Liquidity in competitive dealer markets

Peter Bank

joint work with
Ibrahim Ekren and Johannes Muhle-Karbe

4th Berlin-Princeton-Singapore Workshop on Quantitative Finance
National University of Singapore, 18 - 20 March 2019
Frictions in financial modelling

- Classical Black-Scholes theory: dynamic trading of arbitrary amounts, arbitrarily fast without affect on exogenously given asset prices and without taxes, transaction fees, etc.
- How to account for these nonlinear effects? Formidable challenges at the interfaces between financial modelling, stochastic analysis, and stochastic optimal control
- “Equilibrium models” versus cost specifications
- Illiquidity due to differences in information (Glosten-Milgrom ’85, Kyle ’85) and/or due to inventory risk (Ho-Stoll ’81, Grossman-Miller ’88): \( \leq 3 \) period models
- Dynamic equilibrium type models: Back ’90, Garleanu-Pedersen-Poteshman ’09, Kramkov-Pulido ’16, B.-Kramkov ’15, Sannikov-Skrzypacz ’16, Cetin ’17
- Cost specifications: Soner-Shreve ’94, Almgren-Chriss ’01, Obizhaeva-Wang ’13, Roch-Soner ’13, Bouchard et al. ’18
Frictions in financial modelling

- Classical Black-Scholes theory: dynamic trading of arbitrary amounts, arbitrarily fast without affect on exogenously given asset prices and without taxes, transaction fees, etc.
- How to account for these nonlinear effects? Formidable challenges at the interfaces between financial modelling, stochastic analysis, and stochastic optimal control
- “Equilibrium models” and cost specifications
- Illiquidity due to differences in information (Glosten-Milgrom ’85, Kyle ’85) and/or due to inventory risk (Ho-Stoll ’81, Grossman-Miller ’88): \( \leq 3 \) period models
- Dynamic equilibrium type models: Back ’90, Garleanu-Pedersen-Poteshman ’09, Kramkov-Pulido ’16, B.-Kramkov ’15, Sannikov-Skrzypacz ’16, Cetin ’17
- Cost specifications: Soner-Shreve ’94, Almgren-Chriss ’01, Obizhaeva-Wang ’13, Roch-Soner ’13, Bouchard et al. ’18
Dramatis personae

An FX desk’s business as a *ménage a trois*… (Butz-Oomen ’17):

▶ **Dealers**: compete quoting FX rates, exchange currencies to their clients; transfer inventory to end-users at a finite rate at fundamental exchange rate, thereby incurring search costs and inventory risk

▶ **Clients**: demand currency positions from their dealers; orders get filled at competitive rates

▶ **“End-users”**: accept positions at exogenous, fundamental FX rates; dealers can only find them incurring search costs
Dramatis personae

An FX desk’s business as a *ménage a trois*... (Butz-Oomen ’17):

- **Dealers**: compete quoting FX rates, exchange currencies to their clients; transfer inventory to end-users at a finite rate at fundamental exchange rate, thereby incurring search costs and inventory risk
- **Clients**: demand currency positions from their dealers; orders get filled at competitive rates
- **“End-users”**: accept positions at exogenous, fundamental FX rates; dealers can only find them incurring search costs

Questions:

- How do the **dealers’** prices (FX rates) match demand with supply? How are they related to fundamentals? What role is played by the dealers’ search costs and holding costs?
- How should **clients** choose their demand to manage their exogenously given risk? What if they internalize their impact? Do they benefit from the dealers’ presence?
- Who are the **end-users**?
The dealers’ problem

For FX quotes \((S_t)\) and fundamental FX rates \((V_t)\), the dealers servicing their clients’ requested positions \((K_t)\) and cumulatively transferring \(U_t = \int_0^t u_s \, ds\) to the end-users at costs \(\frac{\lambda}{2} u_t^2 \, dt\) in \(t \in [0, T]\), will generate proceeds

\[
\int_0^T (-K_t) dS_t - (V_T - S_T) K_T + \int_0^T U_t dV_t - \frac{\lambda}{2} \int_0^T u_t^2 \, dt.
\]

Assuming \(V\) is a martingale, i.e., ruling out speculation on FX rates trends etc., we get the dealers’ expected proceeds to be

\[
\mathbb{E}\left[ \int_0^T (-K_t) dS_t - (V_T - S_T) K_T - \frac{\lambda}{2} \int_0^T u_t^2 \, dt \right].
\]

The dealers’ inventory risk is determined by \(U - K\):

\[
\frac{1}{2} \mathbb{E}\left[ \int_0^T (K_t - U_t)^2 \, dt \right].
\]
The dealers’ problem

Dealers’ target functional with holding costs $\gamma_d > 0$:

$$J_d(K, u; S) \triangleq \mathbb{E} \left[ \int_0^T (-K_t) dS_t - (V_T - S_T)K_T - \frac{\lambda}{2} \int_0^T u_t^2 dt \right]$$

$$- \frac{\gamma_d}{2} \mathbb{E} \left[ \int_0^T (K_t - U_t)^2 dt \right] \rightarrow \max_{K, u}$$

Observe: Problem can be addressed in two stages.

Stage 1:
Given $K$, maximization over $u$ is a quadratic tracking problem

$$E \left[ \gamma_d \int_0^T (K_t - U_t)^2 dt + \lambda \int_0^T u_t^2 dt \right] \rightarrow \min u$$

as solved explicitly in B., Soner, Voß'17.

Stage 2:
Given the optimal transfer policy $u_{K}$ for any $K$, optimize over $K$.
The dealers’ problem

Dealers’ target functional with holding costs $\gamma_d > 0$:

$$J_d(K, u; S) \triangleq \mathbb{E} \left[ \int_0^T (-K_t) dS_t - (V_T - S_T)K_T - \frac{\lambda}{2} \int_0^T u_t^2 dt \right]$$

$$- \frac{\gamma_d}{2} \mathbb{E} \left[ \int_0^T (K_t - U_t)^2 dt \right] \rightarrow \max_{K, u}$$

Observe: Problem can be addressed in two stages.

**Stage 1:** Given $K$, maximization over $u$ is a quadratic tracking problem

$$\mathbb{E} \left[ \frac{\gamma_d}{2} \int_0^T (K_t - U_t)^2 dt + \frac{\lambda}{2} \int_0^T u_t^2 dt \right] \rightarrow \min_u$$

as solved explicitly in B., Soner, Voß’17.

**Stage 2:** Given the optimal transfer policy $u^K$ for any $K$, optimize over $K$. 
Quadratic tracking problem

Theorem (B., Soner, Voß’17)

The dealers’ optimal trading rate minimizing

\[ \mathbb{E} \left[ \frac{\gamma d}{2} \int_0^T (K_t - U_t)^2 \, dt + \frac{\lambda}{2} \int_0^T u_t^2 \, dt \right] \]

is

\[ u^K_t \triangleq \frac{d}{dt} U^K_t = \frac{\tanh((T - t)/\sqrt{\kappa})}{\sqrt{\kappa}} (\hat{K}_t - U^K_t) \]

where

\[ \kappa \triangleq \lambda/\gamma_d \text{ and } \hat{K}_t \triangleq \mathbb{E} \left[ \int_t^T K_u \frac{\cosh((T - u)/\sqrt{\kappa})}{\sqrt{\kappa} \sinh((T - t)/\sqrt{\kappa})} \, du \mid \mathcal{F}_t \right] \]

\[ \leadsto \text{ Dealers form a view } \hat{K} \text{ on expected future demand and trade with the end-users towards this ideal position.} \]
Quadratic tracking problem with terminal constraint

Theorem (B., Soner, Voß’17)

The dealers’ optimal trading rate minimizing

$$\mathbb{E} \left[ \frac{\gamma_d}{2} \int_0^T (K_t - U_t)^2 \, dt + \frac{\lambda}{2} \int_0^T u_t^2 \, dt \right]$$

subject to $U_T = K_T$ is

$$u^K_t \triangleq \frac{d}{dt} U^K_t = \coth((T - t)/\sqrt{\kappa}) \left( \hat{K}_t - U^K_t \right)$$

where, as before, $\kappa \triangleq \lambda/\gamma_d$, but now

$$\hat{K}_t = \frac{1}{\cosh(\frac{T-t}{\sqrt{\kappa}})} \mathbb{E} [K_T | \mathcal{F}_t]$$

$$+ \left( 1 - \frac{1}{\cosh(\frac{T-t}{\sqrt{\kappa}})} \right) \mathbb{E} \left[ \int_t^T K_s \frac{\sinh(\frac{T-s}{\sqrt{\kappa}})}{(\cosh(\frac{T-t}{\sqrt{\kappa}}) - 1) \sqrt{\kappa}} \, ds \right] \mathcal{F}_t$$.
Illustration: Deterministic demand expanding midway

**Figure:** Demand $K$ with a jump at $t = T/2$ (blue)
Figure: Demand $K$ with a jump at $t = T/2$ (blue), dealers’ unconstrained (orange, dashed) and constrained (green, dashed) target $\hat{K}$. 
Illustration: Deterministic demand expanding midway

Figure: Demand $K$ with a jump at $t = T/2$ (blue), dealers’ unconstrained (orange, dashed) and constrained (green, dashed) target $\hat{K}$, corresponding unconstrained (orange) and constrained (green) transfer policy $u^K$, and myopic transfer policy (red)
Figure: Demand $K$ with a jump at $t = T/2$ (blue), dealers’ unconstrained (orange, dashed) and constrained (green, dashed) target $\hat{K}$, corresponding unconstrained (orange) and constrained (green) transfer policy $u^K$, and myopic transfer policy (red)
Figure: Demand $K$ with a jump at $t = T/2$ (blue), dealers’ unconstrained (orange, dashed) and constrained (green, dashed) target $\hat{K}$, corresponding unconstrained (orange) and constrained (green) transfer policy $u^K$, and myopic transfer policy (red).
Figure: Demand $K$ with a jump at $t = T/2$ (blue), dealers’ unconstrained (orange, dashed) and constrained (green, dashed) target $\hat{K}$, corresponding unconstrained (orange) and constrained (green) transfer policy $u^K$, and myopic transfer policy (red)
Illustration: Deterministic demand expanding midway

Figure: Demand $K$ with a jump at $t = T/2$ (blue), dealers’ unconstrained (orange, dashed) and constrained (green, dashed) target $\hat{K}$, corresponding unconstrained (orange) and constrained (green) transfer policy $u^K$, and myopic transfer policy (red)
Illustration: Deterministic demand expanding midway

**Figure:** Demand $K$ with a jump at $t = T/2$ (blue), dealers’ unconstrained (orange, dashed) and constrained (green, dashed) target $\hat{K}$, corresponding unconstrained (orange) and constrained (green) transfer policy $u^K$, and myopic transfer policy (red)
Figure: Demand $K$ with a jump at $t = T/2$ (blue), dealers’ unconstrained (orange, dashed) and constrained (green, dashed) target $\hat{K}$, corresponding unconstrained (orange) and constrained (green) transfer policy $u^K$, and myopic transfer policy (red)
Illustration: Deterministic demand expanding midway

**Figure:** Demand $K$ with a jump at $t = T/2$ (blue), dealers’ unconstrained (orange, dashed) and constrained (green, dashed) target $\hat{K}$, corresponding unconstrained (orange) and constrained (green) transfer policy $u^K$, and myopic transfer policy policy (red).
Illustration: Deterministic demand expanding midway

**Figure:** Demand $K$ with a jump at $t = T/2$ (blue), dealers’ unconstrained (orange, dashed) and constrained (green, dashed) target $\hat{K}$, corresponding unconstrained (orange) and constrained (green) transfer policy $u^K$, and myopic transfer policy (red).
Back to our equilibrium considerations . . .
Back to our equilibrium considerations . . .

**Stage 2:** Dealers’ target functional with holding costs $\gamma_d > 0$:

\[
J_d(K; S) \triangleq \mathbb{E} \left[ \int_0^T (-K_t) dS_t - (V_T - S_T)K_T \right] \\
- \mathbb{E} \left[ \frac{\gamma_d}{2} \int_0^T (K_t - U^K_t)^2 dt + \frac{\lambda}{2} \int_0^T (u^K_t)^2 dt \right] \rightarrow \max_K
\]
Back to our equilibrium considerations...  

**Stage 2:** Dealers’ target functional with holding costs $\gamma_d > 0$:

$$J_d(K; S) \triangleq \mathbb{E} \left[ \int_0^T (-K_t) dS_t - (V_T - S_T)K_T \right]$$

$$- \mathbb{E} \left[ \frac{\gamma_d}{2} \int_0^T (K_t - U^K_t)^2 dt + \frac{\lambda}{2} \int_0^T (u^K_t)^2 dt \right] \rightarrow \max_K$$

FX quotes $(S_t)$ will generate an **equilibrium** if at these quotes the dealers’ optimal supply matches their clients’ demand $\mathcal{K}$:

$$\mathcal{K} \in \arg \max_K J_d(K; S)$$
Back to our equilibrium considerations . . .

**Stage 2:** Dealers’ target functional with holding costs $\gamma_d > 0$:

$$J_d(K; S) \triangleq \mathbb{E} \left[ \int_0^T (-K_t) dS_t - (V_T - S_T) K_T \right]$$

$$- \mathbb{E} \left[ \frac{\gamma_d}{2} \int_0^T (K_t - U^K_t)^2 \, dt + \frac{\lambda}{2} \int_0^T (u^K_t)^2 \, dt \right] \rightarrow \max_K$$

FX quotes $(S_t)$ will generate an **equilibrium** if at these quotes the dealers’ optimal supply matches their clients’ demand $K$:

$$\mathcal{K} \in \arg \max_K J_d(K; S)$$

**Theorem**

*Given clients’ demand $\mathcal{K}$, the unique equilibrium quotes $S^K$ are*

$$S^K_t \triangleq V_t + \gamma_d \mathbb{E} \left[ \int_t^T (\mathcal{K}_s - U^K_s) \, ds \bigg| \mathcal{F}_t \right], \quad 0 \leq t \leq T,$$

where $U^K$ describes the dealers’ optimal cumulative transfers to the end-users as determined by B., Soner, Voß ’17.
Equilibrium

\[ S_t^{\mathcal{H}} = V_t + \gamma_d \mathbb{E} \left[ \int_t^T (\mathcal{H}_s - U_s^{\mathcal{H}}) \, ds \mid \mathcal{F}_t \right], \quad 0 \leq t \leq T, \]

- fundamental value \( V \) adjusted for dealers’ effective risk
- adjustment in line with asymptotic expansion for small dealer risk aversion in exponential utility setting by Kramkov-Pulido ’16 (who do not consider end-users)
- small search costs asymptotics of dealers’ surcharge depend on demand regularity:
  - absolutely continuous demand \( \mathcal{H} = \int_0^t \mu_t^{\mathcal{H}} \, dt \):
    \[ \int_0^T K_t d(V_t - S_t^{\mathcal{H}}) = \lambda \int_0^T (\mu_t^{\mathcal{H}})^2 \, dt + o(\lambda) \text{ in } L^1 \text{ as } \lambda \downarrow 0 \]
  - diffusive demand \( \mathcal{H} = \int_0^t (\mu_t^{\mathcal{H}} \, dt + \sigma_t^{\mathcal{H}} \, dW_t) \):
    \[ \int_0^T \mathcal{H}_t d(V_t - S_t^{\mathcal{H}}) = \sqrt{\lambda \gamma_d} \int_0^T (\sigma_t^{\mathcal{H}})^2 \, dt + o(\sqrt{\lambda}) \text{ in } L^1 \text{ as } \lambda \downarrow 0 \]
- endogenous price impact model \textit{with resilience}, in contrast to B.-Kramkov ’15
The clients’ problem

How should the clients choose their demand $\mathcal{H}$ given quotes $(S_t)$?

Quadratic criterion:
Facing exogenous FX exposure $(\zeta_t)$, the clients seek to maximize

$$J_c(K; S) \equiv \mathbb{E}\left[\int_0^T K_t \, dS_t\right] - \gamma_c^2 \mathbb{E}\left[\int_0^T (\zeta_t - K_t)^2 \, dt\right] \to \max_K$$

If $(S_t)$ has drift $(\mu_t)$, this amounts to

$$\mathbb{E}\left[\int_0^T (K_t \mu_t - \gamma_c^2 (\zeta_t - K_t)^2) \, dt\right] \to \max_K$$

i.e. $K^*_t = \zeta_t - \mu_t / \gamma_c$

Given demand $K^*_t$, the equilibrium quotes’ drift is

$$\mu_{K^*_t} = -\gamma_d (K^*_t - U_{K^*_t})$$

which yields the equilibrium demand equation:

$$K^*_t = \gamma_d \gamma_d + \gamma_c U_{K^*_t} + \gamma_c \gamma_d + \gamma_c \zeta_t, \quad t \in [0, T]$$

where, again, $U_{K^*_t}$ is as in B., Soner, Voß ’17.
The clients’ problem

How should the clients choose their demand $\mathcal{K}$ given quotes $(S_t)$?

**Quadratic criterion:** Facing exogenous FX exposure $(\zeta_t)$, the clients seek to maximize

$$J_c(\mathcal{K} ; S) \triangleq \mathbb{E} \left[ \int_0^T \mathcal{K}_t dS_t \right] - \frac{\gamma_c}{2} \mathbb{E} \left[ \int_0^T (\zeta_t - \mathcal{K}_t)^2 dt \right] \to \max_{\mathcal{K}}$$

If $(S_t)$ has drift $(\mu_t)$, this amounts to

$$\mathbb{E} \left[ \int_0^T \left( \mathcal{K}_t \mu_t - \frac{\gamma_c}{2} (\zeta_t - \mathcal{K}_t)^2 \right) dt \right] \to \max_{\mathcal{K}}, \text{ i.e. } \mathcal{K}_t^* = \zeta_t - \mu_t / \gamma_c$$
The clients’ problem

How should the clients choose their demand $\mathcal{K}$ given quotes $(S_t)$?

**Quadratic criterion:** Facing exogenous FX exposure $(\zeta_t)$, the clients seek to maximize

$$J_c(\mathcal{K}; S) \triangleq \mathbb{E} \left[ \int_0^T \mathcal{K}_t dS_t \right] - \frac{\gamma_c}{2} \mathbb{E} \left[ \int_0^T (\zeta_t - \mathcal{K}_t)^2 dt \right] \to \max_{\mathcal{K}}$$

If $(S_t)$ has drift $(\mu_t)$, this amounts to

$$\mathbb{E} \left[ \int_0^T \left( \mathcal{K}_t \mu_t - \frac{\gamma_c}{2} (\zeta_t - \mathcal{K}_t)^2 \right) dt \right] \to \max_{\mathcal{K}}, \text{ i.e. } \mathcal{K}_t^* = \zeta_t - \mu_t / \gamma_c$$

Given demand $\mathcal{K}^*$, the equilibrium quotes’ $S^{\mathcal{K}^*}$ drift is

$$\mu_t^{\mathcal{K}^*} = -\gamma_d (\mathcal{K}_t^* - U_t^{\mathcal{K}^*})$$

which yields the **equilibrium demand equation**:

$$\mathcal{K}_t^* = \frac{\gamma_d}{\gamma_d + \gamma_c} U_t^{\mathcal{K}^*} + \frac{\gamma_c}{\gamma_d + \gamma_c} \zeta_t, \quad t \in [0, T],$$

where, again, $U^{\mathcal{K}^*}$ is as in B., Soner, Voß '17.
Equilibrium demand

The equilibrium demand equation:

$$\mathcal{K}_t^* = \frac{\gamma d}{\gamma d + \gamma c} U_t \mathcal{K}_t^* + \frac{\gamma c}{\gamma d + \gamma c} \zeta_t, \quad t \in [0, T],$$

is an integral equation for \( \mathcal{K}^* \).
Equilibrium demand

The equilibrium demand equation:

\[ \mathcal{H}_t^* = \frac{\gamma_d}{\gamma_d + \gamma_c} U_t^* + \frac{\gamma_c}{\gamma_d + \gamma_c} \zeta_t, \quad t \in [0, T], \]

is an integral equation for \( \mathcal{H}^* \). With

\[ k_t \triangleq \mathcal{H}_t^* - \frac{\gamma_c}{\gamma_d + \gamma_c} \zeta_t \quad \text{and} \quad K_t \triangleq \mathbb{E} \left[ \int_t^T \mathcal{H}_u^* \frac{\cosh((T - u)/\sqrt{\kappa})}{\sqrt{\kappa} \cosh((T - t)/\sqrt{\kappa})} du \ \bigg| \mathcal{F}_t \right] \]

it is equivalent to the linear forward backward stochastic differential equation (FBSDE):

\[ k_0 = 0, \quad dk_t = \left( \frac{\gamma_d}{\gamma_d + \gamma_c} K_t - \frac{\tanh((T - t)/\sqrt{\kappa})}{\sqrt{\kappa}} k_t \right) dt, \]

\[ K_T = 0, \quad dK_t = \left( \frac{\tanh((T - t)/\sqrt{\kappa})}{\sqrt{\kappa}} K_t - \frac{1}{\kappa} \left( k_t + \frac{\gamma_c}{\gamma_d + \gamma_c} \zeta_t \right) \right) dt + dM^K_t \]

for a suitable martingale \( M^K \) determined uniquely by the FBSDE.
Equilibrium demand

Theorem
The unique equilibrium demand is given explicitly by

\[ \mathcal{K}_t^* = \gamma_c \zeta_t + \tilde{U}_t^{\gamma_d/\gamma_d + \gamma_c} \zeta, \quad t \in [0, T] \]

where \( \tilde{U}_t^{\gamma_d/\gamma_d + \gamma_c} \) denotes the tracking portfolio from B., Soner, Voß:

\[ \frac{d}{dt} \tilde{U}_t^{\gamma_d/\gamma_d + \gamma_c} \zeta = \tanh((T - t)/\sqrt{\tilde{\kappa}}) \left( \frac{\gamma_d}{\gamma_d + \gamma_c} \zeta_t - \tilde{U}_t^{\gamma_d/\gamma_d + \gamma_c} \zeta \right), \]

for the aggregate holding costs \( \tilde{\gamma} = (1/\gamma_d + 1/\gamma_c)^{-1} \), i.e.,

\[ \tilde{\kappa} \triangleq \lambda/\tilde{\gamma} \text{ and } \tilde{\zeta}_t \triangleq \mathbb{E} \left[ \int_t^T \zeta_u \frac{\cosh((T - u)/\sqrt{\tilde{\kappa}})}{\sqrt{\tilde{\kappa}} \sinh((T - t)/\sqrt{\tilde{\kappa}})} \, du \, \bigg| \mathcal{F}_t \right]. \]

This balances the clients’ demand for immediacy with their holding costs, taking into account also their dealers’ holding costs and their ability of transferring risk to end-users: \( \tilde{U} \zeta = U \mathcal{K}^* \).
When do the clients really need their dealers?

**Example: Constant target position**

**Figure:** Risk or holding costs vs. search costs when clients are trading through their dealers’ or are searching end-users themselves.
What if the clients are collectively aware of their impact?

In other words: What if the dealers are facing a large trader?
What if the clients are collectively aware of their impact?

In other words: What if the dealers are facing a large trader?

**Quadratic criterion:** Facing exogenous FX cash flow \((\zeta_t)\), the large investor seeks to maximize

\[
J_c(\mathcal{K}) \triangleq \mathbb{E} \left[ \int_0^T \mathcal{K}_t \, dS_t \right] - \frac{\gamma_c}{2} \mathbb{E} \left[ \int_0^T (\zeta_t - \mathcal{K}_t)^2 \, dt \right] \rightarrow \max_{\mathcal{K}}
\]
What if the clients are collectively aware of their impact?

In other words: What if the dealers are facing a large trader?

**Quadratic criterion:** Facing exogenous FX cash flow \( (\zeta_t) \), the large investor seeks to maximize

\[
J_c(\mathcal{K}) \triangleq \mathbb{E} \left[ \int_0^T \mathcal{K}_t dS_t \mathcal{K} \right] - \frac{\gamma_c}{2} \mathbb{E} \left[ \int_0^T (\zeta_t - \mathcal{K}_t)^2 dt \right] \rightarrow \max_{\mathcal{K}}
\]

This is still **concave** in \( \mathcal{K} \) since \( \mathcal{K} \mapsto -\mathbb{E} \left[ \int_0^T \mathcal{K}_t dS_t \mathcal{K} \right] \) is the dealers’ expected profit in equilibrium and thus nonnegative.

\[\leadsto \text{no statistical arbitrage} \] in this model with **endogenously derived market impact**.
What if the clients are collectively aware of their impact?

In other words: What if the dealers are facing a large trader?

**Quadratic criterion:** Facing exogenous FX cash flow \((\zeta_t)\), the large investor seeks to maximize

\[
J_c(\mathcal{K}) \equiv \mathbb{E} \left[ \int_0^T \mathcal{K}_t \, dS_t \mathcal{K} \right] - \frac{\gamma_c}{2} \mathbb{E} \left[ \int_0^T (\zeta_t - \mathcal{K}_t)^2 \, dt \right] \to \max_{\mathcal{K}}
\]

This is still **concave** in \(\mathcal{K}\) since \(\mathcal{K} \mapsto -\mathbb{E} \left[ \int_0^T \mathcal{K}_t \, dS_t \mathcal{K} \right]\) is the dealers’ expected profit in equilibrium and thus nonnegative. \(\mapsto\) no statistical arbitrage in this model with **endogenously derived market impact**.

Remarkably, first order condition for optimality now reads

\[
\mathcal{K}_t^* = \frac{\gamma_d}{\gamma_d + \gamma_c/2} U_t \mathcal{K}^* + \frac{\gamma_c/2}{\gamma_d + \gamma_c/2} \zeta_t, \quad t \in [0, T],
\]

i.e. the **same equilibrium demand equation** as before, albeit with half the clients’ holding costs.
What if the clients are collectively aware of their impact?

In other words: What if the dealers are facing a large trader?

**Quadratic criterion:** Facing exogenous FX cash flow $(\zeta_t)$, the large investor seeks to maximize

$$J_c(\mathcal{K}) \triangleq \mathbb{E} \left[ \int_0^T \mathcal{K}_t dS_t \mathcal{K} \right] - \frac{\gamma_c}{2} \mathbb{E} \left[ \int_0^T (\zeta_t - \mathcal{K}_t)^2 dt \right] \to \max_{\mathcal{K}}$$

This is still **concave** in $\mathcal{K}$ since $\mathcal{K} \mapsto -\mathbb{E} \left[ \int_0^T \mathcal{K}_t dS_t \mathcal{K} \right]$ is the dealers’ expected profit in equilibrium and thus nonnegative. $
\iff$ **no statistical arbitrage** in this model with endogenously derived market impact.

Remarkably, first order condition for optimality now reads

$$\mathcal{K}_t^* = \frac{\gamma_d}{\gamma_d + \gamma_c/2} U_t^{\mathcal{K}^*} + \frac{\gamma_c/2}{\gamma_d + \gamma_c/2} \zeta_t, \quad t \in [0, T],$$

i.e. the **same equilibrium demand equation** as before, albeit with **half** the clients’ holding costs.

“Price of anarchy”: $J_c(\mathcal{K}^*) \geq J_c(\mathcal{K}^*) = J_c(\mathcal{K}^*; S^{\mathcal{K}^*})$
Conclusions

- analyzed dealer market with clients and end-users
- quadratic setting allows for explicit computations following previous optimal tracking results
- equilibrium quotes for arbitrary demand take into account legacy position and expected future positions
- optimization of demand with and without impact awareness
- dealers will be used if their search and holding costs are small compared to those of their clients
- harder to serve sophisticated clients aware of their impact
- endogenously derived impact model ruling out statistical arbitrage
- asymptotic analysis for small search costs
Conclusions

▸ analyzed dealer market with clients and end-users
▸ quadratic setting allows for explicit computations following previous optimal tracking results
▸ equilibrium quotes for arbitrary demand take into account legacy position and expected future positions
▸ optimization of demand with and without impact awareness
▸ dealers will be used if their search and holding costs are small compared to those of their clients
▸ harder to serve sophisticated clients aware of their impact
▸ endogenously derived impact model ruling out statistical arbitrage
▸ asymptotic analysis for small search costs

Thank you very much!