Optimal auction duration: price formation point of view.

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LOB: "a transparent system that matches customer orders on a 'price/time priority' basis".
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  - Benefits by increasing the bid/ask spread and by being the faster.

- **Market takers send market orders.** Their arrival intensities are very sensitive w.r.t. the spread (see e.g. Madhavan, Richardson, and Roomans (1997), Wyart, Bouchaud, Kockelkoren, Potters, and Vetarazzo (2008), Dayri and Rosenbaum (2015)).
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- **Exchanges** aims at increasing the liquidity (make-take fees system).
Concerning the electronic order book mechanism and to quote Farmer and Skouras, 1992:

"One consequence of this trading mechanism which we believe is underappreciated is that it endows a huge advantage to being faster than other traders, creating evolutionary pressures that drive an arms race for ever-more speed."

Continuous limit order books lead to mechanical arbitrages and generate a competition in speed rather than in price between high frequency market makers.
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- No evidence a priori that the CLOB is the optimal market structure for all the market participants having different preferences.

- Suitable remedies: batch auctions.
Auction mechanism

- Auction market is open during a certain time and receives limit and market orders along the auction duration.
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- At the clearing time, a clearing price of the stock is determined to ensure maximal number of transactions.

Some references:
- Du and Zhu (2014): efficiency of an auction market with respect to the auction duration.
- Fricke and Gerig (2018): extensions to multi assets or the presence of liquidity providers.

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  → Only for homogeneous agents.
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Efficient price. $P_s = \sigma_f W_s$, $s \geq 0$. 

Market makers put limit orders with respect to their preferences indexed on i.i.d. noisy estimations of the efficient price centred with same variance $\sigma^2_n$. 

A counting process $N_{mm}$ with fixed intensity $\mu$ and event time $\tau_{mm}$.

Volume sent by the $k$th market maker $S_{kpq}$ with $\tilde{P}_{\tau_{mm}k}$.

Market takers arrive on the market and send market orders in the auction with intensity $\nu$.

Poisson processes $N_a$ and $N_b$ (ask and bid orders resp.).

After the clearing of the $i$th auction, the exchange opens the auction at time $\tau_{op}$ as soon as a market order is set during a duration $\tau_{op}$, with clearing time $\tau_{cl} = \tau_{op}$.

It determines the clearing price ensuring to match the most orders.
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\[ \text{Counting process } N^{mm} \text{ with fixed intensity } \mu \text{ and event time } T_k^{mm}. \]
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- Counting process \( N^{mm} \) with fixed intensity \( \mu \) and event time \( \tau_{k^{mm}} \).

- Volume sent by the \( k \)th market maker

\[
S_k(p) = K(p - \tilde{P}_{\tau_{k^{mm}}}), \quad \text{with} \quad \tilde{P}_{\tau_{k^{mm}}} = P_{\tau_{k^{mm}}} + g_k.
\]
Auction mechanism

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- **Market takers** arrive on the market and send \( \nu \) market orders in the auction with intensity \( \nu. \)
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- Market takers arrive on the market and send \( \nu \) market orders in the auction with intensity \( \nu. \)

- Poisson processes \( N^a \) and \( N^b \) (ask and bid orders resp.)

- After the clearing of the \( i \)th auction, the exchange opens the auction at time \( \tau^{op} \) as soon as a market order is set during a duration \( h \) so that the clearing time is \( \tau^{cl} = \tau^{op} + h. \)

- It determines the clearing price ensuring to match the most orders.
Assumptions:

For each auction $i$:
- $p_{N_i}$, $a_{N_i}$, $b_{q_i}$, and $m_{mm_i}$ are independent of $P$.
- After each auction clearing, the market regenerates.
- The first market maker always arrives before the auction clearing.

Technically speaking, $p_{N_1}$, $m_{mm}$ stays $0$ during $t_{mm} < t_{op}$ has the law of a Poisson process with intensity $\mu$ conditional on $t_{mm}$. Clearing price sets to ensure the larger number of transactions.

What is the optimal duration $t$? Comparison with the LOB ($t^* = 0$).
In a nutshell

Assumptions:

- For each auction \( i \): \( (N^{i,a}, N^{i,b}) \) and \( N^{i,mm} \) are independent of \( P \).
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Technically speaking, \((N_{s,mm}^{1})_{0 \leq s \leq \tau^{op}_1 + h}\) has the law of a Poisson process with intensity \( \mu \) conditional on \( \{\tau_{1,mm}^1 < \tau_{1,a}^1 \wedge \tau_{1,b}^1 + h\} \).
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Clearing price sets to ensure the larger number of transactions.
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- For each auction $i$: $(N^{i,a}, N^{i,b})$ and $N^{i,mm}$ are independent of $P$.
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- Technically speaking, $(N_{s}^{1,mm})_{0 \leq s \leq \tau_{i}^{op} + h}$ has the law of a Poisson process with intensity $\mu$ conditional on $\{\tau_{1}^{mm} < \tau_{1}^{a} \land \tau_{1}^{b} + h\}$.

- Clearing price sets to ensure the larger number of transactions.
- What is the optimal duration $h$? Comparison with the LOB ($h = 0$).
The clearing price $P_{cl}^{i}$ of the $i$th auction is determined as an equilibrium between supply and demand curves. Hence, $P_{cl}^{i}$ is solution to

$$\#_{\text{shares}}\{\text{buyers MM and MT}\}(p, h) = \#_{\text{shares}}\{\text{sellers MM and MT}\}(p, h).$$
The clearing price $P_{\tau_{cl}}^i$ of the $i$th auction is determined as an equilibrium between supply and demand curves. Hence, $P_{\tau_{cl}}^i$ is solution to

$$
\sum_{k=N_{\tau_{\tau_{cl}}-1}^{mm}}^{N_{\tau_{cl}}^{mm}} S_k(p) - \left( l_{\tau_{cl}} - l_{\tau_{op}} \right) = 0,
$$

with $I = vN^a - vN^b$ where $v$ is a (constant) volume $v$ of market orders.
The clearing price $P^{cl}_{\tau_i}$ of the $i$th auction is determined as an equilibrium between supply and demand curves. Hence, $P^{cl}$ is solution to

$$
\sum_{k=N^{mm}_{\tau_{i-1}} + 1}^{N^{mm}_{\tau_i}} S_k(p) - (I^{cl}_{\tau_i} - I^{op-}_{\tau_i}) = 0,
$$

with $I = vN^a - vN^b$ where $v$ is a (constant) volume $v$ of market orders.

Thus:

$$
P^{cl}_{\tau_i} = \frac{1}{N^{mm}_{\tau_i} - N^{mm}_{\tau_{i-1}}} \sum_{k=N^{mm}_{\tau_{i-1}} + 1}^{N^{mm}_{\tau_i}} \tilde{P}_k + \frac{1}{K} \frac{I^{cl}_{\tau_i} - I^{op-}_{\tau_i}}{N^{mm}_{\tau_i} - N^{mm}_{\tau_{i-1}}}.
$$

$= P^{mid}_{\tau_i}$
Optimal duration

\[ P_{\tau_{i}^{cl}} = \frac{1}{N_{\tau_{i}^{op}+h}^{mm} - N_{\tau_{i}^{cl}}^{mm}} \sum_{k=N_{\tau_{i}^{cl}}^{mm}+1}^{N_{\tau_{i}^{op}+h}^{mm}} \tilde{P}_{k} + \frac{1}{K} \frac{l_{\tau_{i}^{op}+h} - l_{\tau_{i}^{op}-}}{N_{\tau_{i}^{op}+h}^{mm} - N_{\tau_{i}^{cl}+1}^{mm}}. \]
Optimal duration

\[
P^{cl}_{\tau^i_{cl}} = \frac{1}{N^{mm}_{\tau^i_{op}} + h - N^{mm}_{\tau^i_{cl}-1}} \sum_{k=N^{mm}_{\tau^i_{cl}-1} + 1}^{N^{mm}_{\tau^i_{op}} + h} \tilde{P}_k + \frac{1}{K} \frac{l_{\tau^i_{op}} + h - l_{\tau^i_{op} - h}}{N^{mm}_{\tau^i_{op}} + h - N^{mm}_{\tau^i_{cl}-1}}.
\]

Criterion to measure the fairness of the transaction price:

Quadratic error between the clearing price and the fundamental price:

"\[ Z^h_t = \int_0^t (P^{cl}_s - P_s)^2 \, ds. \]"
Optimal duration

\[ P_{\tau_i}^{cl} = \frac{1}{N_{\tau_i}^{mm} + h - N_{\tau_{i-1}}^{mm}} \sum_{k=N_{\tau_i}^{mm} + 1}^{N_{\tau_i}^{mm} + h} \tilde{P}_k + \frac{1}{K} \frac{I_{\tau_i}^{op} + h - I_{\tau_i}^{op} -}{N_{\tau_i}^{mm} + h - N_{\tau_{i-1}}^{mm}}. \]

Criterion to measure the fairness of the transaction price:
Quadratic error between the clearing price and the fundamental price:

\[ Z^h_t = \int_0^t (P_s^{cl} - P_s)^2 ds. \]

Lemma

\[ \lim_{t \to +\infty} \frac{Z^h_t}{t} = \mathbb{E}[(P_{\tau_1}^{cl} + h - P_{\tau_1}^{op} + h)^2]. \]

A duration \( h^* \) is optimal when it is a minimizer of the function

\[ h \mapsto E(h) = \mathbb{E}[(P_{\tau_1}^{cl} + h - P_{\tau_1}^{op} + h)^2]. \]
We get

\[ E(h) = E^{mid}(h) + \frac{\mathbb{E}[l_{\tau_1 + h}^2]}{K^2} \frac{e^{\nu h}}{1 - e^{-\mu h}} \frac{\nu}{\nu + \mu} e^{\nu h} \int_h^{+\infty} \nu e^{-\nu t} e^{-\mu t} \int_0^{\mu t} \frac{1}{s} \int_0^s \frac{e^u - 1}{u} \, du \, ds \, dt. \]
We get

\[
E(h) = E^{mid}(h) + \frac{\mathbb{E}[l^2_{\tau_{1}^{op}+h}]}{K^2} \frac{e^{\nu h}}{1 - e^{-\mu h} \frac{\nu}{\nu + \mu}} e^{\nu h} \int_{h}^{+\infty} \nu e^{-\nu t} e^{-\mu t} \int_{0}^{t} \frac{1}{s} \int_{0}^{s} \frac{e^{u} - 1}{u} \, du \, ds \, dt.
\]

- \(E(h)\) depends on the market takers’ behaviours \underline{during the auction} only through \(\mathbb{E}[l^2_{\tau_{1}^{op}+h}]\).
We get

\[
E(h) = E^{mid}(h) + \frac{\mathbb{E}[l_{2\tau_1^{op}+h}^2]}{K^2} \frac{e^{\nu h}}{1 - e^{-\mu h} \frac{\nu}{\nu + \mu}} e^{\nu h} \int_{h}^{+\infty} \nu e^{-\nu t} e^{-\mu t} \int_{0}^{\mu t} \frac{1}{s} \int_{0}^{s} e^{\mu - \frac{1}{u}} dudsdt.
\]

- \( E(h) \) depends on the market takers’ behaviours during the auction only through \( \mathbb{E}[l_{2\tau_1^{op}+h}^2] \).
- Until here, we have considers "unsophisticated" market takers having a fixed intensity \( \nu \) of arrival between AND during the auction.

\[ \mathbb{E}[l_{2\tau_1^{op}+h}^2] = \nu^2 (\nu h + \frac{1}{2}) \]

So that, we can compute \( h^* \) for "unsophisticated" market takers.
Comparison with "sophisticated" market takers

\[ \tau \text{ is still the event time of a Poisson process with intensity } \nu. \]

During the auction, MTs optimize their trading costs w.r.t. their arrival intensities.
Comparison with "sophisticated" market takers

\( \tau^{op}_i \) is still the event time of a Poisson process with intensity \( \nu \).

**During the auction:** MTs optimize their trading costs w.r.t. their arrival intensities.
Comparison with "sophisticated" market takers

\[ \tau_{i-1}^{op} \] is still the event time of a Poisson process with intensity \( \nu \).

During the auction: MTs optimize their trading costs w.r.t. their arrival intensities.
Focus on the inbalance process

$N_t^{cl}$ is the number of auctions cleared before $t$.
The aggregated total trading cost of buyers market takers at time $t$ can be written:

$$C_t^a = \sum_{i=1}^{N_t^{cl}} (N_{\tau_i}^a - N_{\tau_{i-1}}^a) (P_{\tau_i}^{cl} - P_{\tau_{i-1}}^{cl}),$$

and similarly for sellers

$$C_t^b = \sum_{i=1}^{N_t^{cl}} (N_{\tau_i}^b - N_{\tau_{i-1}}^b) (P_{\tau_i}^{cl} - P_{\tau_{i-1}}^{cl}).$$
Focus on the inbalance process

$N_{t}^{cl}$ is the number of auctions cleared before $t$. The aggregated total trading cost of buyers market takers at time $t$ can be written:

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and similarly for sellers

$$C_{t}^{b} = \sum_{i=1}^{N_{t}^{cl}} (N_{\tau_{i}^{cl}}^{b} - N_{\tau_{i-1}^{cl}}^{b})(P_{\tau_{i}^{cl}}^{cl} - P_{\tau_{i}^{cl}}^{cl}).$$

Proposition

**Long term error**

$$\lim_{t \to +\infty} \frac{C_{t}^{a}}{t} \propto \mathbb{E}[N_{h}^{a}(N_{h}^{a} - N_{h}^{b})].$$

$$\lim_{t \to +\infty} \frac{C_{t}^{b}}{t} \propto \mathbb{E}[N_{h}^{b}(N_{h}^{b} - N_{h}^{a})].$$
Sophisticated market takers during the auction:

- Buyers market takers arrive with intensity $\lambda^a$
- Sellers market takers arrive with intensity $\lambda^b$

Induces a family of measures $\mathbb{P}^{a,b}$ with bounds $[\lambda_-, \lambda_+]$. 
Sophisticated market takers during the auction:

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Market takers aims at optimizing simultaneously

\[
\begin{aligned}
(Nash) \quad \left\{ \begin{array}{ll}
\sup_{\lambda^a \in [\lambda_-, \lambda_+]} & \mathbb{E}^{\mathbb{P}^{\lambda^a, \lambda^b}} [N_h^a (N_h^a - N_h^b)] \\
\sup_{\lambda^b \in [\lambda_-, \lambda_+]} & \mathbb{E}^{\mathbb{P}^{\lambda^a, \lambda^b}} [N_h^b (N_h^b - N_h^a)]
\end{array} \right.
\end{aligned}
\]
Sophisticated market takers during the auction:

- Buyers market takers arrive with intensity $\lambda^a$
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Market takers aims at optimizing simultaneously

\[
\begin{align*}
\text{(Nash)}\left\{ \begin{array}{l}
\sup_{\lambda^a \in [\lambda_-, \lambda_+]} \mathbb{E}^{\mathbb{P}^{\lambda^a, \lambda^b}} \left[ N^a_h (N^a_h - N^b_h) \right] \\
\sup_{\lambda^b \in [\lambda_-, \lambda_+]} \mathbb{E}^{\mathbb{P}^{\lambda^a, \lambda^b}} \left[ N^b_h (N^b_h - N^a_h) \right]
\end{array} \right. 
\end{align*}
\]

Verification procedure: it reminds to solve a bang-bang type integro-differential system of PDEs with Hamiltonian:

\[
H^{a,*}(z, \tilde{z}, \varepsilon) = z_1 \lambda^*_a(z) + z_2 \lambda^*_b(\tilde{z}, \varepsilon), \quad H^{b,*}(z, \tilde{z}, \varepsilon) = z_2 \lambda^*_b(z) + z_1 \lambda^*_a(\tilde{z}, \varepsilon),
\]

where

\[
\begin{align*}
\text{(foc)}\left\{ \begin{array}{l}
\lambda^*_a(z, \varepsilon_a) = 1_{z_1 > 0} \lambda_- + 1_{z_1 < 0} \lambda_+ + \varepsilon_a 1_{z_1 = 0} \\
\lambda^*_b(z, \varepsilon_b) = 1_{z_2 > 0} \lambda_- + 1_{z_2 < 0} \lambda_+ + \varepsilon_b 1_{z_2 = 0}.
\end{array} \right.
\end{align*}
\]
Finding Nash equilibrium...

We mollify the functions $\lambda_a^*$ and $\lambda_b^*$ by introducing

\[
\lambda^n(z) = \begin{cases} 
\lambda_+ & \text{if } z \leq -\frac{1}{n} \\
\lambda_- & \text{if } z \geq \frac{1}{n} \\
n\frac{\lambda_- - \lambda_+}{2} z + \frac{\lambda_+ + \lambda_-}{2} & \text{if } z \in (-\frac{1}{n}, \frac{1}{n}).
\end{cases}
\]

so that there exists a solution to a (Lipschitz) system of BSDE which gives a sequence $(\varepsilon^n_a, \varepsilon^n_b)$ of (Markovian) processes converging to a Nash equilibrium.

- We extend Hamadène and Mu (2014) to the Poisson case by using BSDE theory.

- The proof is constructive and gives a way to approach numerically this value.
There exists a Nash equilibrium for the simultaneous optimization problem given by some Markovian controls \((\lambda_a^*, \lambda_b^*)\) and so that

\[
\mathbb{E}[I_{\tau_{op}^+ + h}^2] = V^a_h(\lambda_a^*, \lambda_b^*) + V^b_h(\lambda_a^*, \lambda_b^*),
\]

where \(V^a\) and \(V^b\) are the values of \((\text{Nash})\).
**Unknown variables**

- MTs’ arrival intensities $\nu$ between two auctions.
- MMs’ arrivals given by $\mu$.
- MMs’ supply parameter $K$.

**Euronext Data:** details on the volumes of each stocks exchanged along the day.
Unknown variables
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Euronext Data: details on the volumes of each stocks exchanged along the day.

$\nu = \frac{V}{T}$ where $T$: duration of the trading day, $V$: average daily volume of market orders.
Unknown variables

- MTs’ arrival intensities $\nu$ between two auctions.
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**Euronext Data:** details on the volumes of each stocks exchanged along the day.

$\nu = \frac{V}{T}$ where $T$: duration of the trading day, $V$: average daily volume of market orders.

$K = (2e - \frac{\varsigma}{e})$ and $\mu = \nu(\frac{e}{K} - 1)$, where $e$ and $\varsigma$ are the average and squared volumes (resp.) present in the first limit of the LOB.

For sophisticated MTs we take $\lambda_{-} = \nu/4$ and $\lambda_{+} = \nu$. 
Table: Optimal auction durations of a sample of stocks traded at Euronext (in seconds) and comparison with the LOB $E(0) - E(h^*)$. 

<table>
<thead>
<tr>
<th>Stock Name</th>
<th>DiffNash</th>
<th>DiffPoiss</th>
<th>Poisson</th>
<th>Nash</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aperam</td>
<td>0.389</td>
<td>0.159</td>
<td>536</td>
<td>375.1</td>
</tr>
<tr>
<td>Sodexo</td>
<td>0.072</td>
<td>0.0</td>
<td>0.0</td>
<td>30</td>
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<tr>
<td>Air France - KLM</td>
<td>0.821</td>
<td>0.452</td>
<td>295</td>
<td>218</td>
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<td>Hermes</td>
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<td>295</td>
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<td>ArcelorMittal</td>
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<td>Air Liquide</td>
<td>0.916</td>
<td>0.647</td>
<td>627</td>
<td>459</td>
</tr>
</tbody>
</table>
Thank you.