professor Chi-tat Chong, Director of IMS, gave a brief presentation of our institute’s mission, organizational structure, and highlighted some of our past and upcoming programs. During the visit, Professors Maeda and Chong exchanged ideas to foster further collaboration in research/programs.

![From left: Yuanyuan BAO, Chi-tat CHONG, Kwok Pui CHOI, Yoshiaki Maeda, Kaori Chiba and Shuntaro Takahashi](image)

**Personnel movements at IMS**
Agnes Wu, who had been the Institute’s secretary since it was established in 1 July 2000, left IMS in 6 April 2015 to be full time homemaker. She was on leave for a year from February 2014, and her duties were covered by Maisarah Binte Abu Bakar from 10 February 2014 to 31 July 2015. The Institute takes this opportunity to thank Agnes and Maisarah for their services and wish them success in their future endeavors. Annie Teo, who joined IMS as management assistant officer on 11 May 2015, is the new secretary.

**Launch of IMS Namecard Holder**
IMS has launched the specially designed IMS Namecard Holder for our visitors to purchase as souvenirs. They are available in two designs: the hypercube and the Klein bottle. Purchases can only be made with cash at the IMS.

![IMS Namecard Holder in two designs: Hypercube and Klein Bottle](image)
High Performance and Parallel Computing for Materials Defects and Multiphase Flows (1 January - 31 March 2015) — jointly organized with Department of Mathematics, NUS

Website: http://www2.ims.nus.edu.sg/Programs/015hiper/index.php

Co-chairs:
Weizhu Bao, National University of Singapore
Weiqing Ren, National University of Singapore and Institute of High Performance Computing, A*STAR
Ulrich Rüde, Universität Erlangen-Nürnberg

This program brought together mathematicians, computer scientists, physicists, materials scientists and mechanical engineers to exchange ideas, and to promote interdisciplinary research in high performance and parallel computing with applications to the simulation of materials defects, material-related processes, multiphase flows and complex fluids.

A series of interrelated workshops and tutorials, which offered the participants first-hand knowledge of the latest research, were organized. The three main workshops, with a total of 75 invited talks, focused on recent advances in parallel and high performance computing environments, exascale computer structures, parallel programming platform (i.e. MPI and OpenMP), techniques for parallelization of existing algorithms, as well as mathematical models and exascale enabled algorithms for simulating materials defects and multiphase/complex fluids.

Tutorial lectures on Parallel Domain Decomposition Methods and Parallel Hierarchical Grid (PHG) were delivered by Xiao-Chuan Cai (University of Colorado Boulder, USA) and Linbo Zhang (Chinese Academy of Sciences, China). There was also a winter school which had five tutorials and three special seminars, and covered topics in defects in crystals, Density Functional Theory and the Kohn-Sham model, the discontinuous Galerkin (DG) method, mathematical equations for modeling quantized vortices in superfluidity and superconductivity, and the asymptotic and numerical methods for rarefied gas flows based on kinetic theory.

The tutorial speakers were David Srolovitz (University of Pennsylvania, USA and Penn Institute for Computational Science, USA), Eric Cances (Ecole des Ponts ParisTech and INRIA, France), Jianxian Qiu (Xiamen University, China), Weizhu Bao (National University of Singapore) and Kazuo Aoki (Kyoto University, Japan). About 40 research projects/papers were initiated during the program. There were a total of 141 participants and among them 33 were graduate students.

Continued on page 5
Inaugural Oppenheim Lecture (28 January 2015) and Workshop on Representation Theory & Automorphic Forms (27 – 29 January 2015)
— Jointly organized with Department of Mathematics, NUS

Website: http://www1.math.nus.edu.sg/events/Poster-OppenheimLectures.pdf

The Oppenheim Lectures is a distinguished lecture series jointly organized by the Department of Mathematics and the Institute for Mathematical Sciences at the National University of Singapore (NUS). It will be held annually at the beginning of each Academic Year. This lecture series honours Sir Alexander Oppenheim, who held the position of Professor and first Head of the Department from 1931, at the time of Raffles College, until 1959.

A workshop on “Representation Theory & Automorphic Forms” was held in conjunction with the Oppenheim Lecture from 27 to 29 January 2015. The inaugural Oppenheim Lecture “On the Average Rank of Elliptic Curve Over Function Field” was delivered by Professor Ngô Bảo Châu from the University of Chicago and the Vietnam Institute for Advanced Study in Mathematics on 28 January 2015.

The Oppenheim lecture and the workshop were attended by 70 participants.

Joint international workshop of the National University of Singapore
Institute for Mathematical Sciences and Yong Siew Toh Conservatory of Music
— Jointly organized with Centre for Digital Music, Queen Mary University of London, UK, Science and Technology of Music and Sound Lab, IRCAM, CNRS, UPMC, France

Website: http://www2.ims.nus.edu.sg/Programs/015wmusic/index.php

Program Chairs:
Gérard Assayag, Institut de Recherche et Coordination Acoustique/Musique
Elaine Chew, Queen Mary University of London

This three-day workshop brought together world leading and emerging researchers in the mathematical music sciences, with a special focus on mathematical and computational research in music performance and composition. The organization itself involved collaboration across countries and three institutions, namely the Centre for Digital Music at Queen Mary University of London (United Kingdom), the Sciences and Technologies of Music and Sound Laboratory across the Institut de Recherche Coordination Acoustique/Musique, the Centre National de la Recherche Scientifique, and the Pierre-and-Marie-Curie University (France), and the Institute for Mathematical Sciences and the Yong Siew Toh Conservatory of Music at the National University of
Singapore (Singapore). The collaborations ensured sharing of awareness at institutional levels as well as enabling greater visibility for the event.

There were six plenary sessions which were planned with 19 invited talks. The workshop also had two panel discussions, 17 poster presentations, five concerts and a jam session. The concerts featured Paul Schoenfield’s music, human and machine intelligence, the prevalence of base 12 in music collections, musical canons, musical creativity and range in style from classical to popular and jazz. All of the activities were held at the Yong Siew Toh Conservatory of Music. The interleaving lectures and concerts were designed to appeal to a broad audience. Videos of the lectures and performances were uploaded to YouTube. There were a total of 137 registered/invited participants. From a feedback survey gathered from the participants, there were two research collaborations/papers which were initiated or worked on during/after the workshop. The organizers received many positive comments from both participants and speakers, with many speakers saying that they were particularly pleased with the opportunity to meet each other.

In addition to the stated plans, a pre-workshop study day was conducted at Raffles Institution on 12 February 2015 to reach out to junior college and advanced secondary school students. In Singapore, where music education is convolved with mathematics and science education well into secondary school, such a workshop has great potential for inspiring young minds to delve further into one or more of the disciplines represented.
Sets and Computations (30 March - 30 April 2015)
Website: http://www2.ims.nus.edu.sg/Programs/015set/index.php

Organizing committee:
Raghavan Dilip, National University of Singapore
Sy David Friedman, University of Vienna
Yue Yang, National University of Singapore

Both set theory and computability theory have experienced dramatic growth in Asia in the recent years. This program aimed to bring leading researchers in set theory and computation theory to IMS to investigate newly-emerging and promising valuable connections between these two fields.

Valuable discussion led to 23 research projects/papers initiated during the program. Participants learned about exciting new developments regarding computational complexity in set theory, generalized degree spectra (connecting descriptive set theory and computability theory), computability and Polish group actions. A total of 21 invited talks on Set theory were given in the first two weeks of the program. The following week, with 12 invited talks, focused on the interactions between set theory and computability theory. The last two weeks of the program were devoted to Computability theory with 12 invited talks. There were a total of 73 participants and among them ten were graduate students.

Workshop on Stochastic Processes in Random Media (4 - 15 May 2015)
Website: http://www2.ims.nus.edu.sg/Programs/015wrandom/index.php

Co-Chairs:
Gleb Oshanin, Université Pierre et Marie Curie
Rongfeng Sun, National University of Singapore
Dong Wang, National University of Singapore

There are increasingly more interactions among researchers from different areas and backgrounds: physics, probability, analysis and combinatorics. Such interactions have led to cross-fertilization of ideas that have benefited many topics in statistical physics and probability including interacting particle systems, disordered systems, random matrices, percolation and Ising models. The central themes of the workshop were: dynamics of reaction-diffusion systems; random polymers and related systems; the Kardar-Parisi-Zhang universality class; and random matrix models.

A total of 38 talks and one poster were given in the two-week workshop. Twenty eight senior researchers lectured on progress made in these three main themes in the first week. During the second week, junior researchers gave talks; and time was set aside for discussions. The talks in the workshop had been very informative and created many opportunities for mathematicians and physicists with common interests to exchange information and ideas. Twelve research projects/papers were initiated during the workshop. There were a total of 63 participants and among them 13 were graduate students. In addition to the funding by the IMS, the organizers received supplementary funding of USD 10,000 from the Office of Naval Research Global.
Workshop on New Directions in Stein’s method (18 - 29 May 2015)

Website: http://www2.ims.nus.edu.sg/Programs/015wstein/index.php

Chair:
Larry Goldstein, University of Southern California
Adrian Röllin, National University of Singapore

Stein’s method for both normal and non-normal approximations has found applications in a large variety of areas, including statistics, point process theory, combinatorics, number theory, random graph theory, random matrix theory, and statistical physics.

Building on from two previous programs (2003 and 2008) and a workshop (2009), which were all held at IMS, and considering the diversity and new exciting directions recent advances has brought, this workshop brought a timely opportunity to bring together researchers who worked directly in this area and those who apply Stein’s method in their work.

The two-week workshop consisted of 42 invited talks and 6 short talks. Areas of applications of Stein’s method included stochastic geometry, concentration of measure, number theory, queuing theory, quantum mechanics, telecommunication networks, high-dimensional statistics, change-point analysis, compressed sensing, and random graphs. Due to the high level of interactions the meeting fostered, fourteen research projects were initiated. Graduate students and young scientists participated actively in discussion with workshop participants and a special session was arranged for them to present their research work. There were a total of 80 participants and among them 14 were graduate students.
Conference on 60 Years of Yang-Mills Gauge Field Theories (25 - 28 May 2015)
— Jointly organized with Institute of Advanced Studies, Nanyang Technological University
Website: http://www.ntu.edu.sg/ias/upcomingevents/Yang-Mills60/Pages/default.aspx

Co-chairs:
Lars Brink, Chalmers University of Technology
Phua Kok Khoo, Nanyang Technological University

There were a total of 51 talks, nine poster presentations and a public lecture on “Personal Perspectives on Physics” given by eminent physicists Michael Fisher, David Gross and Chen Ning Yang.

Public lectures:
Professor David J. Srolovitz of University of Pennsylvania, and Penn Institute for Computational Science, USA gave a public lecture, “Bubbles-Foams, Grains-Metals: Curvature Flow in Cellular Materials” at the National Library on 9 February 2015. Professor Srolovitz focused on the fundamental ideas of curvature flow in foams and in solid materials in this public lecture. He first pointed out that curvature flow, in foams and in polycrystalline materials, is driven by surface tension; motion that minimizes the total surface area. And curvature flow has to respect the connectedness of the bubbles and grains--giving rise to interesting issues in topology and its evolution. This type of evolution has interested generations of metallurgists, chemical engineers, physicists, and mathematicians including John von Neumann over a half century ago. Professor Srolovitz concluded his lecture with several recent developments and computer simulations to an enthused group of 60 people in the audience.

Professor Frank den Hollander of Leiden University, The Netherlands delivered a public lecture, “Complex Networks: How Can We Understand Their Behaviour?” at the National Library on 13 May 2015. In the lecture, Professor den Hollander explained that networks were everywhere in our modern society: Internet, Facebook and Twitter, but also road traffic, transportation of merchandise, mobile telephones and electricity grids. A concern of these networks was that they were very complex: huge in size, intricate in structure, very dynamic, often overloaded, sometimes unpredictable, and at times vulnerable. He pointed out the need for new ideas to better understand the complex networks, to model them adequately, and to control and optimize them in an efficient manner. In the lecture, he described a few examples of complex networks and discussed several key questions.
to illustrate what mathematics, as a powerful tool, was able to do. He then described how the combination of “stochastics” (the art of hazard) and “algorithmics” (the art of computation) provided a new perspective on networks with the ultimate goal to design and build intelligent networks. A total of 29 people attended the lecture.

Professor Rod Downey of Victoria University of Wellington, New Zealand delivered a public lecture, “Alan Turing, Computing, Bletchley, and Mathematics” at the National Library on 1 July 2015. In the lecture, Professor Downey provided a more accurate picture of one of the 20th Century geniuses, Alan Turing: his work and his place in history. He focused on Turing’s abstract work in computation, and the work of the wonderful team of cryptanalysts at Bletchley Park. Then he briefly pointed out a number of inaccuracies in the movie “The Imitation Game” about Turing. He concluded his public lecture with a brief survey of Turing’s other work. A total of 68 people attended the lecture.

Activities

• Tutorial I: 3 - 5 June 2015
• The Protein Network Workshop: 8 - 12 June 2015
• Seminars
  - 7 July 2015, Ancestry estimation and control of genetic association studies, Chaolong Wang, Genome Institute of Singapore
  - 24 July 2015, Integrating biological networks into statistical models: from static to dynamic interaction networks, Hyungwon Choi, National University of Singapore
• Tutorial II: 22 - 24 July 2015
• The Phylogenetic Network Workshop: 27 - 31 July 2015

The second part of the program is on the systematic methods for phylogenetic networks. In the study of evolution, network provides biological explanations that go beyond what can be accommodated by the tree model. However, network modeling is extremely challenging. It offers excellent opportunities for mathematicians and statisticians to develop fast and robust programs for inferring an evolutionary network model from sequence data. Statistical methods are required to distinguish genuine horizontal genetic transfers from background noise such as incomplete lineage sorting.

Current Program

Networks in Biological Sciences (1 June - 31 July 2015)  
Website: http://www2.ims.nus.edu.sg/Programs/015bio/index.php

Chair:  
Louxin Zhang, National University of Singapore

The program focuses on the mathematics for network models in biology. After a wide variety of genomes have been fully sequenced and annotated, networks of different types have been used to study cellular biological processes as well as genetic diseases. In this network view, each node represents a molecule, such as a gene, RNA or protein, or an organism. The edge between two nodes represents a relationship, such as an enzymatic reaction, physical interaction, a transcriptional regulation, or an evolutionary relationship. This program aims to bring together researchers in complex networks and systems biology to enable knowledge transfer in the study of cellular networks. Analyses of dynamic and topological properties of complex networks have produced many insightful concepts useful for systems biology. On the other hand, systems biology provides interesting questions and ideas for complex network analysis.

The IMS Graduate Summer School in Logic (15 June - 3 July 2015)  
— jointly organized with Department of Mathematics, NUS
Website: http://www2.ims.nus.edu.sg/Programs/015logics/index.php

The IMS Graduate Summer School in Logic is jointly organized and funded by the Institute for Mathematical Sciences (IMS) and the Department of Mathematics of the National University of Singapore. The Summer School bridges the gap between a general graduate education in mathematical logic and the specific preparation necessary to do research on problems of current interest in the subject.
Activities

• Week 1: Lectures by Hugh Woodin, Harvard University

• Week 2: Lectures by Andrew Marks, California Institute of Technology

• Week 3: Lectures by Theodore A. Slaman, The University of California, Berkeley

Combinatorial and Toric Homotopy (1 - 31 August 2015) — on the occasion of Professor Frederick Cohen’s 70th Birthday

Website: http://www2.ims.nus.edu.sg/Programs/015homo/index.php

Co-chairs:
Jelena Grbic, University of Southampton
Zhi Lu, Fudan University
Jie Wu, National University of Singapore

This program aims to explore toric homotopy theory and combinatorial homotopy theory as well as their connections with other areas of mathematics. Leading experts from different backgrounds will come together to talk about the latest developments in algebraic topology with attention to the applications of algebraic topology to high technology and sciences, and chart out new directions for research in toric and combinatorial homotopy theory.

Frederick R. Cohen, a Professor of Mathematics at the University of Rochester, is a scholar of distinction.

His major research interest is in algebraic topology. He made major contributions in homotopy theory, particularly in the study of loop spaces, and configuration spaces, with connections to braid groups, modular forms and cohomology of groups. His seminal papers with John Moore and Joe Neisendorfer on exponents in homotopy groups were a breakthrough in classical unstable homotopy theory, recognized to this day as one of the highest achievements in homotopy theory.

Professor Cohen is recognized as one of the most influential mathematicians in the area of topology and its applications. He authored and coauthored more than 120 papers, delivered numerous invited addresses in international conferences, including an invited address in the 1983 ICM. While most of his research life was spent in the United States, he had been a frequent traveler and visited and lectured in great many countries around the world, including Canada, Spain, Germany, France, UK, Israel, China, Japan, Mexico and Brazil. His recent work with Anthony Bahri, Martin Bendersky and Samuel Gitler on polyhedral product functors stimulated the birth of a new subject - toric homotopy - the title of our program.

As Professor Cohen celebrates his 70th birthday, we are organizing a program to honor his achievements as a scholar and a teacher.

The program is partially supported by IMS, Department of Mathematics of NUS, and academic research grants of NUS. To date, 84 overseas visitors have confirmed their participation in the program.

Activities

• Young Topologist Seminar: 11 - 19 August 2015

• Workshop on Applied Topology: 20 - 21 August 2015

• International Conference on Combinatorial and Toric Homotopy: 24 - 28 August 2015

Programs & Activities in the Pipeline

Stochastic Methods in Game Theory (16 November - 25 December 2015)

Website: http://www2.ims.nus.edu.sg/Programs/015game/index.php

Chair
Satoru Takahashi, National University of Singapore

The program aims at showing the role of stochastic methods in strategic situations. Three workshops will focus on some aspects of the interaction between strategy and stochastics and its interest from a mathematical viewpoint.
Activities
• Tutorial on Learning: 16 November 2015
• Workshop on Learning: 17 - 20 November 2015
• Tutorial on Stochastic Games: 30 November – 3 December 2015
• Workshop on Stochastic Games: 30 November - 4 December 2015
• Tutorial on Congestion Games: 14 December 2015
• Workshop on Congestion Games: 15 - 18 December 2015

New Challenges in Reverse Mathematics (3 - 16 January 2016)
Website: http://www2.ims.nus.edu.sg/Programs/016reverse/index.php

Organizing Committee
Denis Hirschfeldt, University of Chicago
Richard Shore, Cornell University
Stephen Simpson, Pennsylvania State University
Theodore Slaman, The University of California, Berkeley
Frank Stephan, National University of Singapore
Yue Yang, National University of Singapore

The central theme of Reverse Mathematics is calibrating the strength of classical mathematical theorems in terms of the axioms needed to prove them; this calibration also takes into account recursion-theoretic complexity measures and consistency strength. The area is a very active one that investigates many topics in classical mathematics and involves all major branches of modern mathematics.

Activities
Collaborative Research and Workshop: 3-16 January 2016

Semidefinite and Matrix Methods for Optimization and Communication (18 January - 28 February 2016)
Website: http://www2.ims.nus.edu.sg/Programs/016semi/index.php

Organizing Committee
Rahul Jain, National University of Singapore
Hartmut Klauck, Nanyang Technological University and National University of Singapore
Troy Lee, Nanyang Technological University

Miklos Santha, Université Paris Diderot - Paris 7 and National University of Singapore

Activities
• Tutorial on Learning: 16 November 2015
• Workshop 1 on Log Rank Conjecture: 18 - 22 January 2016
• Workshop 2 on Positive Semidefinite Rank: 1 - 5 February 2016
• Workshop 3 on Approximation Algorithms: 15 - 19 February 2016

The program will cover topics in combinatorial optimization, approximation algorithms, and communication complexity and links connecting these areas. A common approach to hard combinatorial optimizations is to look at relaxations of these problems as linear or semidefinite programs. On the algorithmic side, one hopes to show that these relaxations can provide good approximations to the optimal value. On the hardness side, one hopes to show that (ever more complicated) relaxations are still far from the true value.

New Developments in Representation Theory (6 - 31 March 2016)

New Directions in Combinatorics (9 - 27 May 2016)

International Workshop on Fluid-Structure Interaction Problems (30 May - 3 June 2016)

Empirical Likelihood Based Methods in Statistics (6 June - 1 July 2016)

Mathematics of Shapes and Application (4 - 31 July 2016)

Geometry, Topology and Dynamics of Moduli Spaces (1 - 19 August 2016)
Mathematical Conversations

Benedict Gross: Elliptic Curves, Millennium Problem

Interview of Benedict Gross (Harvard University) by Y.K. Leong

“It is possible to write endlessly on elliptic curves. (This is not a threat.)”

-- Serge Lang (1927–2005)

Benedict Gross made important contributions to number theory, algebraic geometry, modular forms and group representations and also to undergraduate education. Gross received his university education mainly from Harvard University (BA and PhD) with a short interlude in between on a Sheldon travelling fellowship to study music in Asia and Africa and a Marshall Scholarship (for an MSc) at Oxford University. He taught briefly at Princeton University and Brown University before moving to Harvard University where he remains for 30 years and is now George Vasmer Leverett Professor of Mathematics. His research output consists of more than 110 research publications and he has collaborated with more than 60 researchers. He has attracted and advised more than 34 doctoral students. Together with his distinguished colleagues, he keeps alive the famous Harvard tradition in number theory set by John Tate. His dedication to undergraduate teaching was recognized by his appointment as Harvard College Professor. Then he became Dean for Undergraduate Education (Harvard University) and subsequently Dean of Harvard College. In his capacity as dean, he led the first review of Harvard’s undergraduate education in nearly 30 years. He belongs to the rare breed of scholars who can think deeply about esoteric subjects as well as grapple with the more earthly problems of administration.

One of his most important ground-breaking work, done with Don Zagier, produces a limit formula for the first derivative of the \( L \)-series of modular forms at the point \( s = 1 \) in terms of the heights of points on the Jacobians of modular curves. As a result of this work, certain cubic equations with a large number, on average, of solutions modulo primes can be shown to have infinitely many rational solutions. Also, together with the work of Dorian Goldfeld, it has reduced the classical problem of Carl Friedrich Gauss (1777-1855) on the classification of imaginary quadratic fields with a given class number to a finite decision problem.

Gross’ work has a bearing on the original Birch and Swinnerton-Dyer conjecture – the arithmetic and analytic ranks of an elliptic curve over the rationals are equal (Bryan Birch and Peter Swinnerton-Dyer, 1963) – and its famous refined version: The incomplete \( L \)-series of an elliptic curve \( E \) has the form \( L(E,s) \sim c(s − 1)^r \) as \( s \to 1 \), for some nonzero constant \( c \) and \( r \) is the rank of \( E \) over the rationals. This conjecture has been chosen as one of seven important problems for this new millennium by the Clay Mathematics Institute, each problem carrying a prize of one million dollars (US). (One of these problems, the Poincaré conjecture, named after Henri Poincaré (1854-1912), has been solved some years ago by Grigori Perelman who declined the Fields Medal awarded to him by the International Mathematical Union in 2006 and the prize awarded by the Clay Mathematics Institute in 2010.)

He has served on the committees of the American Mathematical Society, given advisory services to the Mathematical Sciences Research Institute, Berlin Mathematical School and Vietnam Institute for Advanced Study in Mathematics, and was on the board of trustees of the Institute for Advanced Study, Princeton. He has served as editors of leading international journals, notably Annals of Mathematics, Compositio Mathematica and Journal of American Mathematical Society.

His honors include the Sloan Fellowship, MacArthur Fellowship and Cole Prize. He is a Fellow of the American...
Academy of Arts and Sciences and a member of the National Academy of Science (US). Together with John Tate, he was a Clay Mathematics Institute Senior Scholar-in-Residence. Gross is a member-at-large of the Executive Committee 2015-2018 of the International Mathematical Union.

He contributed actively to the efforts of Tsinghua University, China in raising the level of interaction between Chinese mathematicians and the international community. In addition to being invited to give talks at conferences, he is one of those few leading mathematicians who are willing and prepared to reach out to schools and the general public. He has also written, jointly with his colleague Joe Harris, a book for the general reader, *The Magic of Numbers*, based on a general education course they taught at Harvard. In the International Congress of Mathematicians 2014 held in Seoul, he delivered the laudation lecture on the work of Fields Medalist Manjul Bhargava who himself was an undergraduate student of Gross in Harvard and with whom Gross had collaborated on several papers.

A conference on number theory and representation theory was held in 2010 at Harvard in honor of the 60th birthday of Gross. It was sponsored by Clay Mathematics Institute, Peter Kwok, Number Theory Foundation, Columbia University Mathematics Department, Harvard Mathematics Department and National Science Foundation (US). The group photograph taken at the 3½-day event shows that it attracted a large number (visibly more than 200) of mathematicians who have been associated with Gross or Harvard. The comments expressed in the blog of a participant and former student painted a picture of some kind of tribal reunion of an esoteric sect engaged in a festive celebration and revelation of the secrets of some inner mystery (number theory).

His interests extend beyond the world of numbers into the world of music (he plays the viola with a string quartet) and the world of sports (he is an avid golfer, tennis player and skier). In 2008 he collaborated with Japanese avant-garde composer and conceptualist artist Ryoji Ikeda in a Paris “arts-meets-science” research lab (*Le Laboratoire*) which is directed by the author and biomedical engineer David Edwards.

Some forty years ago, Gross first set foot in Singapore en route as a travelling scholar of music. Twenty years later, he came back as an invited speaker in a number theory workshop organized by the Department of Mathematics, National University of Singapore (NUS) and the Singapore Mathematical Society. This was followed by two visits as invited speaker in two programs of the Institute for Mathematical Sciences (IMS), NUS on *Representation Theory of Lie Groups* (July 2002-January 2003) and *Branching Laws* (11-31 March 2012). His connection with NUS runs deeper. Part of the Harvard tradition in number theory finds a thread in the work of some NUS faculty who had been his PhD students. It is not an exaggeration to say that Gross (and Roger Howe of Yale University) have now left an indelible mark on the mathematical research environment in NUS.

On behalf of *Imprints*, Y.K. Leong interviewed Gross on 12 March 2012 in IMS. The following is an edited and vetted version of the transcript of the interview in which he traced the path he took from Harvard back to Harvard where he has influenced the development of undergraduate education and played a key role in upholding its status as a world center in number theory. He also gives us an insight into the intellectual attraction of the oldest branch of mathematics (number theory) and conveys some of the excitement of being personally involved in the Birch and Swinnerton-Dyer conjecture.

**Imprints:** After your B.A. at Harvard University, you went to Oxford University for a Masters’ degree before returning to Harvard for your Ph.D. Is there any reason for this detour in your educational path?

**Benedict Gross:** Yes, in fact, I took 3 years between when I was an undergraduate and when I returned for my PhD. The reason was that I was not quite sure I was going to pursue a career in mathematics. I had some doubt and I was given a wonderful opportunity to go on a [Sheldon] travelling fellowship by my university (Harvard University) to travel in Asia and Africa. In fact, I came to Singapore in 1971 (when it looked a lot different than it does now). During that year of travel, I did a lot of reading of my own in mathematics and I decided I wanted to consider going into mathematics. Re-entering into the subject, I had a Marshall Scholarship to study at Oxford [University]. That’s where I went and got my Masters’ degree and at that time, I became very interested in number theory and I decided I wanted to return to the United States to get my doctorate and to go back to Harvard where I could work with my advisor John Tate.

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**I:** Was there anybody in Oxford University who may have had some influence on your thinking?

**G:** Well, it’s a very good question. One of the mathematicians who influenced me the most in my work is Bryan Birch, Professor at Oxford. The year I was doing my Masters at Oxford, Bryan was visiting Harvard University. So there was really almost no one there who was teaching number theory. I worked on my own with another student there who was an undergraduate in number theory. That was Andrew Wiles. We were both very lucky – in that last year, Michael Atiyah [Fields Medal 1966] arrived and gave a beautiful course on modular forms and Hilbert modular forms.

**I:** Is it the same Andrew Wiles [of Fermat’s Last Theorem fame]?

**G:** Exactly. We got to know each other when we were both very young. Then when he came to Harvard as an assistant professor we shared an apartment.

**I:** What made you choose number theory as your area of research? Who was your PhD advisor?

**G:** I was very struck by the beauty of the subject, and when I was in Oxford, Andrew and I had read an early form of an article by John Tate on the arithmetic of elliptic curves, where he sketched out what were the main problems. The remaining problems were very attractive. When I got to Harvard, I asked Tate whether I could work with him. He was such a terrific advisor.

**I:** If I’m not mistaken, the conjecture was based on empirical evidence, isn’t it?

**G:** Yes, I think it is a tremendous thing that for 50 years in number theory the computer is a tool. The computer can’t prove any of these things or conjectures but it can establish patterns beyond any reasonable doubt, and Birch and Swinnerton-Dyer did a lot of computer experimentation before they saw the right guessestimate. Another conjecture in number theory on which I worked is the beautiful unsolved problem of Harold Stark on $L$-functions at zero. He also did a lot of computational work to provide evidence for these conjectures.

**I:** If I’m not mistaken, the computational work of Birch and Swinnerton-Dyer was done before the advent of the modern PCs.

**G:** Absolutely. They did their work 50 years ago and Swinnerton-Dyer was doing all the programming in the very basic machine language on Cambridge computers. I remember when I was young in Oxford I did some computations on some work I was doing, involving punching cards, putting them in a hopper and coming back again later and finding them on card label 25. I don’t think that young people nowadays can really appreciate how complicated computing was at that time.
I: The computer is ideally suited for number crunching. How much has computational methods and ideas contributed to the solution of long-standing problems in number theory?

G: I’m not sure they actually contribute to the solution. Sometimes they can eliminate boundary cases. If you prove that everything works for every number bigger than this, the computer can check it up to that point. But they have been wonderful tools for experimentation and for discovering patterns.

I: Do you still use computers?

G: Not really. I have so many collaborators who are so much better at programming than I do. I periodically turn to the computers and ask questions. I used to program in languages like Algol, Fortran and C, and now I refer computational questions to young colleagues like Noam Elkies and William Stein, who are much more talented in the area.

I: Number theory could be considered to be one of the areas of pure mathematical research par excellence. Have you considered applying your expertise to more “concrete” problems in other disciplines or do you believe that, as G.H. Hardy did, that number theory should be studied for its own sake?

G: I think Hardy will be surprised to find out how many applications the most abstract number theory has nowadays from public key cryptosystems to numerous issues of communications. On the other hand, you have to know where your skills lie. I feel that I am more suited to work on the theoretical aspects of the subject. I have enormous admiration for people who are able to apply these ideas in number theory to real world problems.

I: I think Gauss was a great number theorist and crumper of numbers.

G: Yes, he was a great crumper of numbers. I once saw some note cards in Gauss’s papers, giving evidence for the prime number theorem. Gauss compared his computations of primes from a to b with the integral of $\frac{dx}{\log(x)}$ from a to b. He was a great believer of computation to acquire evidence.

I: I believe that Professor Jean-Pierre Serre has a long-standing association with Harvard and visits Harvard very regularly. How did this association arise?

G: You are right. I think Serre visited Harvard over 30 times. I was very fortunate that in the 4-year period when I was a graduate student he visited Harvard twice, once to give a talk on finite groups, and once to give a course on $L$-functions and analytic number theory. His association with Harvard predates my time there. He started coming certainly in the 60s. He had many friends there; my advisor John Tate was his close friend, Raoul Bott also a good friend. I think Serre liked the atmosphere at Harvard; it was a little more informal than the atmosphere at Collège de France where he lectures every year. He could try out a course on us and then have it perfectly polished when he went back to Paris. It was just incredible having him in Harvard because as a visitor he was available to all of us. I met Serre two weeks after I arrived at Harvard as a graduate student. He found me in the library and asked if I would like to play ping-pong. I played ping-pong many times in my life but he never let me win. After we finished playing ping-pong he asked me what I was thinking about. Although I was studying Weil’s [André Weil (1906-1998)] work on curves over finite fields, he promptly gave me an hour’s lecture on it. Just incredible for students at Harvard to have him visit [Harvard].

I: I believe somebody said that he suffers fools. Is that true?

G: He certainly suffered me. I got my PhD thesis problem out of a remark he made. So I’m very indebted to him. And he has continued to visit during the time I was faculty at Harvard.

I: Are you on Facebook?

G: I used to be on Facebook when I was a dean at Harvard and I communicated with students that way. But after that I got myself out.

I: I think Serre himself is on Facebook.

G: I can believe it. He communicates with everyone.

I: Except for an initial period of 7 years at Princeton and Brown University, you have been at Harvard University since 1985. Is this due to an intellectual attachment or a loyalty of some kind?
G: Well, there are two different things. First, Harvard is a wonderful place to do research in mathematics. It has been a real center in number theory since John Tate arrived there, and Barry Mazur [arrived]. That was a great tradition to be able to follow but also it is a small department. I have many colleagues who are not themselves number theorists. So I had a chance to work with, like Mike Hopkins in homotopy theory and Curtis McMullen in dynamical systems. The students are just fantastic. As a research environment it is second to none. I also felt that the university has done a great deal for me as an undergraduate student and as a graduate student. They gave me this wonderful travelling fellowship after I graduated. So in that sense teaching there is repaying a debt.

I: You are also now a professor at Harvard College, isn’t it?

G: Yes.

I: What’s the difference between Harvard University and Harvard College?

G: Harvard College is the older institution. That was founded in the beginning of the 17th century. Initially it was just a very small school to train people to become theologians in the Commonwealth of Massachusetts. Since the 19th century it has added on a medical school, a law school, a business school and a school of public administration. It is a vast research university and surrounds this ancient college. I was always very involved with the College. That’s where I was Dean. I feel that with all the wonderful things at Harvard – professional schools, graduate schools, research centers – it still remains a college.

I: You were Dean of Undergraduate Education from 2002-2003 and Dean of Harvard College from 2003-2007. What were some of the most memorable experiences from those years of deanship?

G: The main thing we accomplished is that we went through a curricular review of the college and looked at all aspects of the education we give to undergraduates. We revised the program in general education, changed the kind of concentration, even changed the university calendar which had been in place since 1838. So I was very happy to be involved in all that activity. There was a lot of amusing things. That was the time, for example, when Mark Zuckerberg, who founded Facebook, was an undergraduate at Harvard. I think it was an exciting time to be dean.

I: If I remember correctly, not many mathematicians become deans. How did it happen to you?

G: I remember when I started, the dean of Yale College who later became President of Duke University (Richard Brodhead) said to me, “What are you doing?” I didn’t know that much about the faculty – the politics, the things that the dean usually does – but I was very interested in the curriculum review. Having been an undergraduate there myself I just felt that certain aspects of our system were not really working for the undergraduate. In this case, I tried to do something about them.

I: You have had more than 30 graduate students which is a very high number by any standard. What is it about graduate supervision that gives you the greatest satisfaction?

G: The students I have had at Brown and Harvard are all exceptional students. They do not require much supervision. I often think the best thing I have to do is to get out of their way. The most satisfying thing is to see them go on and become creative research mathematicians in their own right. For example, two of my students are now at the National University of Singapore – Wee Teck Gan and Hung Yean Loke. Their work is wonderful. When I look at it, I think I could never have done anything like that. When you see your students go beyond you, that’s a real satisfaction.

I: What is your advice to students who are keen in doing research in number theory?

G: I think that it’s very important to try to understand where the modern subject comes from. Number theory changed a lot in the last 50 years. You can just start at the point of current knowledge and learn all about theory but it really doesn’t make that much sense unless you learn the work of the great number theorists in the 18th and 19th century. For me, I found in my work, the best thing is to place it in the context of the history of the subject, and I advise students who are interested in going into number theory to try to appreciate some of the historical context.

I: I think number theory involves a lot of complex analysis, doesn’t it?
G: It involves everything. I think that's the most exciting thing. I remember once I was on leave and was visiting Paris. I went to Serre's course in Collège de France and he was speaking about local height functions which subject I knew absolutely nothing about, and I remember him talking about local heights over the reals involving Green's function and partial differential equations. I thought to myself, “Fortunately that's something that I will never ever have to use and I don't have to know these at all.” Of course, two years later when I started working with Don Zagier on heights of Heegner points, that was exactly what I needed to know. It's just a wonderful area of the subject; it borrows all kinds of techniques in mathematics. That's why it has been so central and that's why it has interested so many great mathematicians of the past.

I: But there isn't that much geometry in it, isn't it?

G: Now there is because people have gone beyond the study of number fields which are in some sense zero-dimensional geometric objects to the study of algebraic curves over number fields which are one-dimensional and even higher dimensional varieties over number fields. So a certain amount of geometric intuition is one of the ingredients you need to bring into the subject. I think it is also great in that it allows you to collaborate with people on the boundary of the subject. For example, for the algebraic geometry that I worked in I have been very fortunate to collaborate with one of my oldest friends in college, Joe Harris, who does complex algebraic geometry. His area interacts with number theory and other fields, which gives real opportunity to learn other fields.

I: What about probabilistic methods?

G: That has become very, very exciting now. There’s a whole subject my colleague Barry Mazur calls “arithmetic statistics”, and they are working with a brilliant young mathematician who is now a faculty member at Princeton and who studied with me as an undergraduate in Harvard – Manjul Bhargava [Fields Medal 2014]. He brought in all kinds of fantastic methods into the subject to study averages in the sense of probabilities or densities. Of course, that's always been in the background of number theory because problems about prime numbers and their distributions and densities always have a probabilistic aspect. I think it has come to the fore now with the work of a lot of young mathematicians.

I: I should have asked you earlier on as part of an earlier question. What are the prospects of the Birch and Swinnerton-Dyer conjecture being solved in the next ten years?

G: I'm hesitant to say. I think the only thing you can say about the future of number theory is that it will be interesting and it will be surprising. Things that look so hard one day will be solved the next day. We haven’t really made any substantial progress on the Birch and Swinnerton-Dyer conjecture, in my opinion, for almost 25 years now. On the other hand, Manjul Bhargava and his collaborators have now made a lot of progress on the study of the Mordell-Weil [Louis Joel Mordell (1888-1972)] groups on average, not for a given elliptic curve but over all elliptic curves. So maybe a new point of view will come into the subject and that will immediately lead to progress.

I: I was thinking as a layman. Fermat’s Last Theorem arose as a consequence of a larger theory, is that right?

G: That’s correct. That was what was so exciting about the proof that came out. It didn’t just solve that one equation but it involved learning an awful lot about the relationship between elliptic curves and modular forms. That opened up a whole new area of mathematics. So that's what is so great about these classical problems. It is not just the solution that is important; it's the new methods that are brought into the subject.

I: Could it be that the Birch and Swinnerton-Dyer conjecture may be a consequence of some bigger theory?

G: That’s a good question. People have proposed a grand generalization of the Birch and Swinnerton-Dyer conjecture but we haven’t made any progress on the grand generalization than we have on the original conjecture. But perhaps that will be the way it will be resolved. All I can say is that it will be beautiful and it will be surprising.

I: I believe that you were one of the earliest and few mathematicians who contributed to publicizing mathematics by acting as a consultant in the 1980 movie (“It’s My Turn”). Do you think that the situation is now better in the sense that more mathematicians are now willing to reach out to the public?
G: First, I should clarify that when I worked on this movie it wasn’t because I was interested in publicizing mathematics. I was an instructor in Princeton when the woman [Eleanor Bergstein] who wrote the script in the movie was reading it out. She wanted someone in the department to take a look at it to see if the mathematics in it was accurate. The mathematics in it was horribly inaccurate and so she asked me if I would make it into something reasonable. I did and wrote an initial scene where the professor of mathematics, who was played by Jill Clayburgh, gives a proof of the snake lemma at the blackboard. After doing that they involved me in the coaching of the actresses. So that was a fun experience, but I don’t think it gives a very realistic picture of life as a mathematician. On the other hand, I’ve been very impressed with colleagues who now go out and give talks at public forums and have written books on mathematics. I think we need to reach out to the general public more. There’s a general misconception with these people that mathematics is all finished because all they study in school is algebra and calculus – that was resolved by the beginning of the 18th century. We need to communicate to the public the fact that it is an incredibly active field with advances occurring all the time. Our colleagues in the other sciences seem to do this quite well. Physics, astronomy, biology are always in the papers. I think we can make a better case. I encourage colleagues; I now try to give a lot of public talks. I gave one [talk] two months ago in Mumbai and just try to make the case that it is an exciting field and that young people should consider it. It’s valuable for society in the long run. Of course, all art and creativity is wonderful for society to support but mathematics is not just beautiful art and creativity; it leads to some terrific benefits to the society. We can’t predict what they are but it always has that potential.

I: Could the reluctance of mathematicians to do this kind of outreach be due to their inability to communicate with the public?

G: It could be. Look, most of us go into mathematics (I know I did) to get yourself out of the real world. I mean, if you work in mathematics you have this wonderful and beautiful imaginary world in your mind. It’s hard to translate that back to the world that you just try to get out of. But I think we need to make that effort and some people are going to be better at it than others, and others would be just wise to stick to their research. A lot of people can get out there whether it is in the newspapers, whether it’s in the lectures, whether it’s in the movies, or whatever.

I: Some people seem to think that mathematics is kind of esoteric and monastic.

G: In some sense they are right. I think all of us love moments when we can just retreat to our own desks and think quietly about the problem. In fact, that’s how progress in mathematics is made. We interact a great deal if we are attending a wonderful workshop like in IMS [Institute for Mathematical Sciences] when I had a chance to speak with some of the leading people in the field and bounce ideas off them. Now I’m looking forward to going back and sitting at my own desk and trying to think about them quietly. I think we need those two aspects to have real progress in mathematics.
Yum-Tong Siu: Hongkong-Princeton-Harvard, a path of several complex variables

Interview of Yum-Tong Siu (Harvard University) by Y.K. Leong

“Tell me why should symmetry be of importance?”


Yum-Tong Siu made fundamental contributions to complex analysis, algebraic and analytic geometry, and complex differential geometry.

Siu was born in Guangzhou in the midst of World War II, and his father moved the family to Macau and later, Hong Kong, in the aftermath of the communist takeover of the mainland. He received a classical Chinese education from Pui Ching Primary School and Pui Ching Middle School in Macau. It was this early exposure to classical education that sparked his interest in Chinese literature, philosophy, and history, an interest that has turned into a life-long passion. Later he transferred to Pui Ching Middle School in Hong Kong. Every day before taking a ferry to go from Hong Kong Island to the school in Kowloon on the other side of the harbour, he took an early walk with his father, a textile merchant. It was during these walks and over dim sum in a restaurant that his curious father would ask him to share information about academic subjects and current events. Shortly before Siu’s graduation from high school, he almost had to transfer to a public school with lower tuition because of setbacks in his father’s business. Only a timely reduction in tuition fees granted by Pui Ching enabled him to continue and to graduate from high school. (Pui Ching Middle school has an impressive record of students who subsequently went on to achieve excellence in engineering, mathematics and the natural sciences.) Though financially unable to go to the United States for further studies, Siu won a government scholarship to study in the University of Hong Kong, where he excelled in competitive swimming, obtained a BA in mathematics and met his wife-to-be. In a somewhat unexpected way, he went from the University of Minnesota (for a Master’s degree) to Princeton University (for a PhD). He taught briefly at Purdue and Notre Dame before moving on to and rising quickly on the academic ladder at Yale University and Stanford University. In 1982 he went to Harvard University and became the William Elwood Byerly Professor ten years later.

In a long and distinguished career, Siu was for a quarter of a century a world-renowned leading figure in complex analysis, having (according to the citation for his award of the Bergman Prize in 1993) “settled a long and impressive list of problems and opened new directions of research through highly imaginative and original use of sheaf theory, partial differential equations and differential geometry”. The earlier part of his work (in the theory of several complex variables) is on the theory of extension of coherent subsheaves (joint work with G. Trautman) and coherent sheaves, the structure and extension of closed positive currents and the extension of meromorphic maps across subvarieties.

His later research is conducted at the interface of complex analysis, differential geometry and algebraic geometry and has resolved numerous outstanding problems and conjectures: among others, his work on geometric strong rigidity, his joint work with N. Mok and S. K. Yeung on super-rigidity, Frankel conjecture (with S.T. Yau, Fields Medal 1982), effective results in algebraic geometry such as the freeness part of the Fujita conjecture (with U. Angehrn) and the effective Matsusaka big theorem, the conjecture of the deformational invariance of plurigenera, and the conjecture on the hyperbolicity of generic hypersurfaces of sufficiently high degree in complex projective space.

In addition to a large research output, he has written several
monographs on the topics of his research interest. He has served on the editorial boards of *Annals of Mathematics* and *Journal of Differential Geometry*. He has been invited to many universities throughout the world and was invited to give talks at the International Congress of Mathematicians three times, two of which were plenary lectures. His awards include the Bergman Prize, Guggenheim Fellowship, Sloan Fellowship and honorary doctorates from the University of Hong Kong, University of Bochum (Germany) and University of Macau. He is a member of the American Academy of Arts and Sciences, U.S. National Academy of Sciences, Academia Sinica (Taiwan), Corresponding Member of the Göttingen Academy of Sciences and Foreign Member of the Chinese Academy of Sciences. He is actively involved in scientific committees to promote and maintain professional awareness in mathematics. In particular, he has travelled frequently to various countries in Asia, contributing to their efforts in raising international interaction. He was on the Advisory Committee for the Shaw Prize (established by Hong Kong philanthropist Run Run Shaw) in Mathematical Sciences. He was Chair of the National Committee for Mathematics (National Research Council, National Academy of Science). He was a member of the Scientific Advisory Board of the Clay Mathematics Institute. Currently he is on the Scientific Advisory Board for the Institute for Mathematics Sciences (IMS), National University of Singapore (NUS) and for the Institute of Advanced Studies, Nanyang Technological University (NTU), Singapore.

Siu has a long association with NUS. He has been a familiar visitor to the Department of Mathematics since the era of the University of Singapore and is a well-respected friend of the faculty of NUS and NTU. Since 2009 when he became, first as a member of the Scientific Advisory Board of IMS and subsequently as its chairman, he has been visiting NUS annually. He was an invited speaker for the IMS program on Complex Geometry (22 July - 9 August 2013). On behalf of *Imprints*, Y.K. Leong took the opportunity to interview him on 1 August 2013. The following is an edited and enhanced version of the transcript of the interview in which he traced his path from Hong Kong to Harvard, reminisced about his formative years in Minnesota and Princeton, and talked about doing research, teaching students and his projections of mathematical developments in Asia. It also gives us a glimpse of the influence of private Chinese school education on the post-war generation of mathematicians and scientists from Hong Kong.

*Imprints*: You went from BA (Hong Kong University or HKU) to PhD (Princeton University). Please tell us how you came to take this path?

*Yum-Tong Siu*: At the University of Hong Kong I majored in mathematics. It was offered inside the Faculty of Arts.

*I*: Isn’t it offered in the Faculty of Science?

*S*: Yes, in those days, mathematics was offered in both the Faculty of Science and Faculty of Arts. In the former, mathematics must be combined with another science subject such as physics and chemistry. It was only in the Faculty of Arts that one could major in mathematics alone. In the old British system, a major in just one subject meant taking 9 papers in the final examination. I took 9 papers in mathematics. At that time I was thinking of going to Germany for further study. As preparation, I took German in my first year. The instructor was the cultural attaché from the German consulate in Hong Kong. The class was very small as not too many HKU students were interested in German. The main job of the instructor was to recruit Hong Kong students to eventually study in Germany. After graduating from HKU, I was offered a German Academic Exchange Service or DAAD scholarship. (DAAD stands for Deutscher Akademischer Austauschdienst). However, my dream of studying in Germany was thwarted. When I contacted the German university that I was interested in, I found out that the HKU system and German university system were not compatible. I had to start my study in Germany from the level of the *Arbitur* which is the high school graduation exam in Germany. If I thought I already had the education up to the level of their *Vordiplom*, I was allowed to take the exams right away. But I figured I wouldn’t pass right away their *Vordiplom* exam and it made no sense to me to start from the level of high school graduation all over again. I thought I should study in the United States but I had no application form. By that time, it was rather late in terms of applying to graduate study elsewhere. I am talking about pre-computer days. All correspondence with overseas universities had to be carried out by snail mail, not email or even fax. Fortunately, my HKU classmate Tsit Yuen Lam had several application forms left. He had already been accepted by Columbia University. (Lam is now a professor emeritus in the Department of Mathematics at UC Berkeley.) I took the application forms to show my professor (Professor Yung-Chow Wong) and asked for his advice. He recommended...
the University of Minnesota because Eugenio Calabi was there. So I applied and was accepted. I went to Minnesota, studied with Calabi, and earned a Master’s degree before deciding to leave. After that year, Calabi left Minnesota and went to University of Pennsylvania. I think he stayed there till he retired. Another reason for my not wanting to continue with my PhD study in Minnesota was the brutal winter weather there.

I: Why didn’t you go to Pennsylvania?

S: While at the University of Minnesota, I actually had trouble understanding Calabi’s lectures. I was his only student in class; the rest were professors and postdocs. I was not really the targeted audience but then outside of class he helped me tremendously. Whenever I asked him questions privately after class, he would explain a lot of things to me. I really learned a lot from him; he was extremely knowledgeable. Still, I felt that his style was very different from what I was used to in Hong Kong. He had a really panoramic view, which certainly helped me conceptually, but he often skipped technical details. At that time I felt a need for more foundational content, so I decided to go to Princeton instead of following Calabi to Pennsylvania. The Princeton style was also different. There were no basic courses in Princeton and the professors would only lecture about their own work, sort of seminar style. Students of comparable background would form small study groups in order to learn the basic material together. I worked with three fellow students.

I: How did you come to choose functions of several complex variables as the area of research for your PhD?

S: When I went to Princeton, besides student seminars, I also attended a seminar run by Bochner [Salomon Bochner (1899-1982)]. Gunning [Robert Clifford Gunning] was also in the seminar. I also took a course with Gunning. I found the subject very interesting and Gunning was also a very good and encouraging teacher. He had many students at that time. Having also learned from the other students of his, gradually I was drawn more into the field of several complex variables. During my stay at Princeton (1964-1966) the field was blessed with many breakthrough activities, for example, exciting results obtained by Grauert [Hans Grauert (1930-2011)] and others in Germany and France. At Princeton, Gunning was lecturing from material in the book ‘Functions of Several Complex Variables’ that he wrote with Rossi [Hugo Rossi]. I found the subject fascinating. The material available in book form was presented well. My fellow students were helpful. It was in this academic atmosphere that I chose several complex variables as my research interest.

I: Were you the only student from Asia at that time?

S: I was not the only Asian mathematics student at Princeton. The other student was K.Y. Lam [Kee Yuen Lam], the older brother of T.Y. Lam. K.Y. Lam, a student of Professor Steenrod [Norman Earl Steenrod (1910-1971)], was a couple of years ahead of me in HKU. Whereas T.Y. Lam is in algebra, K.Y. Lam is in topology. Yes, there were only two Chinese mathematics graduate students at the time, but Wu-Yi Hsiang, a visitor to the Institute of Advanced Study at Princeton, would show up in seminars of Princeton University. Yiu-Hung Chan and Hung-Hsi Wu were also around at the Institute for Advanced Study. Both Hung-Hsi Wu and Yiu-Hung Chan were from my high school, four years ahead of me. Yiu-Hung Chan later quit mathematics or maybe just quit research in mathematics. He went back to Hong Kong to teach but was not active in research anymore. Hung-Hsi Wu later became Professor of Mathematics at UC Berkeley.

I: He’s in differential geometry, I think.

S: Yes, he’s now retired from Berkeley.

I: He’s very keen in education.

S: Oh, you know him well.

I: In fact, I interviewed him two years ago. He’s now almost totally engaged in training teachers.

S: Yes, he has always been interested in teaching and then got more and more involved in teaching, trying to reach out to those at the fundamental level, like even primary school pupils.

I: You have been on the faculty of Harvard University for the past 30 years or so. What is it that makes you so attached to Harvard?
S: Well, Harvard has both excellent students and very distinguished faculty members. It’s thus a very exciting place. I first taught at Purdue for one year and then 3 years at Notre Dame and then I taught at Yale for 8 years. Actually I left Yale because of the weather. There was a big snow storm, blizzard of ’78, so I left in ’78 to go to Stanford. Weather was not a consideration, however, when I went to Harvard. A main reason for my departure from Stanford was my wife’s career. She finished her doctorate in social work (Columbia University) in 1982 and had difficulty finding social work faculty position at or near Stanford. I received offers from Harvard, MIT and Princeton. The greater Boston area with its over 50 colleges would provide more job opportunities for my wife. I chose to accept Harvard’s offer. After a year, my wife did find a faculty position at Wheelock College of Boston. She taught there for 22 years and was Chair of the Department of Social Work for ten years. She chose to retire in 2005. Well, we have lived in the Boston area for over 30 years. I think we have made the right decision, not only professionally for me but for my family as well. Harvard students are extremely good because Harvard remains a popular choice for the most talented students. For example, freshmen enrolled in Math 55 (a special course for the most talented students) are a pleasure to teach. The main part of the course consists of challenging weekly assignments. I taught this course a couple of times and found my task to be time-consuming but highly rewarding.

I: I remember reading some comments from the blog of some of your students that they find it to be a tough course.

S: I think it’s not only mine. That particular Harvard mathematics course is meant to be tough. The choice of material for the course is to serve two purposes. One is to challenge students. Another is not to discourage them. One seeks a balance between the two. I believe that gearing the course content to the level of the students is true of all courses.

I: Probably your expectations were higher.

S: Actually it depends on the class composition of a given year. Math 55 is a completely dynamic situation. In this only-one-of-its-kind course, you sort of look at what the students in the class have accomplished, and based on feedback from students, you then channel the students to more and more difficult material, with the goal of providing maximum challenge without discouraging them. To me, it is a good thing if students find the course to be challenging. After all, it is supposed to be tough! Student opinions about the course can be short-term (immediately after finishing the course) and long-term as they reflect on the learning experience much later. I know most Math 55 students, despite the hard work involved, enjoyed both the learning outcome and the process. This unique course has become an important tool for Harvard to recruit gifted students who aspire to be mathematicians.

I: I believe that you have settled a number of outstanding conjectures in analysis and algebraic geometry. What is the style of your research? Do you pick specific difficult problems to work on or do you try and relate and synthesize different concepts and theories?

S: For research it’s hard to fully know beforehand the nature of the problem and likelihood of successfully solving it. It’s not that I pick this problem, I want to solve it because, at least in my own case, my work depends so much on what other people have done in the past. My view of my research is that it is akin to putting together a jigsaw puzzle. Sometimes you see already quite a number of pieces in place and you think you have some way of putting in more. Whether I pursue a research topic depends on what is interesting, what is available, and what is feasible at that time. It’s just like investment. You do several things at the same time. There is no telling which one would bear fruit. So when I work on things, it depends on what is interesting but it also depends on my background. At the inception of my career, I started out in differential geometry. As I told you, I went to Minnesota and learnt from Calabi. Later when I went to Princeton, at that time the work of Grothendieck [Alexander Grothendieck (1928-2014), Fields Medal 1966] was fashionable, so I learnt algebraic geometry, all the language and so forth. Later I picked up several complex variables which interface with several areas, including the methods of algebraic geometry and methods of PDE (partial differential equations). At that time the method of the so-called complex Neumann problem or \( \overline{\partial} \)-estimates became available, as did methods of integral kernels and methods of global differential geometry. Thus, I worked at the interface of these areas and on quite a number of conjectures. I was lucky enough to solve some. Of course, we are all aware what the prevailing problems are. I simply choose the ones that I feel are feasible at that point, given my background.
and the available methods. I leave behind other problems on which I cannot launch a meaningful attack. That’s my way of approach to research. I suspect that probably most mathematicians do it the same way. In the interface or bridge between algebra and analysis, I usually use differential geometric, algebraic geometric and also PDE methods involving hard analysis.

I: Have you ever tried to do applications of what you have done to other fields?

S: You mean, that would affect the livelihood of people?

I: I mean, physics or . . .

S: Yes, actually some of the problems were originally proposed by people in other areas. But I actually never go directly to other areas to see how these things are being used. Now with globalisation not only geographically but also in terms of subject matter, research has become more interdisciplinary, resulting in more interactions among researchers. There are people who serve as bridges at various stages, some closer to mathematics, some closer to other fields. Many of the mathematical developments have been spurred by other areas. Some of my own work has been motivated by other fields. The field of differential equations certainly consists of practical aspects, and in geometry now physics plays a greater role.

I: A question about undergraduate curriculum. In the undergraduate mathematics curriculum, a course on real analysis (functions of a real variable) is usually followed by one on real multivariate calculus (functions of several real variables) whereas a first course on complex analysis is very rarely followed by one on functions of several complex variables. Is this due to pedagogical reasons or deeper mathematical reasons?

S: Usually there are two kinds of undergraduate curriculum. The first is of a service nature, that is, a curriculum serving people in other fields. For various reasons, some students in other fields need to learn mathematics at an undergraduate level. The curriculum varies according to the kind of mathematics they need. Sometimes they need only the results without the theory, sometimes a combination of theory and results. The other curriculum is for our own mathematics students. We are using this kind of curriculum to train a new generation of mathematicians who will replace us at some point in the future.

In the service curriculum, the main purpose is to help other fields, so we just do whatever people in other disciplines want us to provide. Although we certainly could provide suggestions and feedback to the other departments about course content, our primary role is to serve them. In the other situation where we are grooming future mathematicians, the curriculum content beyond the basics depends very much on faculty members of a particular mathematics department. After mastering foundation materials, second-stage mathematics majors are steered towards courses and seminars offered by existing faculty members. The curriculum very much depends on the composition of the department and the professional bias of faculty members. When I teach, of course, I would like students to understand what I consider interesting, so I try to give more information about several complex variables. Such knowledge of several complex variables would help students gain a different perspective of one complex variable.

After so many years of teaching, I have concluded that it’s important to achieve a good balance between abstract rigorous reasoning and concrete mathematical structures such as computational formulas. Students exposed to an overemphasis on the abstract side may only know how to make logical arguments but lack enough knowledge about structure, actual formulas, and specific examples. Interestingly, as I become older, I tend to favour knowledge about examples of actual structures or formulas. I believe that’s what mathematics is. Of course, it’s always good to know the general theory but then the theory is, to start with, built up from specific examples. Being able to work with specific examples would lead to a true understanding of what is really going on and increase the probability for fruitful research.

I: Results in complex dynamics, such as fractals, often give rise to and are conversely suggested by computer simulation. Has the computer played any role in the theory of functions of several complex variables?

S: Yes, as you said, in the case of complex dynamics, the computer is of great benefit. It helps in performing computations. In the past, we did long computations by hand, which was not only time-consuming but also prone
to making mistakes. Computers in a way are more reliable provided that the programme is correctly written. From the computer, one can quickly obtain a lot more examples, which are presented well visually, sometimes with colors. These examples may provide motivation to go forth in one’s research. But there is a caveat. Even with the use of computers, in the final analysis, you still have to get back to pure logical reasoning – mathematically rigorous reasoning.

I: Did you use the computer for your research?

S: I did it a number of times for checking computations, but I trust my own logical reasoning more. At one time when I computed curvature, especially with the situation of symmetric Riemannian manifolds, I did use the computer to help me guess what the final result is likely to look like. But then eventually I still had to do it by hand to be completely satisfied that there were no errors. One reason is my lack of trust in my own ability to program and, more importantly, to debug the programme.

I: You have spent many years lecturing and sharing your experiences with mathematicians in countries like China, Taiwan, Hong Kong, South Korea and Singapore among others. What are some of your most memorable experiences of your visits in Asia?

S: First, let me say that recently there has been a proliferation of activity in terms of conferences and special programmes in all these countries you mentioned. This development is very exciting, providing a lot of opportunities for young people. One difference between young people here and the United States and Europe is that Asians are generally more hard working. It’s just the way they are brought up or the way the education system works, I think. In some ways, it is more satisfying for me to work with Asians because of their work ethics. They are simply more willing to put in effort. On the other hand, Asians out of their respect for elders may tend to discuss mathematics less directly. Perhaps, this impression is due to my usually brief stays in Asia. Even though I have made many visits, the duration of each visit is not long enough to interact in depth with young mathematicians. Or perhaps the age difference has put some distance between them and me. The interactions between them and their local senior mathematicians may well be different, resulting in a different level of discussion.

I: Maybe they are differential towards their elders.

S: Yes, I believe that may be one of the reasons. Also it’s changing. It’s good to see a really international mix. You also see here in the workshops and programmes a lot of mathematicians from Europe and North America. Eventually everything is going to even out. Besides, there are more Asian students, postdocs and mathematicians visiting Europe and North America. With the kind of funding Asian countries are putting into mathematical research, the rate of increase in research will certainly far outpace what is currently going on in Europe and North America.

I: Now that the last ICM [International Congress of Mathematicians] 2010 was held in India and the coming ICM 2014 will be held in South Korea, what are your expectations of the next phase of mathematical development in Asia, in general, and East Asia, in particular?

S: Oh, I think the pace will pick up faster and faster. You also see the number of institutes and meeting venues springing up everywhere in Asia. And globalization is really accelerating. Besides the examples you just cited of the ICM in India, there was also the ICM 2002 in China. There is simply more and more activity at the international level in Asia. I believe that globalization will make the whole world more homogeneous so far as mathematical research activity is concerned. Such a development is not only inevitable but desirable as well.

I: Do you think that at the regional level mathematical research in Asia-Pacific will be comparable to that in Europe?

S: Of course, Europe has a really long history. It takes time to develop a mathematical tradition and provide role models to students. For example, in China for a long time, the development was more in geometry and analysis because of their work ethics. They are simply more willing to put in effort. On the other hand, Asians out of their respect for elders may tend to discuss mathematics less directly. Perhaps, this impression is due to my usually brief stays in Asia. Even though I have made many visits, the duration of each visit is not long enough to interact in depth with young mathematicians. Or perhaps the age difference has put some distance between them and me. The interactions between them and their local senior mathematicians may well be different, resulting in a different level of discussion.

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I: Can I be a bit more specific? I think Hong Kong has produced a number of very distinguished mathematicians during your generation.

S: Actually from my own high school I could count quite a number of people. For example, Hung-Hsi Wu, myself, S.T. Yau [Shing Tung Yau], S.Y. Cheng [Shiu-Yuen Cheng], Lawrence Ein, Kai Yuen Hu (who was on the faculty here and retired a number of years ago), Pit-Mann Wong, Bun Wong. What is unique about Pui Ching Middle School in Hong Kong is that it is not only private but also uses Chinese as a medium of instruction, unlike the mainstream Anglo public and private schools in Hong Kong. I came to Hong Kong after the communists took over and established the People's Republic. Many in the intellectual community in China also relocated to Hong Kong. Their academic degrees might have been earned in China and in countries which are not part of the British Commonwealth (England, Canada and Australia). As a result they were considered not qualified by the British government in Hong Kong to teach in Hong Kong. Those who could not find desirable teaching jobs in the Anglo schools ended up teaching in Pui Ching and similar schools. As a private school, Pui Ching did not have to meet the hiring criteria set up by the government. I consider it a blessing to have had very good teachers, who not only imparted their wealth of knowledge but also their worldview. High school students are usually quite easily influenced by good teachers. I was one of them. My teachers attempted to motivate us in an interesting way by using as role models former Pui Ching students who have gone to study in the United States. Because Pui Ching school was not part of the mainstream, its graduates usually could not directly go to the University of Hong Kong for their college education. I and a couple of my classmates were exceptions. Some of my classmates went to the United States right after high school graduation, eventually ending up in institutions like Caltech and UC Berkeley. The teachers always cited Pui Ching alumni as examples for current students to follow, pointing out their areas of study, namely physics, mathematics and engineering. A lot of my classmates actually ended up as engineers.

In those days, the winning of the Nobel Prize by Lee Tsung-Dao and Yang Chen-Ning was a big event. Also the results and developments in physics were very exciting to me and my peers. I remember clearly the incident of Tsien Hsue-Shen [Qian Xuesen (1911-2009), aerospace scientist, “Father of Chinese rocketry”]. He went back to China and he was allowed to leave the United States in exchange for some airmen who were held captive by the North Koreans after their planes were shot down. When I was young, I read news about this exchange and saw photos of Tsien Hsue-Shen and his family (two children and his wife) crossing the Lo Wu Bridge as the airmen walked across the bridge from China into Hong Kong. Students in my high school in those days were all thinking about mathematical sciences, engineering and similar fields. In contrast, the mainstream schools in Hong Kong produced students for the University of Hong Kong who, upon graduation became educational officers, administrative officers and medical officers to help the British govern Hong Kong. Their career paths were totally different from the kind of career paths which I and my high school classmates took.

I: It's rather strange because you would expect them to go to England, right?

S: Yes, but then we were not in the mainstream because we were in a private school that used Chinese for instruction. We had no particular identification with the British! Students from Taiwan also went to the United States but only for graduate school because at that time they had to finish their military service first after completing their undergraduate degrees. So the phenomenon I described represents a unique situation within a very specific timeframe and political context.

I: Many, if not most of, the programmes of IMS during the first 10 years of its operation, tend to be of an applied nature or closely related to applied fields. (The previous director Louis Chen is a probabilist.) Now that both the present director (Chong Chi Tat) and the new SAB Chairman (namely yourself) are “pure mathematicians”, will there be a slight change in emphasis, if not direction, in the offering of IMS programmes in the future?

S: I don’t think so. Roger Howe, the chairman before me and my former Yale colleague, is a pure mathematician. The then director Louis, of course, is a probabilist. However, if you look at the current advisory board, you can see that most are applied or related to applied areas. Douglas Arnold (from Minnesota) was the president of SIAM and certainly would be applied. Then you have Fan Jianqing (he's professor of finance), very applied. Then Wolfgang Hackbusch from
Max Planck Institute in Germany, he’s also more in PDE, numerical analysis. Of course, there’s Hugh Woodin in logic, and myself in pure mathematics. Louis Chen still sits on the board, as does Chong Chi Tat. I think it’s a good balance and a continuation of what has been working. I succeeded Roger Howe as Chair of SAB. I assume that I am Chair because I’m so much older than the others. The field is not the primary consideration when selecting the SAB chair; age probably is. In summary, I don’t think there’s going to be any change in the direction of IMS. We always look for the most exciting areas in mathematical sciences, whether pure or applied probability, PDE, numerical mathematics, finance, computing and so forth.

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The Institute for Mathematical Sciences (IMS) of the National University of Singapore (NUS) invites submissions of pre-proposals for April 2018 to March 2019 from researchers in the academia and industry in Singapore or overseas. The pre-proposals are for organizing thematic programs to be held wholly or partly at IMS with funding from the Institute. These programs, each lasting for one to six months, should have a well-defined theme or themes that are at the forefront of current research in an area of mathematical science or its applications, and should be of international interest as well as of interest or relevance to the local scientific community. Typically, a program should involve both international and local organizers. Pre-proposals on interdisciplinary programs in areas that interface with the mathematical sciences are welcome.

A soft copy of the pre-proposal should be sent to the Director of the Institute at imsdir@nus.edu.sg not later than 31 May 2016. The exposition of a pre-proposal should be aimed at the non-specialist and will be evaluated by a panel. Pre-proposals on interdisciplinary programs should indicate how the program would benefit the intended audience with diverse backgrounds and facilitate research collaboration.

Information on the Institute and its activities, as well as a detailed format for pre-proposals are available on the IMS website http://www.ims.nus.edu.sg/. Enquiries may be directed to imssec@nus.edu.sg.