Dynamic Treatment Regimes and “SMART” Design

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The World is Aging!

Source: U.S. Department of State Archive
Global Shift in Causes of Death...

Towards Chronic, Noncommunicable Diseases

1990

- Injuries: 9%
- Comm/Mater/Neonat/Nutr: 34%
- Non Communicable Diseases: 57%

46.5 million

2010

- Injuries: 10%
- Comm/Mater/Neonat/Nutr: 25%
- Non Communicable Diseases: 65%

52.7 million

Source: Institute for Health Metrics and Evaluation, University of Washington
Similar Trend in Developing and Developed Countries

Source: U.S. Department of State Archive
Outline

1. Chronic Diseases, Personalized Medicine and Dynamic Treatment Regimes

2. SMART Design
   - Primary Analysis and Sample Size

3. SMART in Mobile Health (mHealth)
   - Non-Inferiority Testing
   - Adaptive SMART for Mobile Health

4. Estimation of Optimal DTRs
   - Q-learning
   - Analysis of Smoking Cessation Data: A Simple Case Study

5. Discussion
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Personalized Medicine for Chronic Diseases

Believed by many as the future of medicine ...

Source: http://www.personalizedmedicine.com/

Often refers to tailoring by genetic profile, but it’s also common to personalize or stratify treatments to patients based on more “macro” level characteristics, some of which are time-varying
Personalized Medicine for Chronic Diseases

- Paradigm shift from “one size fits all” to individualized, patient-centric care
  - Can address inherent heterogeneity across patients
  - Can also address variability within patient, over time
  - Can increase patient compliance or adherence, thus increasing the chance of treatment success
  - Likely to reduce the overall cost of health care

- Overarching Methodological Questions:
  - How to decide on the optimal treatment for an individual patient?
  - How to make these treatment decisions evidence-based or data-driven?
One perspective in personalized medicine: given an individual patient’s characteristics, can we identify a treatment, among the available options, that is most likely to confer the most benefit?

A treatment regime is a rule (or rules) that specifies how to allocate treatment based on an individual patient’s characteristics.

If treatment is administered only once, then there is a single decision point, often the time of diagnosis.

In case of chronic diseases, treatments are typically administered at multiple decision points.
Dynamic Treatment Regime(n)s

- A **dynamic treatment regime (DTR)** is a sequence of decision rules, one per decision point (or stage), that specify how to adapt the type, dosage and timing of treatment according to the ongoing information on an individual patient.
  
  - Each decision rule takes a patient’s treatment and covariate history as inputs, and outputs a recommended treatment.

- Decision points can occur at regular intervals (e.g. yearly follow-up visits) or clinical events (e.g. remission, relapse, complication).
Dynamic Treatment Regimes (DTRs)

- DTRs offer a data-driven framework for operationalizing the adaptive clinical decision-making in a time-varying setting, and thereby potentially improving it
  - Clinical decision support systems for treating chronic diseases

- A subject’s stage-specific treatment is not known at the start of a dynamic regime, since treatment depends on time-varying variables
ADHD Example: Treatment Scenarios

ADHD: Attention Deficit Hyperactivity Disorder

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“Give **Low-intensity BMOD** initially; if the subject **responds**, then continue **BMOD**, otherwise prescribe **BMOD + MEDS**”
ADHD Example: One Not-so-simple DTR

- **Stage-1 Rule**: “If the baseline level of impairment is greater than a threshold (say, $\psi$), prescribe MEDS; otherwise prescribe BMOD”

- **Stage-2 Rule**: “If the subject is a responder to initial treatment, continue the same treatment; if non-responder, prescribe BMOD + MEDS”

How to specify $\psi$?
Treatment Regime vs. Realized Treatment Experience

- Subjects following the same DTR can have different realized treatment experiences:
  - Subject 1 experiences “Low-intensity BMOD, followed by response, followed by Low-intensity BMOD”
  - Subject 2 experiences “Low-intensity BMOD, followed by non-response, followed by BMOD + MEDS”
Notation and Data Structure

It’s a bit more than standard longitudinal data!

- **K** stages (or decision points) on a single patient:
  \[ O_1, A_1, \ldots, O_K, A_K, O_{K+1} \]

  - \( O_j \): Observation (pre-treatment) at the \( j \)-th stage
  - \( A_j \): Treatment (action) at the \( j \)-th stage, \( A_j \in A_j \)
  - \( H_j \): History at the \( j \)-th stage, \( H_j = \{O_1, A_1, \ldots, O_{j-1}, A_{j-1}, O_j\} \)
  - \( Y \): Primary Outcome (assume larger is better, without loss of generality)

- A DTR is a sequence of decision rules:
  \[ d \equiv (d_1, \ldots, d_K) \text{ with } d_j(h_j) \in A_j \]
The Big Scientific Questions in DTR Research

- What would be the mean outcome if the population were to follow a particular pre-conceived DTR?

- How do the mean outcomes compare among two or more DTRs?

- What is the optimal DTR in terms of the mean outcome?
  - What individual information (tailoring or prescriptive variables) do we use to make these decisions?
The Big Statistical Questions

1. What is the right kind of data for comparing two or more DTRs, or estimating optimal DTRs? What is the appropriate study design?
   - Sequential Multiple Assignment Randomized Trial (SMART)

2. How can we compare pre-conceived, embedded DTRs?
   - primary analysis of SMART data

3. How can we estimate the “optimal” DTR for a given patient?
   - secondary analysis of SMART data
   - e.g. Q-learning, a stagewise regression-based approach, originally developed in computer science
DTR Discovery: Observational Data

- People extensively use observational longitudinal data (including electronic health records) – data collection is cheap!

- Usual concerns about observational data, e.g. confounding and other hidden biases (Rubin, 1974; Rosenbaum, 1991)

- Need unverifiable assumptions to make causal inference about treatment effects

- Analysis is more complex in general, and also in the DTR context (see, e.g. Robins, 2004; Moodie, Chakraborty and Kramer, 2012)
DTR Discovery: Experimental Data

- As is well-known, experimental data are generally better than observational data (of course, if you can afford it)

- Standard *randomized controlled trials* (RCTs) assess the effectiveness of a single dose-level of a single treatment, as compared to another

- Estimating the sequence of treatments that optimizes response in a longitudinal setting requires studying the elements in the sequence

- Can this be accomplished by a series of single-stage RCTs?
Example: Treating MDD

- Suppose we wish to compare both front-line and second-line treatment of major depressive disorder (MDD):
  - Front-line options: citalopram (Cit) or cognitive behavioral therapy (CBT)
  - Second-line options: treatment switch to Cit, CBT, or Lithium (Li)
  - All responders to first-line therapy will continue with maintenance and follow-up

- Remember: The goal is to find the optimal treatment sequence
Example: Treating MDD

SMART Design

Maintenance dose +
telephone monitoring

Responder

Non-responder

Telephone monitoring

Responder

Non-responder

CBT

Cit

Li

R

R

CBT

Cit

Li

R
Example: Treating MDD

- **SMART Design**
- **Example:** Treating MDD
- **Maintenance dose + telephone monitoring**
- **Responder**
- **Non-responder**
- **Responder**
- **Non-responder**

**Diagram:**
- **R** to **Cit**
- **R** to **CBT**
- **R** to **Li**
- **Telephone monitoring**
- **Responsive**
- **Non-responsive**

**Legend:**
- R: Responder
- CBT: Cognitive Behavioral Therapy
- Li: Lithium
Suppose we observe 60% response with Cit, and only 50% with CBT

Conclude: **Cit is the best front-line therapy**

Now run another **one-stage trial** amongst Cit non-responders
Second-line Treatment of MDD

- We now observe 40% response to CBT and 20% to Li

- Conclude: CBT is the best second-line therapy

- Final treatment sequence: "Cit, followed by CBT for non-responders"
  - Under this regimen, we expect to see 76% of patients respond
**Delayed Effect**

- What if initial treatment with CBT increases treatment adherence \( \Rightarrow \) subsequent therapies more successful?

  - Optimal DTR: “CBT, followed by Cit for non-responders”; 80% response expected
Two single-stage trials would not have detected the best overall strategy for treatment

Instead, we should have used a two-stage trial

Such two-stage trials are known as SMARTs (Lavori and Dawson, 2004; Murphy, 2005), i.e.

**Sequential Multiple Assignment Randomized Trials**
Susan Murphy won the MacArthur Foundation “Genius Grant” in 2013 for inventing the SMART design:

http://www.macfound.org/fellows/898/
SMARTs, in general

- **Multi-stage** trials with a goal to inform the development of DTRs

- Same subjects participate **throughout** (they are followed through stages of treatment)

- Each stage corresponds to a treatment decision

- At each stage the patient is **randomized** to one of the available treatment options

- Treatment options at randomization may be **restricted** on ethical grounds, depending on intermediate outcome and/or treatment history
SMART: The Gains

- Ability to detect
  - delayed therapeutic effects (treatment interactions)
  - diagnostic effects

- More generalizable than standard RCT?

- Better recruitment and retention potential than standard RCT?
  - By virtue of the option to alter a non-functioning treatment while in the trial
SMART: The Costs

Unfortunately there is no free lunch!!

- More **expensive** than a single-stage RCT
  - Longer follow-up
  - Need more participants if the trial is powered to compare whole sequences of treatments – but there exist less expensive alternatives to power SMARTs!

- More **complex methods** for planning and analysis: requires an experienced statistician, or one with time to learn new methods

- May require **additional work** to get funded: still relatively new, unfamiliar
SMART vs. Crossover Trial Designs

- Operationally, SMARTs look similar to crossover trials

- However, conceptually they are very different
  - Unlike SMART, Crossover trials aim to evaluate stand-alone treatments, not DTRs
  - Unlike a crossover trial, treatment allocation in a SMART is typically adaptive to a subject’s intermediate outcome
  - Crossover trials attempt to “wash out” the “carry-over” effect while SMARTs attempt to capture it
SMART Design

SMART vs. Usual Adaptive Trial Designs

- SMARTs are different from usual adaptive trials
  - Within-subject vs. between-subject adaptation

- In an (outcome-) adaptive trial, the randomization probabilities can change during the course of the trial depending on the relative success of already-recruited patients on each of the treatments
  - Adaptive trails typically aim to reduce the number of trial participants exposed to the inferior treatment
  - Adaptive trials are most useful for examining questions where the outcome follows fairly quickly after treatment and/or the recruitment process is (relatively) slow
  - SMARTs are fixed trial designs – while randomization probabilities may depend on covariates, those probabilities remain constant over the course of the trial
Does anyone actually use SMARTs?

ADHD Trial (PI: Pelham; see *Lei et al.*, 2012, for design details)

<table>
<thead>
<tr>
<th>History</th>
<th>Initial Treatment</th>
<th>Intermediate Outcome</th>
<th>Secondary Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children With ADHD</td>
<td>Low-intensity BMOD</td>
<td>Responder</td>
<td>Low-intensity BMOD</td>
</tr>
<tr>
<td>R</td>
<td>Low-intensity MEDS</td>
<td>Non-responder</td>
<td>BMOD + MEDS</td>
</tr>
<tr>
<td>R</td>
<td>Non-responder</td>
<td>R</td>
<td>Intensify BMOD</td>
</tr>
<tr>
<td>R</td>
<td>Responders</td>
<td>R</td>
<td>Intensify MEDS</td>
</tr>
</tbody>
</table>

R = Randomization

Primary Outcome: Teacher-rated Impairment Rating Scale (TIRS)
Other Examples of SMART Studies

- **Schizophrenia:** CATIE (*Schneider et al., 2001*)
- **Depression:** STAR*D (*Rush et al., 2003*)
- **Prostate Cancer:** Trials at MD Anderson Cancer Center (e.g., *Thall et al., 2000*)
- **Leukemia:** CALGB Protocol 8923 (e.g., *Stone et al., 1995; Wahed and Tsiatis, 2004*)
- **Smoking Cessation:** Project Quit (*Strecher et al., 2008; Chakraborty et al., 2010*)
- **Alcohol Dependence:** *Oslin et al.* (see, e.g., *Lei et al., 2012*)

Many recent examples available at the Methodology Center, PennState University website:

http://methodology.psu.edu/ra/smart/projects
SMART Design Principles

Primary and Secondary Hypotheses

- Choose scientifically important primary hypotheses that also aid in developing DTRs
  - Power trial to address these hypotheses

- Choose secondary hypotheses that further develop the DTR, and use randomization to eliminate confounding
  - Trial is not necessarily powered to address these hypotheses
  - Still better than post hoc observational analyses
  - Underpowered randomizations can be viewed as pilot studies for future full-blown comparisons

- From a funding perspective, it may be more feasible to design a standard two-arm trial as primary goal, and add the secondary randomizations and DTR estimation as a secondary goal
SMART Design: Tailoring

- At each stage, restrict the class of treatments only by ethical, feasibility or strong scientific considerations.

- Use a low-dimension summary (e.g. responder status) instead of all intermediate outcomes to determine randomization and restrict class of next treatments:
  - This will often be a key tailoring variable.
  - The definition must be concrete, e.g., determined using specific measure(s) at a specific time.
  - Generally, this variable should be regularly available in clinical practice, so that tailoring with this variable is feasible.
  - In mental illness studies, feasibility considerations may force investigators to use preference in this low dimensional summary (e.g. STAR*D).

- Collect intermediate outcomes potentially useful in further tailoring of treatment at the analysis stage.
SMART Design: Embedded DTRs

Embedded DTR-1 ($d_1$)

**History**
- Children With ADHD

**Initial Treatment**
- Low-intensity BMOD
- Low-intensity MEDS

**Intermediate Outcome**
- Responder
- Non-responder

**Secondary Treatment**
- Responder
  - Low-intensity BMOD
  - Low-intensity MEDS
  - BMOD + MEDS
- Non-responder
  - BMOD + MEDS
  - Intensify MEDS
  - Intensify BMOD

R indicates a route taken based on response.
SMART Design: Embedded DTRs

Embedded DTR-2 ($d_2$)

- **History**: Children With ADHD
  - **Initial Treatment**: Low-intensity BMOD
  - **Intermediate Outcome**: Responder, Non-responder
    - Responder: Low-intensity BMOD, Low-intensity MEDS
    - Non-responder: BMOD + MEDS, BMOD + MEDS
- **Secondary Treatment**: Low-intensity BMOD, BMOD + MEDS, Intensify BMOD, Low-intensity MEDS, BMOD + MEDS, Intensify MEDS
SMART Design: Embedded DTRs

Embedded DTR-3 ($d_3$)

<table>
<thead>
<tr>
<th>History</th>
<th>Initial Treatment</th>
<th>Intermediate Outcome</th>
<th>Secondary Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children With ADHD</td>
<td>Low-intensity BMOD</td>
<td>Responder</td>
<td>Low-intensity BMOD</td>
</tr>
<tr>
<td></td>
<td>Low-intensity MEDS</td>
<td>Non-responder</td>
<td>BMOD + MEDS</td>
</tr>
<tr>
<td></td>
<td>Low-intensity BMOD</td>
<td>Responder</td>
<td>Intensify BMOD</td>
</tr>
<tr>
<td></td>
<td>Low-intensity MEDS</td>
<td>Non-responder</td>
<td>BMOD + MEDS</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Intensify MEDS</td>
</tr>
</tbody>
</table>
SMART Design: Embedded DTRs

Embedded DTR-4 ($d_4$)

- History: Children With ADHD
  - Initial Treatment: Low-intensity BMOD
    - Intermediate Outcome: Responders
      - Secondary Treatment: Low-intensity BMOD
      - Secondary Treatment: BMOD + MEDS
    - Intermediate Outcome: Non-responders
      - Secondary Treatment: Intensify BMOD
  - Intermediate Outcome: Low-intensity MEDS
    - Secondary Treatment: Low-intensity MEDS
    - Secondary Treatment: BMOD + MEDS
  - Intermediate Outcome: Non-responders
    - Secondary Treatment: Intensify MEDS
SMART Design: Simplified Data Structure

\((O_{1i}, A_{1i}, O_{2i}, A_{2i}, Y_i)\) for \(i = 1, \cdots, N\), where

- \(O_1 = \emptyset\)
- \(A_1 \in \{BMOD, MEDS\}\)
- \(O_2 = R\) is the response indicator, where \(R = 1\) denotes responder and \(R = 0\) denotes non-responder
- \(A_2 \in \{BMOD, BMOD^+, MEDS, MEDS^+, BMOD + MEDS\}\)
- \(Y\) is the \text{continuous} end-of-trial primary outcome
- \(N\) is the total sample size of the trial
SMART Design: Embedded DTRs (Formally)

- The 4 embedded DTRs in the above SMART are:

\[
\begin{align*}
    d_1 & = \left( BMOD, BMOD^R (BMOD + MEDS)^{1-R} \right) \\
    d_2 & = \left( BMOD, BMOD^R (BMOD^+)^{1-R} \right) \\
    d_3 & = \left( MEDS, MEDS^R (BMOD + MEDS)^{1-R} \right) \\
    d_4 & = \left( MEDS, MEDS^R (MEDS^+)^{1-R} \right)
\end{align*}
\]

- Key Questions:
  - How can we measure the performance of an embedded DTR?
  - How can we compare two embedded DTRs in the SMART?
Regime Mean or Value Function: The Key Estimand

- What would be the mean outcome if the entire population follows a DTR $d$? ⇒ Call this the "Value" of $d$ and denote by $\mu_d = E_d(Y)$

- Recall the general data structure, $K$ stages (or decision points) on a single patient: $O_1, A_1, \ldots, O_K, A_K, O_{K+1}$

- If $\pi_j(a_j|h_j)$’s are used to allocate the treatments, then the data likelihood (or, joint distribution) $P_\pi$ is:

$$f_1(o_1)\pi_1(a_1|o_1) \prod_{j=2}^K f_j(o_j|h_{j-1}, a_{j-1})\pi_j(a_j|h_j)f_{K+1}(o_{K+1}|h_K, a_K)$$

- What is the likelihood of the data under an arbitrary set of deterministic decision rules $d_j(h_j)$’s for allocating treatments? Call it $P_d$:

$$f_1(o_1)\mathbb{I}\{a_1 = d_1(o_1)\} \prod_{j=2}^K f_j(o_j|h_{j-1}, a_{j-1})\mathbb{I}\{a_j = d_j(h_j)\}f_{K+1}(o_{K+1}|h_K, a_K)$$
The notation $\mu_d = E_d(Y)$ actually means expectation with respect to the distribution $P_d$ even though the data were actually generated by $P_{\pi}$.

Computing $E_d(Y)$ non-parametrically involves a change of probability measure, under the assumption that $P_d$ is absolutely continuous or feasible with respect to $P_{\pi}$.

- Any data trajectory that can result by implementing $d$ must also have positive probability of occurring under the generative distribution $\pi$.

Under the feasibility assumption, $E_d(Y) = \int Y dP_d = \int Y \left( \frac{dP_d}{dP_{\pi}} \right) dP_{\pi}$ where $\frac{dP_d}{dP_{\pi}}$ is a version of the Radon-Nikodym derivative and is given by the ratio of the two likelihoods:

$$\frac{dP_d}{dP_{\pi}} = \prod_{j=1}^{K} \frac{\mathbb{I}\{a_j = d_j(h_j)\}}{\pi_j(a_j|h_j)}$$
Regime Mean or Value Function: The Key Estimand

- Embedded regimes are, by design, feasible

- For the embedded regime $d_1 = \left( BMOD, BMOD^R(BMOD + MEDS)^{1-R} \right)$, the value function is

  $$
  \mu_{d_1} = E_{d_1}(Y) \\
  = E\left[ \frac{\mathbb{I}\{A_1 = BMOD, A_2 = BMOD^R(BMOD + MEDS)^{1-R}\}}{\pi_1(A_1)\pi_2(A_2|A_1, R)} Y \right] \\
  = E(W^{d_1} Y), \text{ say}
  $$

- $\pi_1$ and $\pi_2$ are the randomization probabilities

- Inverse probability weighting (IPW) approach

- Same underlying idea as in the classical Horvitz-Thompson estimator

- Similarly, one can write expressions for other regimes $d_2, d_3, d_4$
Estimation of Regime Mean

- The regime mean can be estimated by the IPW estimator (Robins et al., 2000):

\[
\bar{Y}_{d1} = \frac{\sum_{i=1}^{N} W_i d_1 Y_i}{\sum_{i=1}^{N} W_i d_1}
\]

- Also, for large samples (Murphy, 2005):

\[
\hat{Var}(\bar{Y}_{d1}) = \frac{1}{N^2} \sum_{i=1}^{N} (W_i d_1)^2 (Y_i - \bar{Y}_{d1})^2
\]
Primary Analysis

- Depending on the research question, it could be a comparison of two or more groups corresponding to two or more DTRs embedded in the SMART, or components thereof.

- Standard methods of analysis, involving the above means and variances.

- Standard software like SAS PROC GENMOD can be employed\(^2\)

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Primary Hypothesis and Sample Size: Scenario 1

Hypothesize that averaging over the secondary treatments, the initial treatment BMOD is as good as the initial treatment MEDS

– Sample size formula is same as that for a two-group comparison
Primary Hypothesis and Sample Size: Scenario 2

Hypothesize that among non-responders an augmentation (BMOD+MEDS) is as good as an intensification of treatment

- Sample size formula is same as that for a two-group comparison of non-responders (overall sample size depends on the presumed non-response rate)
Primary Hypothesis and Sample Size: Scenario 3

Hypothesize that the “red” DTR ($d_2$) is as good as the “green” DTR ($d_3$)

- Sample size formula involves a two-group comparison of “weighted” means (overall sample size depends on the presumed non-response rate)
Sample Size Requirements

Assume continuous outcome, e.g., Teacher-rated Impairment Rating Scale in case of ADHD

Key Design Parameters:

- Effect Size = \( \frac{\Delta \mu}{\sigma} \) (Cohen’s \( d \))
- Type I Error Rate = \( \alpha = 0.05 \)
- Desired Power = \( 1 - \beta = 0.8 \)
- Initial Response Rate = \( \gamma = 0.5 \)

Trial Size:

<table>
<thead>
<tr>
<th>Effect Size</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>( N_1 = 350 )</td>
<td>( N_2 = \frac{N_1}{(1-\gamma)} = 700 )</td>
<td>( N_3 = N_1 \times (2 - \gamma) = 525 )</td>
</tr>
<tr>
<td>0.5</td>
<td>( N_1 = 128 )</td>
<td>( N_2 = \frac{N_1}{(1-\gamma)} = 256 )</td>
<td>( N_3 = N_1 \times (2 - \gamma) = 192 )</td>
</tr>
<tr>
<td>0.8</td>
<td>( N_1 = 52 )</td>
<td>( N_2 = \frac{N_1}{(1-\gamma)} = 104 )</td>
<td>( N_3 = N_1 \times (2 - \gamma) = 78 )</td>
</tr>
</tbody>
</table>
Comparing embedded DTR-1 ($d_1$) with embedded DTR-2 ($d_2$)

- $d_1$ and $d_2$ share the initial treatment of BMOD
- Patients who respond to BMOD are consistent with both $d_1$ and $d_2$, and contribute information on the overall response to both regimens
- Specialized methods are needed to account for “re-using” their information
SMART Design: Other Comparisons

- Alternatively, we may wish to:
  
  compare $d_1$ vs. $d_2$ vs. $d_3$ vs. $d_4$ to see which of the four regimes leads to the best expected outcome

- The above question requires more specialized methods to account for both multiple comparisons as well as the re-use of data for patients who are consistent with more than one regime
How about Comparing DTRs with Time-to-event Outcomes?

- Same principles of **inverse probability weighting** apply (e.g., weighted Kaplan-Meier estimator, weighted log-rank test, etc.)

- Under the **proportional hazards** assumption, Li and Murphy\(^3\) provided a conservative sample size formula using **weighted log-rank** test for comparing \(d_1\) and \(d_3\):

\[
 n \leq \left\{ \frac{1}{\pi_1 \pi_2} + \frac{1}{(1 - \pi_1) \pi_2} \right\} \frac{(z_1 - \frac{\alpha}{2} + z_1 - \beta)^2}{\xi^2 \cdot P_{d_1} \text{(event)}}
\]

where \(\xi\) is the **log hazard ratio** between the times-to-event under the two DTRs, and \(P_{d_1} \text{(event)}\) denotes the probability of observing an event before the end of the study among subjects following \(d_1\) (**event rate**)

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How about Comparing DTRs with Time-to-event Outcomes?

- Li and Murphy’s formula builds on sample size formula based on log rank test for classical survival analysis\(^4\)

- Li and Murphy also developed an alternative formula based on weighted Kaplan-Meier estimator, but that requires more inputs at the design stage

- The log rank test based formula has been implemented in the online sample size calculator (web app):
  http://methodologymedia.psu.edu/logranktest/samplesize

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5. Discussion
Life is Getting Digital...

Image courtesy of Gary Bennett

Source: Duke Global Health Institute
Mobile Phone Subscriptions, Globally

Mobile phone subscriptions

- Developed
- Developing

Countries with the highest increases in subscriptions per 100 inhabitants (2005–2015):

- Botswana: +139
- Maldives: +138
- Mali: +133
- Costa Rica: +125
- Cambodia: +125

Source: International Telecommunications Union (2016)
Countries with Shortage of Healthcare Providers

Source: Duke Global Health Institute
The Global Epidemic of Diabetes

Source: Duke Global Health Institute
mHealth Interventions for Diabetes Management

- Good glycaemic control is key to managing diabetes and its many complications

- Treatments for Type 2 Diabetes Mellitus (T2DM) include diet, exercise, oral medications and insulin injections

- Initiation of insulin in insulin-naive patients traditionally involves multiple visits to a physician to adjust insulin dose which can be costly and time-consuming
  - Delay in scheduling can result delay in achieving glycaemic control

- Self-titration for insulin dose calculation is also an option but many patients hesitate and express uncertainty
The “Diabetes Pal” App

- A smartphone app, developed in Singapore (Duke-NUS and SGH), with an in-built algorithm to compute daily insulin dosage based on lowest of the previous 3 fasting blood glucose (FBG) readings (patients are required to measure their FBG daily and input into the app)
The “Diabetes Pal” App

The feasibility of the app to deliver the insulin titration algorithm in insulin-naive patients has been validated in a pilot RCT ($n = 66$):

The “Diabetes Pal” App: What’s Next?

- Not everyone in the population is good with technology (e.g., app); some may need human intervention (e.g., nurse)
  - But human intervention adds more cost to the healthcare system

- Perhaps a stepped-care approach involving the app may be cost-effective
  - Start with a low-cost, low-intensity intervention (app) first; and then step up to a high-cost, high-intensity intervention (e.g., nurse) only for patients who show early signs of non-response to low-cost, low-intensity intervention

- It would be interesting to study if a “stepped-care regime” is almost as good as (non-inferior to) a “more aggressive or resource-intensive regime” – if yes, then the stepped-care regime should be selected

- This sort of research question can be investigated using a SMART design
A SMART Design involving The Diabetes Pal App
Non-Inferiority Testing Framework

- Embedded regimes are:
  \[ d_1 = (\text{App}, \text{App}^R \text{Nurse}^{1-R}), \quad d_2 = (\text{App}, \text{App}^R (\text{App} + \text{Nurse})^{1-R}) \] and
  \[ d_3 = (\text{Nurse}, \text{Nurse}^R \text{Nurse}^{1-R}) \]

- Here \( d_1 \) and \( d_2 \) are stepped-care regimes, whereas \( d_3 \) is the active control regime.

- In non-inferiority testing\(^5\), generally the goal is to show that the efficacy of the experimental treatment (regime), when compared to an active control treatment (regime), is not below the pre-specified non-inferiority margin.

- In practice, a non-inferiority margin is the maximum clinically acceptable difference from the active control on average that researchers agree to accept in exchange of the secondary benefits (e.g. cost, burden, side-effects) of the new treatment.

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Non-Inferiority Testing in SMARTs

- For two regimes $d_1$ and $d_3$ with corresponding regime means $\mu_{d_1}$ and $\mu_{d_3}$ respectively, the hypothesis for the non-inferiority test is

$$H_0 : \mu_{d_1} \leq \mu_{d_3} - \theta \ vs \ H_1 : \mu_{d_1} > \mu_{d_3} - \theta,$$

where $\theta$ is a pre-specified non-inferiority margin ($\theta > 0$)

- This hypothesis tests that the average efficacy of regime $d_1$ is not inferior to that of the regime $d_3$, with the non-inferiority margin $\theta$

- The choice of $\theta$ depends on both statistical reasoning and clinical judgment

- The unscaled test statistic is $\bar{Y}_{d_1} - \bar{Y}_{d_3}$, with mean $\mu_{d_1} - \mu_{d_3}$ and variance $\nu_{d_1d_3}$ (obtained from Murphy’s calculations)
Test Statistic and Sample Size Calculation

- Under $H_0$, the large-sample distribution of the test statistic

$$Z_{d_1d_3} = \frac{(\bar{Y}_{d_1} - \bar{Y}_{d_3}) - (\mu_{d_1} - \mu_{d_3})}{\sqrt{\text{var}(\bar{Y}_{d_1} - \bar{Y}_{d_3})}} = \frac{\bar{Y}_{d_1} - \bar{Y}_{d_3} + \theta}{\sqrt{\nu_{d_1d_3}}} \to N(0, 1)$$

- Reject $H_0$ at level $\alpha$ and conclude non-inferiority if $Z_{d_1d_3} > z_{\alpha}$ (one-sided)

- Deriving sample size formula requires large sample approximations and some assumptions

- Given a postulated effect size $\hat{\delta} = \mu_{d_3} - \mu_{d_1}$, the required $n$ depends on the difference between $\theta$ and $\delta$, rather than their individual values
“Realistic” Synthetic Data Analysis

- Data simulated from the pilot study sample, with $Y$ taken as $-\text{HbA1c}$

- Response rate to app = 51% and response rate to nurse = 69% (according to Deloitte Global Mobile Consumer Survey UK Edition)

- $\bar{Y}_{d_3} = -8.107$ with s.d. = 0.086; $\bar{Y}_{d_1} = -8.269$ with s.d. = 0.491

- For $\theta = 0.5$, test statistic = 0.6781 < $z_\alpha = 1.645$, so non-inferiority can’t be concluded

- For $\theta = 1$, test statistic = 1.6811 > $z_\alpha = 1.645$, so non-inferiority can be concluded

- Choice of $\theta$ is tricky!
From SMART to SMART-AR

- Technologies evolve very quickly, so interventions using mobile devices must be evaluated and disseminated very quickly; otherwise they will lose relevance.

- This is a very different setting from traditional RCTs (or even traditional SMARTs).
  - SMARTs can be made more useful by incorporating adaptive randomization.
  - In modern contexts like mHealth, SMART with Adaptive Randomization (SMART-AR) has been developed.
  - SMART-AR is also ethically appealing even in a non-mHealth context.
  - In general, how best to do this is not known yet.

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5. Discussion
Secondary Analysis of SMART Data

- Goal: To find **optimal** treatment sequence for each individual patient by **deeply tailoring** on their time-varying covariates and intermediate outcomes
  - This is to develop the **evidence-based decision support system** for clinicians (they can apply these regimens while treating future patients)

- Methodologically challenging – custom-made analytic tools necessary
  - One popular approach is **Q-learning**, a stage-wise regression-based method

  - We developed an **R package** called **qLearn** (*Xin et al., 2012*) that conducts Q-learning (Freely available at CRAN):

    [http://cran.r-project.org/web/packages/qLearn/](http://cran.r-project.org/web/packages/qLearn/)
Q-learning: Introduction

- **Q-learning** (*Watkins, 1989*)
  - A popular method from Reinforcement (Machine) Learning
  - A generalization of least squares regression to multistage decision problems (*Murphy, 2005*)
  - Adapted to and implemented in the DTR context with several variations (*Zhao et al., 2009; Chakraborty et al., 2010; Schulte et al., 2012; Song et al., 2014*)
  - Relatively easy to understand and implement, and forms a good basis for more complex approaches

- The intuition comes from **dynamic programming** (*Bellman, 1957*) in case the multivariate distribution of the data is known
  - Q-learning is an approximate dynamic programming approach
Notation and Data Structure (Recap)

*K* stages (or decision points) on a single patient:

\[ O_1, A_1, \ldots, O_K, A_K, O_{K+1} \]

- \( O_j \) : Observation (pre-treatment) at the \( j \)-th stage
- \( A_j \) : Treatment (action) at the \( j \)-th stage, \( A_j \in \{-1, 1\} \)
- \( H_j \) : History at the \( j \)-th stage, \( H_j = \{O_1, A_1, \ldots, O_{j-1}, A_{j-1}, O_j\} \)
- \( Y \) : Primary Outcome (assume larger is better, without loss of generality)

A DTR is a sequence of decision rules:

\[ d \equiv (d_1, \ldots, d_K) \text{ with } d_j(h_j) \in A_j \]
Dynamic Programming (DP): The Background

Two Stages

- Move backward in time to take care of the delayed effects
- Define the “Quality of treatment”, Q-functions:

\[
Q_2(h_2, a_2) = \mathbb{E}[Y \mid H_2 = h_2, A_2 = a_2]
\]

\[
Q_1(h_1, a_1) = \mathbb{E}\left[\max_{a_2} Q_2(H_2, a_2) \mid H_1 = h_1, A_1 = a_1\right]
\]

- Optimal DTR:

\[
d_j(h_j) = \arg \max_{a_j} Q_j(h_j, a_j), \ j = 1, 2
\]

What if the true Q-functions are unknown?
From DP to Q-learning

- DP requires modeling the joint multivariate distribution (likelihood) of time-varying covariates and outcome

- This is hard!
  - The knowledge needed to model the joint distribution is often unavailable
  - Model mis-specification can lead to incorrect conclusions

- Turn to semi-parametric methods

- In particular, Q-learning estimates the true Q-functions from data using regression models
Q-learning: Typical Implementation ($K = 2$)

- Linear regression models for Q-functions:
  \[ Q_j(H_j, A_j; \beta_j, \psi_j) = \beta_j^T H_j + (\psi_j^T H_j)A_j, \ j = 1, 2, \]

- At stage 2, regress $Y$ on $(H_2, H_2A_2)$ to obtain $(\hat{\beta}_2, \hat{\psi}_2)$

- Construct stage-1 Pseudo-outcome:
  \[ \tilde{Y}_{1i} = \max_{a_2} Q_2(H_{2i}, a_2; \hat{\beta}_2, \hat{\psi}_2), \ i = 1, \ldots, n \]

- At stage 1, regress $\tilde{Y}_1$ on $(H_1, H_1A_1)$ to obtain $(\hat{\beta}_1, \hat{\psi}_1)$

- Estimated Optimal DTR:
  \[ \hat{d}_j(h_j) = \arg \max_{a_j} Q_j(h_j, a_j; \hat{\beta}_j, \hat{\psi}_j) = \text{sign}(\hat{\psi}_j^T h_j) \]
Q-learning: Remarks

- Q-learning is appealing because it is easy to implement in standard software, and easy to explain to clinical collaborators who are familiar with regression.

- The approach has several limitations, e.g.:
  - Not robust to model mis-specification
  - Only limited results are available for discrete outcomes

- More sophisticated approaches (e.g., A-learning or outcome-weighted learning) exist, at least for some treatment/outcome types.
Why move through stages as in Q-learning? Why not run an “all-at-once” multivariable regression?

Berkson’s Paradox or Collider-stratification Bias: There may be non-causal association(s) even with randomized data, leading to biased stage-1 effects (*Berkson, 1946; Greenland, 2003; Murphy, 2005*).
Project Quit: A Smoking Cessation Trial (Simplified)

Two-stage Web-based (*eHealth*) Behavioral Intervention Trial for Smoking Cessation conducted at the University of Michigan

Stage-1 Covariate: education (≤ high school vs. > high school)

Stage-1 Intervention: tailoring of success story, low vs. high
(in addition to free nicotine patch)

Stage-2 Covariates: quit status at 6 months (1 = quit, 0 = not quit), months non-smoking over 6 months

Stage-2 Intervention: booster prevention vs. control

Primary Outcome: months non-smoking over 12 months

*Center for Health Communications Research, University of Michigan*

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SMART Design Schematic (Simplified)
Secondary Research Questions

- **Stage-1 question:** (In future) How should a web-based smoking cessation intervention be designed so as to maximize each individual’s chance of quitting over the two stages? Should this intervention be adapted to the smoker’s baseline education?

- **Stage-2 question:** Should the stage-2 intervention be adapted to either the stage-1 intervention the smoker has already received and/or the smoker’s intermediate outcome (e.g., stage-1 quit status)?
Results from Q-learning Analysis

No significant stage-2 treatment effect ($n = 479$)

Stage-1 Analysis Summary ($n = 1848$)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>95% Bootstrap CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>education</td>
<td>0.01</td>
<td>(-0.18, 0.20)</td>
</tr>
<tr>
<td>high vs. low tailoring</td>
<td>0.07</td>
<td>(-0.01, 0.11)</td>
</tr>
<tr>
<td>tailoring:education</td>
<td>-0.11</td>
<td>(-0.24, -0.00)*</td>
</tr>
</tbody>
</table>

- The “highly individually tailored” level of story appears more effective for subjects with less education ($\leq$ high school)
- This finding is consistent with that of Strecher et al. (2008) – a logistic regression analysis of the stage-1 quit status (binary) data
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Take-home Messages

- **SMART** is a cutting-edge trial design that formalizes the sequential decision making in clinical practice
  - Very relevant for chronic conditions
  - Does not necessarily require a lot of sample size (depends on the primary question)

- **SMARTs** are useful in the modern context of mHealth

- Non-inferiority testing in **SMART** is very appealing from a cost-effectiveness perspective

- Analysis methods exist for continuous, binary and time-to-event data coming from a **SMART**

- Relevant softwares are also available

- Lots of open problems, thus many opportunities for statisticians!
Shoot your questions, comments, criticisms, request for slides to:
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