Estimating time-varying causal effect moderation in mobile health with binary outcomes

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BariFit MRT

- A micro-randomized trial (MRT) to promote weight maintenance among people who received bariatric surgery.
- Data collected from:
  - Fitbit tracker (step count)
  - user self-report (weight, calories intake)
- mHealth intervention components:
  - daily step goals
  - actionable activity suggestions
  - reminders to track food intake
  - ...
- This talk: assess the effect of
Data in an MRT

- On each individual: $O_1, A_1, Y_2, \ldots, O_T, A_T, Y_{T+1}$.
- $t$: decision point.
- $A_t$: treatment indicator at decision point $t$.
- $O_t$: observation accrued between decision point $t - 1$ and decision point $t$.
- History $H_t = (O_1, A_1, Y_2, \ldots, O_t)$: information accrued prior to decision point $t$. 
Decision Points $t$

- Times at which a treatment might be provided
- Times that the treatment is likely to be beneficial
- **BariFit**: food track reminder may be sent every morning. $t = 1, 2, \ldots, 112$ (112 days)
Treatment indicator $A_t$

- Whether a treatment is provided at decision point $t$
- (What type of treatment)
- Here we assume binary ($A_t \in \{0, 1\}$)
- Randomization probability $p_t(H_t) := P(A_t = 1 \mid H_t)$
- BariFit: whether a text message of food track reminder is sent. $p_t(H_t) = 0.5$.
Proximal outcome \( Y_{t+1} \)

- Outcome measured after decision point \( t \) (assumed to be binary here)
- Something that the treatment is directly targeting
- BariFit: whether the individual completes food log on that day
- Note the subscript!
Observation $O_t$

- Observation accrued between decision point $t - 1$ and decision point $t$.
- $O_1$ includes baseline variables.
- **BariFit**: Fitbit tracker (step count) user self-report (e.g., weekly weight) baseline variables (e.g., age, gender)
Availability $l_t$

- Treatment $A_t$ can only be delivered at a decision point if an individual is available.
- Available: $l_t = 1$; unavailable: $l_t = 0$. $l_t \in O_t$.
- Safety and ethical consideration: e.g., an individual is unavailable for a physical activity suggestion message if she is driving.
- Treatment effect is defined conditional on availability. (later)
- BariFit: for food track reminder, individuals are always available.
- Availability is different from adherence!
Conceptual models

- Data: \( O_1, A_1, Y_2, \ldots, O_T, A_T, Y_{T+1} \)
- \( H_t = (O_1, A_1, Y_2, \ldots, O_t) \)
- Usually data analysts fit a series of models
  \[
  Y_{t+1} \sim g(H_t)^T \alpha + \beta_0 A_t,
  \]
  \[
  Y_{t+1} \sim g(H_t)^T \alpha + \beta_0 A_t + \beta_1 A_t S_t,
  \]
  \[
  \ldots
  \]

- \( g(H_t) \): summaries from \( H_t \); “control variables”
- \( S_t \): potential moderators (e.g., day in the study)
- \( \beta_0, \beta_1 \): quantities of interest
- ‘ \( \sim \)’: e.g., logit or log for binary \( Y \)
Goal

- Develop statistical methods to model and estimate the treatment effect
- Be consistent with the scientific understanding of the $\beta$ coefficients
- Use control variables $g(H_t)$ for noise reduction in a robust way
Potential outcomes

- To mathematize the problem, we use potential outcomes notation (e.g., Rubin (1974))
- Define $\bar{a}_t = (a_1, \ldots, a_t)$ where $a_1, \ldots, a_t \in \{0, 1\}$
- $O_t(\bar{a}_{t-1})$: $O_t$ that would have been observed if individual received treatment history $\bar{a}_{t-1}$.
- Similarly, $Y_{t+1}(\bar{a}_t), H_t(\bar{a}_{t-1})$
Causal excursion effect

\[ Y_{t+1}(\bar{A}_{t-1}, 1) \]
Causal excursion effect

\[
\frac{Y_{t+1}(\bar{A}_{t-1}, 1)}{Y_{t+1}(\bar{A}_{t-1}, 0)}
\]
Causal excursion effect

\[
\begin{align*}
E\{Y_{t+1}(\bar{A}_{t-1}, 1) \} \\
E\{Y_{t+1}(\bar{A}_{t-1}, 0) \}
\end{align*}
\]

- Contrasting two excursions: following \( \bar{A}_{t-1} \), then receive treatment \( (A_t = 1) \) vs. no treatment \( (A_t = 0) \) at time \( t \).
Causal excursion effect

\[
\begin{align*}
E\{ Y_{t+1}(\bar{A}_{t-1}, 1) | S_t(\bar{A}_{t-1}) \} \\
E\{ Y_{t+1}(\bar{A}_{t-1}, 0) | S_t(\bar{A}_{t-1}) \}
\end{align*}
\]

- Contrasting two excursions: following $\bar{A}_{t-1}$, then receive treatment ($A_t = 1$) vs. no treatment ($A_t = 0$) at time $t$.
- $S_t(\bar{A}_{t-1}) \subseteq H_t(\bar{A}_{t-1})$: a vector of summary variables chosen from $H_t(\bar{A}_{t-1})$.
- Effect is marginal over all variables in $H_t(\bar{A}_{t-1})$ that are not in $S_t(\bar{A}_{t-1})$.
Causal excursion effect

\[
\frac{E\{Y_{t+1}(\bar{A}_{t-1}, 1) \mid S_t(\bar{A}_{t-1}), I_t(\bar{A}_{t-1}) = 1\}}{E\{Y_{t+1}(\bar{A}_{t-1}, 0) \mid S_t(\bar{A}_{t-1}), I_t(\bar{A}_{t-1}) = 1\}}
\]

- Contrasting two excursions: following \(\bar{A}_{t-1}\), then receive treatment (\(A_t = 1\)) vs. no treatment (\(A_t = 0\)) at time \(t\).
- \(S_t(\bar{A}_{t-1}) \subset H_t(\bar{A}_{t-1})\): a vector of summary variables chosen from \(H_t(\bar{A}_{t-1})\).
- Effect is marginal over all variables in \(H_t(\bar{A}_{t-1})\) that are not in \(S_t(\bar{A}_{t-1})\).
- Conditional on being available: \(I_t(\bar{A}_{t-1}) = 1\).
Causal excursion effect

\[
\log \frac{E\{ Y_{t+1}(\bar{A}_{t-1}, 1) \mid S_t(\bar{A}_{t-1}), I_t(\bar{A}_{t-1}) = 1 \}}{E\{ Y_{t+1}(\bar{A}_{t-1}, 0) \mid S_t(\bar{A}_{t-1}), I_t(\bar{A}_{t-1}) = 1 \}}
\]

- Contrasting two excursions: following \( \bar{A}_{t-1} \), then receive treatment \( (A_t = 1) \) vs. no treatment \( (A_t = 0) \) at time \( t \).
- \( S_t(\bar{A}_{t-1}) \subset H_t(\bar{A}_{t-1}) \): a vector of summary variables chosen from \( H_t(\bar{A}_{t-1}) \).
- Effect is marginal over all variables in \( H_t(\bar{A}_{t-1}) \) that are not in \( S_t(\bar{A}_{t-1}) \).
- Conditional on being available: \( I_t(\bar{A}_{t-1}) = 1 \).
• $S_t(\bar{A}_{t-1}) = 1$: average treatment effect

\[
\log \frac{E\{ Y_{t+1}(\bar{A}_{t-1}, 1) \mid l_t(\bar{A}_{t-1}) = 1 \}}{E\{ Y_{t+1}(\bar{A}_{t-1}, 0) \mid l_t(\bar{A}_{t-1}) = 1 \}}
\]

• $S_t(\bar{A}_{t-1}) = (1, \text{ day in study})$

\[
\log \frac{E\{ Y_{t+1}(\bar{A}_{t-1}, 1) \mid \text{day}_t, l_t(\bar{A}_{t-1}) = 1 \}}{E\{ Y_{t+1}(\bar{A}_{t-1}, 0) \mid \text{day}_t, l_t(\bar{A}_{t-1}) = 1 \}}
\]
Identifiability assumptions

<table>
<thead>
<tr>
<th><strong>Assumption (consistency)</strong></th>
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<tbody>
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<td>The observed data equals the potential outcome under observed treatment assignment: $O_t = O_t(\bar{A}_{t-1})$ for every $t$.</td>
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## Identifiability assumptions

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<td>For every $t$, for every possible history $H_t$ with $I_t = 1$, $P(A_t = a \mid H_t, I_t = 1) &gt; 0$ for $a \in {0, 1}$.</td>
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</table>
Identifiability assumptions

Assumption (consistency)

The observed data equals the potential outcome under observed treatment assignment: \( O_t = O_t(\bar{A}_{t-1}) \) for every \( t \).

Assumption (positivity)

For every \( t \), for every possible history \( H_t \) with \( I_t = 1 \),
\[
P(A_t = a \mid H_t, I_t = 1) > 0 \quad \text{for} \quad a \in \{0, 1\}.
\]

Assumption (sequential ignorability)

For every \( t \), the potential outcomes \( \{O_{t+1}(\bar{a}_t), A_{t+1}(\bar{a}_t), \ldots, O_{T+1}(\bar{a}_T) : \bar{a}_T \in \{0, 1\}^T \} \) are independent of \( A_t \) conditional on \( H_t \).
Identifiability

Under assumptions on previous slide,

$$\log \frac{E\{Y_{t+1}(\bar{A}_{t-1}, 1) \mid S_t(\bar{A}_{t-1}), I_t(\bar{A}_{t-1}) = 1\}}{E\{Y_{t+1}(\bar{A}_{t-1}, 0) \mid S_t(\bar{A}_{t-1}), I_t(\bar{A}_{t-1}) = 1\}}$$

$$= \log \frac{E\{E(Y_{t+1} \mid A_t = 1, H_t) \mid S_t, I_t = 1\}}{E\{E(Y_{t+1} \mid A_t = 0, H_t) \mid S_t, I_t = 1\}}$$

$$= \log \frac{E \left\{ \frac{\mathbb{1}(A_t=1) Y_{t+1}}{p_t(H_t)} \right\} \mid S_t, I_t = 1}{E \left\{ \frac{\mathbb{1}(A_t=0) Y_{t+1}}{1-p_t(H_t)} \right\} \mid S_t, I_t = 1}$$
Introduction

2 Special case: conditional on $H_t$

3 A simple and robust estimator

4 Simulation study

5 Analysis of BariFit

6 Extension: proximal outcome defined over a duration

7 Summary
Special case: conditional on $H_t$

Suppose for all $t$,

$$\log \frac{E(Y_{t+1} \mid A_t = 1, H_t, I_t = 1)}{E(Y_{t+1} \mid A_t = 0, H_t, I_t = 1)} = S_t^T \beta$$

holds for some $S_t \subset H_t$ and some parameter $\beta$. 
Special case: conditional on $H_t$

\[
\log \frac{E(Y_{t+1} \mid A_t = 1, H_t, I_t = 1)}{E(Y_{t+1} \mid A_t = 0, H_t, I_t = 1)} = S_t^T \beta
\]

- working model $\exp\{g(H_t)^T \alpha\}$
Special case: conditional on $H_t$

\[ \log \frac{E(Y_{t+1} | A_t = 1, H_t, I_t = 1)}{E(Y_{t+1} | A_t = 0, H_t, I_t = 1)} = S_t^T \beta \]

- working model $\exp \{g(H_t)^T \alpha \}$
- If working model is correct, then

\[ E(Y_{t+1} | H_t, I_t = 1) = e^{g(H_t)^T \alpha + A_t S_t^T \beta}, \tag{1} \]

and one can use GEE to estimate $\alpha$ and $\beta$. 
Special case: conditional on $H_t$

\[
\log \frac{E(Y_{t+1} \mid A_t = 1, H_t, I_t = 1)}{E(Y_{t+1} \mid A_t = 0, H_t, I_t = 1)} = S_t^T \beta
\]

- working model $\exp\{g(H_t)^T \alpha\}$
- If working model is correct, then

\[
E(Y_{t+1} \mid H_t, I_t = 1) = e^{g(H_t)^T \alpha + A_t S_t^T \beta}, \quad (1)
\]

and one can use GEE to estimate $\alpha$ and $\beta$.

- However, (1) is required to guarantee the consistency of GEE.
Semiparametric model

\[
\log \frac{E(Y_{t+1} \mid A_t = 1, H_t, I_t = 1)}{E(Y_{t+1} \mid A_t = 0, H_t, I_t = 1)} = S_t^T \beta
\]

\[
E(Y_{t+1} \mid A_t = 0, H_t, I_t = 1) = e^{g(H_t)^T \alpha}
\]
Semiparametric model

- Assume this (parametric part)

$$\log \frac{E(Y_{t+1} \mid A_t = 1, H_t, I_t = 1)}{E(Y_{t+1} \mid A_t = 0, H_t, I_t = 1)} = S_t^T \beta$$

- Don’t assume this; this becomes nonparametric

$$E(Y_{t+1} \mid A_t = 0, H_t, I_t = 1) = e^{g(H_t)^T \alpha}$$
Semiparametric model

- Assume this (parametric part)
  \[
  \log \frac{E(Y_{t+1} \mid A_t = 1, H_t, I_t = 1)}{E(Y_{t+1} \mid A_t = 0, H_t, I_t = 1)} = S_t^T \beta
  \]

- Don’t assume this; this becomes nonparametric
  \[
  E(Y_{t+1} \mid A_t = 0, H_t, I_t = 1) = e^{g(H_t)^T \alpha}
  \]

- “semi-parametric” model; Newey (1990), Tsiatis (2007)
- Robins (1994), structural nested mean model
Estimate causal excursion effects

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Introduction

Conditional on \( H_t \)

Estimator

Simulation

BariFit

Extension

Summary

References

Semiparametric estimator

• The following estimator for \( \beta \), derived based on Robins (1994), is semiparametric locally efficient:

\[
\mathbb{P}_{n} \sum_{t=1}^{T} I_t e^{-A_t S_t^T \beta} (Y_{t+1} - eg(H_t)^T \alpha + A_t S_t^T \beta) V_t \begin{bmatrix} g(H_t) \\ (A_t - pt(H_t)) S_t \end{bmatrix} = 0
\]

\[
\Rightarrow (\hat{\alpha}, \hat{\beta})
\]

• Robust: \( \hat{\beta} \) is consistent for \( \beta \) with any choice of control variables \( g(H_t) \)

• “Locally efficient”: \( \hat{\beta} \) has the smallest asymptotic variance (among all semiparametric regular and asymptotically linear estimators) if \( eg(H_t)^T \alpha \) is a correct model for \( E(Y_{t+1} \mid H_t, A_t = 0, I_t = 1) \).

\[
V_t := \frac{e^{S_t^T \beta}}{e^{S_t^T \beta \{1 - eg(H_t)^T \alpha\} p_t(H_t) + \{1 - eg(H_t)^T \alpha + S_t^T \beta\} (1 - p_t(H_t))}}.
\]
Intuition for robustness

\[
\mathbb{P}_n \sum_{t=1}^{T} l_t e^{-A_t S_t^T \beta} \left( Y_{t+1} - e^{g(H_t)^T \alpha + A_t S_t^T \beta} \right) V_t \begin{bmatrix} g(H_t) \\ (A_t - p_t(H_t)) S_t \end{bmatrix} = 0
\]
Intuition for robustness

\[
\mathbb{P} \sum_{t=1}^{T} l_t e^{-A_t S_t^T \beta} (Y_{t+1} - e g(H_t)^T \alpha + A_t S_t^T \beta) V_t \left[ g(H_t) (A_t - p_t(H_t)) S_t \right] = 0
\]

- \( e^{-A_t S_t^T \beta} Y_{t+1} \): “blipped-down” outcome

\[
E(e^{-A_t S_t^T \beta} Y_{t+1} | H_t, A_t) = E(Y_{t+1}(\bar{A}_{t-1}, 0) | H_t, A_t)
\]
Intuition for robustness

\[
\mathbb{P}_n \sum_{t=1}^{T} I_t e^{-A_t S_t^T \beta} (Y_{t+1} - e^{g(H_t)^T \alpha + A_t S_t^T \beta}) V_t \begin{bmatrix} g(H_t) \\ (A_t - p_t(H_t)) S_t \end{bmatrix} = 0
\]

- \( e^{-A_t S_t^T \beta} Y_{t+1} \): “blipped-down” outcome

\[
E(e^{-A_t S_t^T \beta} Y_{t+1} \mid H_t, A_t) = E\{Y_{t+1}(\bar{A}_t - 1, 0) \mid H_t, A_t\}
\]

- \( e^{g(H_t)^T \alpha} \): a function of \( H_t \)
Intuition for robustness

\[
\prod_{n} \sum_{t=1}^{T} I_t \begin{bmatrix}
    e^{-A_t S_t^T \beta} (Y_{t+1} - eg(H_t)^T \alpha + A_t S_t^T \beta) \\
    V_t \\
    g(H_t) \\
    (A_t - p_t(H_t)) S_t
\end{bmatrix} = 0
\]

- \(e^{-A_t S_t^T \beta} Y_{t+1}\): “blipped-down” outcome
  \[E(e^{-A_t S_t^T \beta} Y_{t+1} \mid H_t, A_t) = E\{Y_{t+1}(\bar{A}_{t-1}, 0) \mid H_t, A_t\}\]
- \(eg(H_t)^T \alpha\): a function of \(H_t\)
- \(A_t - p_t(H_t)\): centered treatment assignment
Intuition for robustness

\[
\mathbb{P}_n \sum_{t=1}^{T} I_t e^{-A_t S_t^T \beta} (Y_{t+1} - e^{g(H_t)^T \alpha + A_t S_t^T \beta}) V_t \begin{bmatrix} g(H_t) \\ (A_t - p_t(H_t)) S_t \end{bmatrix} = 0
\]

- \(e^{-A_t S_t^T \beta} Y_{t+1}\): “blipped-down” outcome

\[
E(e^{-A_t S_t^T \beta} Y_{t+1} \mid H_t, A_t) = E\{ Y_{t+1}(\bar{A}_{t-1}, 0) \mid H_t, A_t \}
\]

- \(e^{g(H_t)^T \alpha}\): a function of \(H_t\)

- \(A_t - p_t(H_t)\): centered treatment assignment

\(\implies\) The blue term and the red term are orthogonal to each other (with any \(g(H_t)\)).

\(\implies\) robustness
T. Qian

**Treatment effect of interest**

- Special case considered so far: fully conditional on $H_t$

$$\log \frac{E(Y_{t+1} \mid A_t = 1, H_t, I_t = 1)}{E(Y_{t+1} \mid A_t = 0, H_t, I_t = 1)} = S_t^T \beta$$

- What makes more scientific sense: marginal over variables in $H_t$ but not in $S_t$

$$\log \frac{E\left\{ E(Y_{t+1} \mid A_t = 1, H_t) \mid S_t, I_t = 1 \right\}}{E\left\{ E(Y_{t+1} \mid A_t = 0, H_t) \mid S_t, I_t = 1 \right\}} = S_t^T \beta$$
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A simple and robust estimator for marginalized effect

- Control variables: \( \exp\{g(H_t)^T \alpha\} \)

\[
P_n \sum_{t=1}^{T} l_t e^{-A_t S_t^T \beta} (Y_{t+1} - e^{g(H_t)^T \alpha + A_t S_t^T \beta}) V_t \begin{bmatrix} g(H_t) \\ (A_t - p_t(H_t)) S_t \end{bmatrix} = 0
\]

- Because the model assumption is now on the marginalized treatment effect, the \textcolor{blue}{blue term} and the \textcolor{red}{red term} are no longer orthogonal.
A simple and robust estimator for marginalized effect

- Control variables: \( \exp\{g(H_t)^T \alpha \} \)

\[
\mathbb{P}_n \sum_{t=1}^T l_t e^{-A_t S_t^T \beta} (Y_{t+1} - e^{g(H_t)^T \alpha + A_t S_t^T \beta}) \begin{bmatrix} g(H_t) \\ S_t \end{bmatrix} = 0
\]

- Choose \( \tilde{p}_t(s) \in (0, 1) \)

- Form weights: \( W_t = \left( \frac{\tilde{p}_t(S_t)}{p_t(H_t)} \right)^{A_t} \left( \frac{1 - \tilde{p}_t(S_t)}{1 - p_t(H_t)} \right)^{1-A_t} \)

- Center treatment: \( A_t \rightarrow (A_t - \tilde{p}_t(S_t)) \)
A simple and robust estimator for marginalized effect

- Control variables: $\exp\{g(H_t)^T \alpha\}$

$$\mathbb{P}_n \sum_{t=1}^T l_t e^{-A_t S_t^T \beta} (Y_{t+1} - e^{g(H_t)^T \alpha + A_t S_t^T \beta}) W_t \left[ \begin{array}{c} g(H_t) \\ (A_t - \tilde{p}_t(S_t)) S_t \end{array} \right] = 0$$

- Choose $\tilde{p}_t(s) \in (0, 1)$
- Form weights: $W_t = \left( \frac{\tilde{p}_t(S_t)}{p_t(H_t)} \right)^{A_t} \left( \frac{1 - \tilde{p}_t(S_t)}{1 - p_t(H_t)} \right)^{1-A_t}$
- Center treatment: $A_t \rightarrow (A_t - \tilde{p}_t(S_t))$
- $W_t$ and $\tilde{p}_t(S_t)$ make the blue term and the red term orthogonal to each other.
- Boruvka et al. (2018)
A simple and robust estimator for marginalized effect

- Suppose \((\hat{\alpha}, \hat{\beta})\) solve the estimating equation:

\[
\mathbb{P}_n \sum_{t=1}^{T} I_t e^{-A_t S_t^T \beta} (Y_{t+1} - e^{g(H_t)^T \alpha + A_t S_t^T \beta}) W_t \left[ g(H_t) \right] = 0
\]

- Under moment conditions, \(\hat{\beta}\) is consistent for \(\beta\) and is asymptotically normal if

\[
\log \frac{E\{E(Y_{t+1} \mid A_t = 1, H_t) \mid S_t, I_t = 1\}}{E\{E(Y_{t+1} \mid A_t = 0, H_t) \mid S_t, I_t = 1\}} = S_t^T \beta
\]

- **Robustness**: consistency of \(\hat{\beta}\) doesn’t require \(e^{g(H_t)^T \alpha}\) to be a correct model for \(E(Y_{t+1} \mid A_t = 0, H_t, I_t = 1)\)
Choice of $\tilde{p}_t$

- Choice of $\tilde{p}_t(S_t)$ determines marginalization over time under model misspecification of treatment effect.
- For example, if $S_t = 1$, $\tilde{p}_t(S_t) = \tilde{p}$ for some $\tilde{p} \in (0, 1)$, then $\hat{\beta}$ converges to

$$
\beta' = \log \frac{\sum_{t=1}^{T} E\{E(Y_{t+1} \mid H_t, A_t = 1) \mid I_t = 1\}}{\sum_{t=1}^{T} E\{E(Y_{t+1} \mid H_t, A_t = 0) \mid I_t = 1\}},
$$

• Choice of $\tilde{p}_t(S_t)$ determines marginalization over time under model misspecification of treatment effect.
• For example, if $S_t = 1$, $\tilde{p}_t(S_t) = \tilde{p}$ for some $\tilde{p} \in (0, 1)$, then $\hat{\beta}$ converges to

$$
\beta' = \log \frac{\sum_{t=1}^{T} E\{E(Y_{t+1} \mid H_t, A_t = 1) \mid I_t = 1\}}{\sum_{t=1}^{T} E\{E(Y_{t+1} \mid H_t, A_t = 0) \mid I_t = 1\}},
$$
Outline

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7 Summary
Simulation: generative model

- $E(Y_{t+1} | H_t, A_t) = f(Z_t) \exp\{A_t(0.1 + 0.3Z_t)\}$
- Covariate $Z_t$: takes value from 0, 1, 2 with equal probability
- $f(Z_t) = 0.2\mathbb{1}(Z_t = 0) + 0.5\mathbb{1}(Z_t = 1) + 0.4\mathbb{1}(Z_t = 2)$
- $P(A_t = 1 | H_t) = 0.2$
- $I_t = 1$: always available
True treatment effect

\[
\log \frac{E(Y_{t+1} \mid A_t = 1, H_t, I_t = 1)}{E(Y_{t+1} \mid A_t = 0, H_t, I_t = 1)} = 0.1 + 0.3Z_t
\]

\[
\log \frac{E\{E(Y_{t+1} \mid A_t = 1, H_t) \mid I_t = 1\}}{E\{E(Y_{t+1} \mid A_t = 0, H_t) \mid I_t = 1\}} = 0.477
\]

So if we let \( S_t = 1 \) in the analysis model, the semiparametric locally efficient estimator would be inconsistent, and the robust estimator would be consistent.
## Result

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Sample size</th>
<th>Bias</th>
<th>SD</th>
<th>RMSE</th>
<th>CP</th>
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<td>0.072</td>
<td>0.072</td>
<td>0.96</td>
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<tr>
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<td>0.94</td>
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<tr>
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<td>0.78</td>
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* SD: standard deviation. RMSE: root mean squared error. CP: 95% confidence interval coverage probability.
Outline

1. Introduction
2. Special case: conditional on $H_t$
3. A simple and robust estimator
4. Simulation study
5. Analysis of BariFit
6. Extension: proximal outcome defined over a duration
7. Summary
BariFit food track reminder

- $n = 45$ participants:
- 112 days $= 112$ decision points
- $A_t$: food track reminder is sent as text message with probability $0.5$ every morning
- $Y_{t+1}$: binary indicator of whether the individual completes food log on that day ($Y_{t+1} = 1$ if logged calories $> 0$)
Estimated effect

\[ \log E(Y_{t+1}) \sim g(H_t)^T \alpha + \beta_0 A_t \]

- Control variables \( g(H_t) \): day in study, gender, food log completion on previous day
- Estimation result for \( \beta_0 \):

<table>
<thead>
<tr>
<th>Method</th>
<th>Estimate</th>
<th>SE</th>
<th>95% CI</th>
<th>( p )-value</th>
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<td>0.014</td>
<td>(-0.017, 0.039)</td>
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* SE: standard error. CI: confidence interval.
Initial conclusion

- The data indicates that there is no detectable effect of the food track reminder text message on the food log completion of that day.
- For the next iteration of BariFit...
  - Implement the reminder as part of a native app (instead of text messages) — to improve effectiveness
  - Or, combine it with other text messages (such as daily step goal) that are sent to the individuals in the morning — to reduce burden
Introduction

Special case: conditional on $H_t$

A simple and robust estimator

Simulation study

Analysis of BariFit

Extension: proximal outcome defined over a duration

Summary
Proximal outcome defined over a duration of time

- Sometimes the proximal outcome is measured over a duration of time during which other treatments may occur.
- On each individual: $O_1, A_1, \ldots, O_T, A_T, O_{T+1}$.
- Proximal outcome $Y_{t+\Delta}$, is a known function of the individual’s data within a subsequent window of length $\Delta$; i.e., $Y_{t+\Delta} = y(O_{t+1}, A_{t+1}, \ldots, O_{t+\Delta-1}, A_{t+\Delta-1}, O_{t+\Delta})$ for some known function $y(\cdot)$.
- Previously, $Y_{t+1} = y(O_{t+1})$. 
Causal excursion effect

Let $\bar{0}$ be a vector of length $\Delta - 1$.

$$\log \frac{E\{Y_{t+\Delta}(\bar{A}_{t-1}, 1, \bar{0}) \mid S_t(\bar{A}_{t-1}), I_t(\bar{A}_{t-1}) = 1\}}{E\{Y_{t+\Delta}(\bar{A}_{t-1}, 0, \bar{0}) \mid S_t(\bar{A}_{t-1}), I_t(\bar{A}_{t-1}) = 1\}} = S_t^T \beta$$

Estimating equation for $\beta$:

$$P_n \sum_{t=1}^{T} I_t e^{-A_t S_t^T \beta} (Y_{t+1} - e^{g(H_t)^T \alpha + A_t S_t^T \beta}) \tilde{W}_t \left[ \begin{array}{c} g(H_t) \\ (A_t - \tilde{\rho}_t(S_t))S_t \end{array} \right] = 0,$$

where

$$\tilde{W}_t = W_t \times \prod_{j=t+1}^{t+\Delta-1} \frac{1(A_j = 0)}{1 - p_j(H_j)}$$
Summary

• Definition of causal excursion effect for binary outcome
• A semiparametric locally efficient estimator for the effect conditional on history observed up to that time point, $H_t$
• A simple and robust estimator for the effect marginalized over all but a small subset $S_t$ of $H_t$
• An extension to settings where the proximal outcome is defined over a duration of time during which other treatments may occur
• An analysis of marginal effect of food track reminder in BariFit MRT


