Speculative Trade and Market Newcomers

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Motivation and Contribution

• Inexperienced, young or immigrant turned investors share markets with veteran traders

• Economists and finance scholars have two conflicting opinions about the role of experienced investors:
  1. That they abate deviations of prices from fundamentals or even burst bubbles
  2. That they systematically take advantage of newcomers and sell them assets above fundamental values

• Sufficient conditions for a perpetual bubble of this type?

• This is my contribution
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When Do Such Bubbles Occur?

• Generally speaking, my paper predicts such bubbles if
  (1) conditional on same information, most experienced cohort
  does not tend to be most optimistic about fundamentals
  (2) short-selling is limited

• Proved in deliberately simple, tractable model

• But underlying intuition has real-world appeal
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Intuition

1. Asset prices ≥ subjective fundamental valuations of most optimistic traders

2. Anticipate to get premium at least at end of their lives
   - Eventually in public contingencies become relatively pessimistic (by assumption)
   - Short-sales constraints ⇒ possibility to sell assets above their updated fundamental valuations

3. Willing to pay above their fundamental valuations today

4. Asset prices > most optimistic fundamental valuations

5. Perpetual bubble
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Room for Such Disagreement of Opinion?

- Claimed conditions for bubble require disagreement of opinion between veterans and novices conditional on public information.
- Possible foundation in terms of newcomers' prior beliefs.
- They are over future at time of market entry.
- They are not over entire history.
- They incorporate information about past rather coarsely.
- My model generates heterogeneous beliefs this way even though traders are identical at time of market entry.
- Traders learn identically as time goes by.
- Heterogeneity $\equiv$ variation in duration of market participation and learning.
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Pessimistic Veteran Investors?

Claimed conditions for bubble require most experienced cohort to be relatively pessimistic

How plausible?

My model: for nonnegligible set of parameter values

Somewhat compatible empirical evidence

Stock prices reaching two-year lows, last week of March 2001

Investors above 60 were more likely to say stocks were overvalued (39% vs. 25%)

No significant differences from bullish market in 1998

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Timing in My Model
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- Trader enters market
- Identical trader enters

\[ \forall \tau \]

\[ 0 \]

\[ \text{Time (R)} \]
• Believes asset pays $1 dividend at uncertain time \( \theta \in (0, \infty) \)
• Single asset in my model
• This asset actually does not pay dividends
• Finite and strictly positive number of units of asset
• Beliefs about dividend time \( \theta \) are Gamma \((\alpha, \beta)\)
• Density \(f_{\alpha, \beta}(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\theta/\beta}\)
• Cumulative distribution \(F_{\alpha, \beta}(t) = \int_0^t f_{\alpha, \beta}(\theta) d\theta\)
Time 0 Entrant

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- Beliefs about dividend time $\theta$ are Gamma ($\alpha, \beta$)
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Time 0 Entrant’s Valuation

- Fundamental valuation measures
- Willingness to pay for asset if obliged to hold it forever
- Beliefs + risk-free rate, $r > 0$
- Fundamental valuation of asset
- Constant over time
- Same for all traders

At every $t \in (0, \infty)$, fundamental valuation is

$$V_0(t) = \frac{1}{1 - F_{\alpha,\beta}(t)} \int_{\infty}^{t} e^{r(t - \theta)} f_{\alpha,\beta}(\theta) d\theta$$

At 0, fundamental valuation is

$$V_0(0) = \lim_{t \to 0^+} V_0(t) = \left( \frac{\beta r + 1}{\alpha} \right)^{9/21}$$
Time 0 Entrant’s Valuation

- Fundamental valuation measures willingness to pay for asset if obliged to hold it forever
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  \]
Time $\tau$ Entrant

• Enters after past entrants observe that asset does not pay at $\tau$
• Believes that asset pays dividend at uncertain time $\theta \in (\tau, \infty)$
• Fundamental valuation at every $t \in [\tau, \infty)$ is $V_\tau(t) = V_0(t - \tau)$
Time $\tau$ Entrant

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V_\tau(t) = V_0(t - \tau)
\]
Market Exit

• All traders stay in market for fixed duration $T \in (0, \infty)$
• Time $\tau$ entrant leaves at $\tau + T$ after new trader enters
• Traders do not know for how long they will live
• Traders can plan to hold asset forever
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Trade and Prices

Trade occurs with period $\Delta \in (0, T)$ at time points $\Delta Z$.

- Strictly after new traders enter.
- Strictly before old traders leave.

- Steady-state price $p \in \mathbb{R}$.

- Limited short-selling.

- Every trader's demand for asset when trader participates in market is a simple function of trader's reservation prices $\varphi_q$.

- Expected discounted returns of holding asset.
Trade and Prices

- Trade occurs with period $\Delta \in (0, T)$ at time points $\Delta \mathbb{Z}$.
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Expected Discounted Returns
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- Time 0 entrant
Expected Discounted Returns

• Time 0 entrant
  • At every $t \in (0, \infty)$,
    for holding asset to arbitrary $s \in (t, \infty)$:

  \[
  R_0(t, s) = \frac{1}{1 - F_{\alpha,\beta}(t)} \int_t^s e^{r(t-\theta)} f_{\alpha,\beta}(\theta) \, d\theta + \frac{1 - F_{\alpha,\beta}(s)}{1 - F_{\alpha,\beta}(t)} e^{r(t-s)} p
  \]

  \[
  = V_0(t) + \frac{1 - F_{\alpha,\beta}(s)}{1 - F_{\alpha,\beta}(t)} e^{r(t-s)} (p - V_0(s))
  \]
Expected Discounted Returns

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    \]

- At 0, for holding asset to arbitrary $s \in (0, \infty)$:
  \[
  R_0(0, s) = V_0(0) + (1 - F_{\alpha,\beta}(s)) e^{-rs} (p - V_0(s))
  \]
Expected Discounted Returns

- Time 0 entrant
  - At every $t \in (0, \infty)$, for holding asset to arbitrary $s \in (t, \infty)$:

$$R_0(t, s)$$

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- At 0, for holding asset to arbitrary $s \in (0, \infty)$:

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- Time $\tau$ entrant
Expected Discounted Returns

• **Time 0 entrant**
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• **Time $\tau$ entrant**
  • At every $t \in [\tau, \infty)$, for holding to arbitrary $s \in (t, \infty)$:
    \[
    R_\tau(t, s) = R_0(t - \tau, s - \tau)
    \]
Reservation Prices

• Reservation prices, for every time $\tau$ entrant, at every $t \in [\tau, \tau + T) \cap (\Delta Z)$

• For holding asset to arbitrary $s \in (t, \infty) \cap (\Delta Z)$:
  
  \[ P_{\tau}(t, s) = R_{\tau}(t, s) \]

• For holding asset forever:
  
  \[ P_{\tau}(t, \infty) = V_{\tau}(t) \]
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Reservation Prices

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- For holding asset forever:
  \[
P_\tau (t, \infty) = V_\tau (t)
  \]
Demand

Infinite demand

Zero demand

Arbitrary demand

\[ p = P_{\tau}(t, s) \]

Demand by time \( \tau \) entrant at \( t \) for holding asset to \( s \) \( s \in (t, \infty) \cap (\Delta Z) \cup \{\infty\} \)
Demand

\[ p = P_\tau(t, s) \]

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Harrison-Kreps Equilibrium Price
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- $p$ clears market at $t \in \Delta \mathbb{Z} \iff p$ is max of reservation prices

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\{P_\tau(t, s) : \tau \in [t - T, t], s \in ((t, \infty) \cap (\Delta \mathbb{Z})) \cup \{\infty\}\}
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= \{P_\tau(0, s) : \tau \in [-T, 0], s \in ((0, \infty) \cap (\Delta \mathbb{Z})) \cup \{\infty\}\}
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• Due to above identity,
  $p$ clears market at $0 \iff p$ clears market at all $t \in \Delta \mathbb{Z}$
Harrison-Kreps Equilibrium Price

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\end{align*}$$

- Due to above identity,
  $p$ clears market at $0 \iff p$ clears market at all $t \in \Delta \mathbb{Z}$

- Any such price is called Harrison-Kreps equilibrium price
Proposition

The shape of the valuation function $V_0$ depends on parameter $\alpha$ as follows:

- If $\alpha < 1$, then $V_0$ is strictly decreasing.
- If $\alpha = 1$, then $V_0$ is constant.
- If $\alpha > 1$, then $V_0$ is strictly increasing.

<table>
<thead>
<tr>
<th>$\alpha$</th>
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<th>Claim that bubble occurs</th>
</tr>
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<td>yes</td>
</tr>
<tr>
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Claimed conditions for bubble hold in my model $\iff \alpha < 1$. 
Proposition

*Shape of valuation function* $V_0 : [0, \infty) \to \mathbb{R}$

depends on parameter $\alpha$ as follows:

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Optimism as Function of Experience

Proposition

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Table: Relative optimism of most experienced trader as function of $\alpha$

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Claimed conditions for bubble hold in my model $\iff \alpha < 1$
Sufficiency Formally

Proposition (BUBBLE)

*If* \( \alpha < 1 \) *and* \( p \) *is Harrison-Kreps equilibrium price, then* \( p > V_0(t), \forall t \geq 0 \)
Existence and Properties of Equilibrium

Proposition

If $\alpha < 1$, then unique Harrison-Kreps equilibrium price is

$$P(0, \Delta) = V_0(\Delta) - (1 - F_{\alpha, \beta}(\Delta)) e^{-r \Delta} \frac{1}{1 - (1 - F_{\alpha, \beta}(\Delta)) e^{-r \Delta}}$$

In this equilibrium, at every trading time point $t \in \Delta \subset \mathbb{Z}$ only time $t$ entrant holds asset

Proposition

Mapping $\Delta \mapsto P(0, \Delta)$ on $(0, \infty)$ into $\mathbb{R}$ is strictly decreasing — i.e., bubble is increasing in trading frequency.
Existence and Properties of Equilibrium

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If $\alpha < 1$, then unique Harrison-Kreps equilibrium price is

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P (0, \Delta) = \frac{V_0 (0) - (1 - F_{\alpha,\beta} (\Delta)) e^{-r\Delta} V_0 (\Delta)}{1 - (1 - F_{\alpha,\beta} (\Delta)) e^{-r\Delta}}
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Case without Bubble

Proposition
If $\alpha = 1$, then unique Harrison-Kreps equilibrium price is $V_0(0)$. In this equilibrium, at every trading time point $t \in \Delta Z$, any trader can hold asset.

Proposition
If $\alpha > 1$, then unique Harrison-Kreps equilibrium price is $V_0(T)$. In this equilibrium, at every trading time $t \in \Delta Z$, only time $t - T$ entrant holds asset.
Case without Bubble

Proposition
If $\alpha = 1$, then unique Harrison-Kreps equilibrium price is $V_0(0)$.
In this equilibrium, at every trading time point $t \in \Delta \mathbb{Z}$ any trader can hold asset.
Case without Bubble

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If $\alpha = 1$,
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If $\alpha > 1$,
then unique Harrison-Kreps equilibrium price is $V_0(T)$
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at every trading time $t \in \Delta \mathbb{Z}$ only time $t - T$ entrant holds asset
Take-away

• Relatively optimistic inexperienced investors are prey for relatively pessimistic veteran traders
• Limited short-selling enables success of this perpetual scheme
• Result is perpetual bubble
• My paper gives formal proofs of this intuitive conjecture in simple model
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