Bootstrap Percolation: Critical Probability and Speed

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Bootstrap percolation, introduced by Chalupa, Leath and Reich in 1979, is one of the simplest cellular automata. Its most basic variant is bootstrap percolation on a (finite or infinite) graph $G$ with infection parameter $r$. This starts with a set $A_0$ of initially infected sites (vertices); at each time step $t = 1, 2, \ldots$ the set $A_{t-1}$ of sites infected at time $t-1$ gets updated as follows: sites infected at time $t-1$ remain infected, and every uninfected site with at least $r$ infected neighbours (i.e. at least $r$ neighbours in the set $A_{t-1}$) also gets infected. Thus $A_0 \subset A_1 \subset \cdots \subset A_t \subset V(G)$. We say that $A_0$ percolates (with parameter $r$) if eventually all sites get infected, i.e. $A_t = V(G)$ for some $t$; the time of percolation is the minimal $t$ with $A_t = V(G)$, and the speed is the number of vertices divided by the time.

Over the years, much attention has been paid to bootstrap percolation on lattices and grids in which the initial configuration is selected at random, with a certain probability $p$ of selecting each site, independently of each other. There are some deep results about the critical probability, the probability $p_0$ such that for $p < p_0$ percolation is unlikely, and for $p > p_0$ it is likely. In the talk I shall review some of the fundamental results concerning the critical probability, due to Aizenman, Lebowitz, Schonman, Holroyd and others, and will present a number of results I have obtained recently with Balogh, Morris, Duminil-Copin, Riordan, Holmgren, Smith and Uzzell about the critical probability and the speed of percolation.