Supersymmetric Curvepoles in 6D

Ori Ganor

Berkeley Center for Theoretical Physics, UC Berkeley

December 14, 2018

arXiv:1710.06880, and thanks to Kevin Schaeffer for a discussion in 2012 that provided motivation for this project.
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(New effects on M5-branes in strong flux?)

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Dipoles and Curvepoles

**Curvepole** – counterpart of E&M dipole, interacting with a 2-form.
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Two endpoints interacting with a 1-form gauge field $A$ with opposite charges.
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**Curvepole:**

Tensor field

$\Sigma$

$C$
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Has closed curve boundary $C = \partial \Sigma$ charged under 2-form gauge field $B$. 
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**E&M**

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**Tensor field**

**Curvepole:**
Has closed curve boundary $C = \partial \Sigma$ charged under 2-form gauge field $B$. Bulk of $\Sigma$ is inert.

Focus on 6D where (anti-)selfduality can impose restrictions.
The Goal

Construct a supersymmetric second quantized theory of curvepoles interacting with a 6D tensor multiplet of (1, 0) SUSY.
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\textbf{NOTE:} The shape \( C \) of these curvepoles is fixed! \n\( C \) is an external parameter of the theory.
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![Curvepoles Diagram]

**NOTE:** The shape \( C \) of these curvepoles is fixed! \( C \) is an external parameter of the theory.

**Does SUSY impose any restrictions on \( C \)?
Preview of results

Construct action order by order:

\[ L = L_2 + L_3 + L_4 + \cdots \]

\[ \delta_{\text{SUSY}} = \delta_0 + \delta_1 + \delta_2 + \cdots \]
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Require SUSY and Hollowness.

*Hollowness* – theory depends only on \( C = \partial \Sigma \), but not on \( \Sigma \).
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– I’ll present explicit formulas for up to quartic interactions.
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◊ If \( C \) is planar, both SUSY and Hollowness hold* (up to 4\(^{th}\) order).
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All of this is at the “classical” level.  
I’ll briefly mention issues regarding anomalies at the end.
A digression: Connection to M-theory

$T^3$

$\text{Vol}(T^3) \to 0$
A digression: Connection to M-theory

\[ \text{Vol}(T^3) \to 0 \]
A digression: Connection to M-theory

\[ T^3 \]
\[ \text{Vol}(T^3) \rightarrow 0 \]

U-duality

\[ \text{Wrapped M5: free hyper 'plet \& tensor 'plet} \]
\[ (1,0) \text{ multiplets} \]

\[ \text{U-dual } T^3 \]
\[ \text{Vol(U-dual } T^3) \rightarrow \infty \]
A digression: Connection to M-theory

\[ T^3 \]
\[ \text{Vol}(T^3) \to 0 \]

Insert \( \Omega \in \text{Spin}(5) \) twist

Scaling limit: \( \Omega \to I \) with \( (\Omega - I)/M_p^4\text{Vol}^{2/3} \to \text{finite} \)

\[ \text{Vol}(\text{U-dual } T^3) \to \infty \]

Wrapped M5:
- free hyper 'plet
- tensor 'plet
- (1,0) multiplets
A digression: Connection to M-theory

\[ \text{U-duality} \]

\[ T^3 \quad \text{Vol}(T^3) \rightarrow 0 \]

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\[ \text{Scaling limit: } \Omega \rightarrow I \text{ with } (\Omega - I)/M_p^4 \text{Vol}^{2/3} \rightarrow \text{finite} \]

\[ \text{Wrapped M5: free curvepole, hyper 'plet, tensor 'plet (1,0) multiplets} \]
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\[ \text{Wrapped M5: free curvepole} \]

\[ \text{hyper 'plet} \]

\[ \text{& tensor 'plet} \]

\[ (1,0) \text{ multiplets} \]

\[ [\text{Bergman \\& OG, 2000; Bergman \\& Dasgupta \\& Karczmarek \\& Rajesh \\& OG, 2001; Alishahiha \\& OG 2003}] \]

[\text{c.f. [Douglas \\& Hull, 1997; Connes \\& Douglas \\& Schwarz, 1997]}

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M-theory construction (more details)
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\[ 0 \leq x_i \leq 2\pi R_i \]

\[ 0 \leq \tilde{x}_i \leq 2\pi \tilde{R}_i \]

\[ V = (2\pi)^3 R_3 R_4 R_5 \quad \xrightarrow{U} \quad \tilde{V} = (2\pi)^3 \tilde{R}_3 \tilde{R}_4 \tilde{R}_5 = \frac{(2\pi)^6}{M_p^6 V} \]
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Twist: \( x_5 \rightarrow x_5 + 2\pi R_5 \)

accompanied by \( e^{i\theta} \in U(1) \subset SO(5)_{6,\ldots,10} \)
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Momentum quantization: \( P_5 R_5 \in \mathbb{Z} + \frac{\theta J}{2\pi} \)
M-theory construction (more details)

\[ \begin{align*}
\mathcal{M}^6 & \quad \xrightarrow{U} \quad \mathcal{M}^5 \\
T^3 & \quad \xrightarrow{U} \quad \tilde{T}^3
\end{align*} \]

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0 \leq x_i & \leq 2\pi R_i \\
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Dual to: \( \text{M2-area} \in \mathbb{Z}(2\pi)^2 \tilde{R}_3 \tilde{R}_4 + \underbrace{(2\pi \theta \tilde{R}_3 \tilde{R}_4) J}_{\text{keep finite}} \)
M-theory construction (more details)

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keep finite
M-theory construction - what are the curvepoles?

Curvepoles can also be explained (heuristically) by a Myers effect (dielectric open M2’s)

[Bergman & OG, 2000; Bergman & Dasgupta & Karczmarek & Rajesh & OG, 2001]
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![Diagram of M5 brane with strong G_{1267} flux and x_6, x_7 labels]
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![Strong $G_{1267}$ flux](image)
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Strong $G_{1267}$ flux

See also [Witten, 1997; Aganagic & Park & Popscu & Schwarz, 1997; Bergshoeff & Berman, & van der Schaar & Sundell, 2000; Berman & Tadrowski, 2007; Lambert & Orlando & Reffert, 2014; Lambert & Orlando & Reffert & Sekiguchi, 2018]
Deforming the free theory to Curvepole Theory
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Free 6D (2, 0) tensor 'plet
Deforming the free theory to Curvepole Theory

Free 6D $(2, 0)$ tensor 'plet

$(1, 0)$ tensor $(B, \chi^i, \varphi)$

$SU(2)_R$ index: $i = 1, 2$

$(1, 0)$ hyper $(\phi^i, \psi)$
Deforming the free theory to Curvepole Theory

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$$H^{(+)} := \frac{1}{2}(dB + *dB) = 0$$
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want their quanta to become curvepoles:

$$H^{(+)} := \frac{1}{2}(dB + *dB) = 0$$
Deforming the free theory to Curvepole Theory

Free 6D (2, 0) tensor ’plet

(1, 0) tensor $(B, \chi^i, \varphi)$

want their quanta to stay pointlike.

$SU(2)_R$ index:

$i = 1, 2$

(1, 0) hyper $(\phi^i, \psi)$

want their quanta to become curvepoles:

\[ H^{(+)} := \frac{1}{2} (dB + *dB) = 0 \]
Recall: 2nd quantized dipoles

Dipole:
Recall: 2nd quantized dipoles

Dipole:

\[ \begin{array}{c}
\, \\
- \\
\end{array} \quad \times \quad \begin{array}{c}
\, \\
+ \\
\end{array} \]
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$L$ is a fixed vector.
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\[ \Phi \text{ is 2}\text{nd quantized field of dipole.} \]

\( L \) is a fixed vector.

\( \Phi \) is 2\text{nd} quantized field of dipole.

Modified covariant derivative:

\[
D_\mu \Phi(x) = \partial_\mu \Phi(x) + iA_\mu(x + \frac{L}{2})\Phi(x) - iA_\mu(x - \frac{L}{2})\Phi(x)
\]
Recall: 2nd quantized dipoles

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\[ [\text{Bergman} \& \text{OG, Bergman} \& \text{Dasgupta} \& \text{Karczmarek} \& \text{Rajesh} \& \text{OG}] \]

Attach Wilson line:

\[ P_x \]

\[ P \text{ is a fixed path from } -\frac{L}{2} \text{ to } \frac{L}{2}. \]

\[ \tilde{\Phi}(x) := e^{-i \int_{P_x} A} \Phi(x) \text{ is gauge invariant.} \]

\[ P_x := x + P \text{ is "} P \text{ translated by } x\text{."} \]
Recall: 2nd quantized dipoles

**Dipole:**

\[
\begin{array}{cc}
- & + \\
\hline
x & L \\
\end{array}
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[Bergman & OG, Bergman & Dasgupta & Karczmarek & Rajesh & OG]

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\[
D_\mu \tilde{\Phi}(x) = \partial_\mu \tilde{\Phi}(x) + i\tilde{\Phi}(x) \int_{P_x} F_{\nu \mu} dx^\nu
\]
Curvepole Theory (first try)

We want quanta of $\phi$ to be curvepoles:

$[\text{interacting with } (H^{(-)}, \chi^i, \varphi)]$
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$\Sigma$ – fixed shape (open surface) around origin.
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**Step 1:** “covariant” curvepole derivative

$$\partial_\mu \phi \rightarrow D_\mu \phi(x) := \partial_\mu \phi(x) + i \phi(x) \int_{\partial \Sigma} B_{\mu \nu}(x + y) dy^\nu$$

[Schaeffer, 2012 (unpublished)], (Implicit in [Alishahiha & OG, 2003])
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But action should depend only on $H^{(-)} := \frac{1}{2}(dB - *dB)$. 
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But action should depend only on $H^{(-)} := \frac{1}{2} (dB - *dB)$.

For free tensor: $H^{(+)} := \frac{1}{2} (dB + *dB) = 0 \Rightarrow H^{(-)} = dB$,

a field redefinition $\tilde{\phi}(x) := e^{i \int_{\Sigma} B_{\mu \nu}(x + y) dy^\mu \wedge dy^\nu} \phi(x)$ does it:
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Curvepole Theory (first try)

We want quanta of $\phi$ to be curvepoles:

[interacting with $(H^\text{(-)}, \chi^i, \varphi)$]

$\Sigma$ – fixed shape (open surface) around origin.

**Step 1:** “covariant” curvepole derivative

$$\partial_\mu \phi \rightarrow D_\mu \phi(x) := \partial_\mu \phi(x) + i \phi(x) \int_{\partial \Sigma} B_{\mu \nu}(x + y) dy^\nu$$

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But action should depend only on $H^\text{(-)} := \frac{1}{2} (dB - * dB)$.

For free tensor: $H^\text{(+)} := \frac{1}{2} (dB + * dB) = 0 \Rightarrow H^\text{(-)} = dB$,

a field redefinition $\bar{\phi}(x) := e^{i \int_{\Sigma} B_{\mu \nu}(x + y) dy^\mu \wedge dy^\nu} \phi(x)$ does it:

$$\Rightarrow D_\mu \bar{\phi}(x) := \partial_\mu \bar{\phi}(x) + i \bar{\phi}(x) \int_{\Sigma} (dB)_{\mu \nu \sigma}(x + y) dy^\nu \wedge dy^\sigma$$

But interactions $\Rightarrow H^\text{(+)} := \frac{1}{2} (dB + * dB) = K^\text{(+)} = (\phi\text{-contributions})$
Curvepole Theory

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$\Sigma$ – fixed shape (open surface) around origin.

**Step 1’**: Modified “covariant” curvepole derivative

$\partial_{\mu} \phi \rightarrow D_{\mu} \phi(x) := \partial_{\mu} \phi(x) + iV_{\mu}(x)\phi(x)$

$V_{\mu}(x) := \frac{1}{2} \int_{\Sigma} dy^\nu \wedge dy^\sigma H_{\mu\nu\sigma}^{(-)}(x + y)$
Curvepole Theory

We want quanta of $\phi$ to be curvepoles:

$[\text{interacting with } (H^{(-)}, \chi^i, \varphi)]$

$\Sigma$ – fixed shape (open surface) around origin.

**Step 1’**: Modified “covariant” curvepole derivative

$$\partial_\mu \phi \rightarrow D_\mu \phi(x) := \partial_\mu \phi(x) + iV_\mu(x)\phi(x)$$

$$V_\mu(x) := \frac{1}{2} \int_\Sigma dy^\nu \wedge dy^\sigma H^{(-)}_{\mu\nu\sigma}(x + y) - \frac{1}{2} \int_{\partial\Sigma} dy_\mu \varphi(x + y)$$

required for SUSY
Curvepole Theory

We want quanta of $\phi$ to be curvepoles: [interacting with $(H^{(-)}, \chi^i, \varphi)$]

$\Sigma$ – fixed shape (open surface) around origin.

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Additional terms in $L$ required for SUSY
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Additional terms in \( L \)

required for SUSY

A condition on \( \Sigma \)
What is the condition on $\Sigma$?

Define the *curvepole integrals*:
What is the condition on $\Sigma$?

Define the **curvepole integrals**:

\[
\psi^{\mu\nu} := \int_{\Sigma} \psi(x + y) dy^\mu \wedge dy^\nu
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\[
\overline{\psi}^{\mu\nu} := \int_{\Sigma} \psi(x - y) dy^\mu \wedge dy^\nu
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A condition on $\Sigma$ appears by requiring SUSY at quartic order.
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\Psi_{\alpha\beta}^{\gamma\delta} = \Psi_{\gamma\delta}^{\alpha\beta}
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for arbitrary $\Psi$
What is the condition on $\Sigma$?

Define the \textit{curvepole integrals}:

\[ \Psi_{\mu\nu} := \int_\Sigma \Psi(x + y) dy^\mu \wedge dy^\nu \]
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A condition on $\Sigma$ appears by requiring SUSY at quartic order.

**SUSY preserved if:**

\[ \Psi_{\alpha\beta}^{\gamma\delta} = \Psi_{\gamma\delta}^{\alpha\beta} \]

for arbitrary $\Psi$

This is a geometric condition on $\Sigma$.

Let’s call such $\Sigma$’s \textbf{balanced curvepoles}.
Balanced Curvepoles

\[ \Psi_{\mu\nu} := \int_{\Sigma} \Psi(x + y) dy^\mu \wedge dy^\nu \]

\[ \overline{\Psi}_{\mu\nu} := \int_{\Sigma} \Psi(x - y) dy^\mu \wedge dy^\nu \]

**Balanced curvepole:**

\[ \boxed{\Psi}_{\alpha\beta}^{\gamma\delta} = \boxed{\Psi}_{\gamma\delta}^{\alpha\beta} \]

for arbitrary \( \Psi \)
Balanced Curvepoles

\[ \Psi^{\mu\nu} := \int_\Sigma \Psi(x + y) dy^\mu \wedge dy^\nu \]

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Balanced curvepole: \( \begin{array}{c|c}
\Psi_{\alpha\beta} & = & \Psi_{\gamma\delta}^{\alpha\beta} \\
\end{array} \) for arbitrary \( \Psi \)

Equivalent to: \( \int d^6x \Psi_{\alpha\beta} \Phi_{\gamma\delta} = \int d^6x \Psi_{\gamma\delta} \Phi_{\alpha\beta} \)
for arbitrary \( \Psi, \Phi \)
Balanced Curvepoles

\[ \Psi^{\mu\nu} := \int_{\Sigma} \Psi(x + y) dy^\mu \wedge dy^\nu \]

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Balanced curvepole:  
\[ \Psi_{\alpha\beta}^{\gamma\delta} = \Psi_{\gamma\delta}^{\alpha\beta} \]

for arbitrary \( \Psi \)

Equivalent to:  
\[ \int d^6x \Psi_{\alpha\beta}^{\gamma\delta} \Phi = \int d^6x \Psi_{\gamma\delta}^{\alpha\beta} \Phi \]

for arbitrary \( \Psi, \Phi \)

\[ \Psi_{\alpha\beta}^{\gamma\delta} \Phi = \Psi_{\gamma\delta}^{\alpha\beta} \Phi \]

(This is a stronger condition)
Balanced Curvepoles

\[ \Psi^{\mu\nu} := \int_{\Sigma} \Psi(x + y) dy^\mu \wedge dy^\nu \]

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Balanced curvepole: \[ \Psi_{\alpha\beta}^{\gamma\delta} = \Psi_{\gamma\delta}^{\alpha\beta} \]
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Planar curvepole \( (\Sigma \subset \mathbb{R}^2 \subset \mathbb{R}^6) \)
\[ \Psi_{\alpha\beta} \Phi_{\gamma\delta} = \Psi_{\gamma\delta} \Phi_{\alpha\beta} \]
(This is a stronger condition)
Balanced and Unbalanced Curvepoles

\[ \Psi^{\mu\nu} := \int_\Sigma \Psi(x + y)dy^\mu \wedge dy^\nu \]

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Balanced curvepole: \[ \Psi_{\alpha\beta}^{\gamma\delta} = \Psi_{\gamma\delta}^{\alpha\beta} \] for arbitrary \( \Psi \)
Balanced and Unbalanced Curvepoles

\[ \psi_{\mu\nu} := \int_{\Sigma} \psi(x + y) dy^{\mu} \wedge dy^{\nu} \]

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Balanced curvepole: \[ \boxed{\psi_{\alpha\beta}}_{\gamma\delta} = \boxed{\psi_{\gamma\delta}}_{\alpha\beta} \]

for arbitrary \( \psi \)

Parity invariant (\( \Sigma = -\Sigma \)) (balanced)
Balanced and Unbalanced Curvepoles

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\[ \Psi^{\mu\nu} := \int_{\Sigma} \Psi(x - y) dy^\mu \wedge dy^\nu \]

**Balanced curvepole:**

\[ \Psi_{\alpha\beta}^{\gamma\delta} = \Psi_{\gamma\delta}^{\alpha\beta} \quad \text{for arbitrary } \Psi \]

Parity invariant \((\Sigma = -\Sigma)\) \hspace{1cm} (balanced)

Planar \((\Sigma \subset \mathbb{R}^2 \subset \mathbb{R}^6)\) \hspace{1cm} (balanced)
Balanced and Unbalanced Curvepoles

\[ \Psi^{\mu\nu} := \int_{\Sigma} \Psi(x + y) dy^\mu \wedge dy^\nu \]

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Balanced curvepole: \[ \Psi_{\alpha\beta\gamma\delta} = \Psi_{\gamma\delta\alpha\beta} \] for arbitrary \( \Psi \)

Parity invariant (\( \Sigma = -\Sigma \)) (balanced)

Planar (\( \Sigma \subset \mathbb{R}^2 \subset \mathbb{R}^6 \)) (balanced)

(a chair) (not balanced)
The action - quadratic terms

The bosonic quadratic terms are:

\[ L^{(\text{bosonic})}_2 = \frac{1}{12\pi} H_{\mu\nu\sigma} H^{\mu\nu\sigma} + \frac{1}{4\pi} \partial_\mu \varphi \partial^\mu \varphi + \partial_\mu \phi_i \partial^\mu \phi^i \]

where

\[ H_{\mu\nu\sigma} := 3 \partial_{[\mu} B_{\nu\sigma]} \]

3-form field strength

\[ H_{\mu\nu\sigma}^{(\pm)} := \frac{1}{2} (H_{\mu\nu\sigma} \pm \frac{i}{6} \epsilon_{\mu\nu\sigma\alpha\beta\gamma} H^{\alpha\beta\gamma}) \]

selfdual and anti-selfdual components
The action - cubic terms

The bosonic cubic terms are:

$$L_3^{(bosonic)} = - J^\mu V^\mu$$

where

$$J^\mu := i(\bar{\phi}_i \partial^\mu \phi^i - \partial^\mu \bar{\phi}_i \phi^i) \quad U(1) \text{ current}$$

$$V^\mu := \frac{1}{2} \begin{array}{c} H^{(-)} \end{array}_{\mu\nu\sigma}^{\nu\sigma} - \frac{1}{2} \begin{array}{c} \partial^\nu \phi \end{array}_{\mu\nu} \quad \text{effective gauge field defined above}$$
The action - quartic terms

The additional bosonic quartic terms are:

\[ L_4^{(bosonic)} = \mathbf{V}_\mu \mathbf{V}^{\mu} \bar{\phi}_i \phi^i - \frac{3\pi}{16} \mathbf{J}_{[\mu}^{[\sigma}] \mathbf{J}^{\sigma\nu]}_{\mu\nu} + \frac{\pi}{2} \left( \partial^\mu \mathbf{M}_i^j \right)_{\mu\sigma} \partial_\nu \mathbf{M}_j^i \nu\sigma \]

where

\[ J_\mu := i(\bar{\phi}_i \partial_\mu \phi^i - \partial_\mu \bar{\phi}_i \phi^i) \]

\[ U(1) \text{ current} \]

\[ M_i^j := \bar{\phi}_i \phi^j - \frac{1}{2} \delta_i^j \bar{\phi}_k \phi^k \]

\[ SU(2)_R \text{ triplet} \]

\[ \mathbf{V}_\mu := \frac{1}{2} \mathbf{H}_{\mu\nu\sigma}^{(-)} \nu\sigma - \frac{1}{2} \partial_\nu \phi^i_{\mu\nu} \]

effective gauge field defined above
Hollowness

\[ C = \partial \Sigma \]
It would be nice if the theory depended only on $C$; not on $\Sigma$. 
Hollowness

\[ C = \partial \Sigma \]

It would be nice if the theory depended only on \( C \); not on \( \Sigma \).

If \( \partial \Sigma = \partial \Sigma' \), we’d like \( \text{theory}(\Sigma) \simeq \text{theory}(\Sigma') \)

up to a field redefinition.
Hollowness

$\Sigma$

$C = \partial \Sigma$

It would be nice if the theory depended only on $C$; not on $\Sigma$.

If $\partial \Sigma = \partial \Sigma'$, we'd like theory$(\Sigma) \simeq$ theory$(\Sigma')$

up to a field redefinition.

That turns out to be true . . . well . . . sort of.
Hollowness

Recall dipole case:
Hollowness

Recall dipole case:

\[ \times \]

\[ \begin{array}{c}
\text{+} \\
\text{−}
\end{array} \]
Hollowness

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Hollowness

Recall dipole case:

\[ D_\mu \Phi(x) = \partial_\mu \Phi(x) + i[A_\mu(x + \frac{L}{2}) - A_\mu(x - \frac{L}{2})]\Phi(x) \]
Recall dipole case:

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Field redefinition (attach Wilson line):

\[ \tilde{\Phi}(x) := e^{-i \int_{P_x} A} \Phi(x) \] is gauge invariant.
Hollowness

Recall dipole case:

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D_\mu \Phi(x) = \partial_\mu \Phi(x) + i [A_\mu(x + \frac{L}{2}) - A_\mu(x - \frac{L}{2})] \Phi(x)
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\[
D_\mu \tilde{\Phi}(x) = \partial_\mu \tilde{\Phi}(x) + i \tilde{\Phi}(x) \int_{P_x} F_{\nu \mu} d\chi^\nu
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Recall dipole case:

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\( P_x \rightarrow P'_x \) can be compensated by a field redefinition.
Hollowness for cuvepoles
Hollowness for cuvepoles

\[ \sum \]
Hollowness for cuvepoles
Hollowness for cuvepoles

\[ \xi : \Sigma \rightarrow \mathbb{R}^6 \]

is the deformation vector

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\delta_{\xi} \Phi^{\mu\nu} = \xi^\sigma \partial_\sigma \Phi^{\mu\nu} + \xi^\mu \partial_\sigma \Phi^{\nu\sigma} - \xi^\nu \partial_\sigma \Phi^{\mu\sigma}
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Hollowness for cuvepoles

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\[ V_\mu := \frac{1}{2} H^{(\cdot)}_{\mu\nu\sigma}^{\nu\sigma} - \frac{1}{2} \partial^\nu \varphi^{\mu\nu} \]
Hollowness for cuvepoles

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\[ \mathbf{V}_\mu := \frac{1}{2} H_{\mu\nu\sigma}^{(-)} \nu^\sigma - \frac{1}{2} \partial^\nu \varphi_{\mu\nu} \]

\[ \partial^\nu \varphi_{\mu\nu} = \int_{\Sigma} \partial^\nu \varphi (x + y) dy_\mu \wedge dy_\nu = \int_{\partial \Sigma} \varphi (x + y) dy_\mu \text{ is hollow.} \]
Hollowness for cuvepoles

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\[ \delta \xi \Phi^{\mu \nu} = \xi^\sigma \partial_\sigma \Phi^{\mu \nu} + \xi^\mu \partial_\sigma \Phi^{\nu \sigma} - \xi^\nu \partial_\sigma \Phi^{\mu \sigma} \]

\[ \mathbf{V}_\mu := \frac{1}{2} [H_{\mu \nu \sigma}^{(-)}]^{\nu \sigma} - \frac{1}{2} [\partial^\nu \varphi]_{\mu \nu} \]

\[ \partial^\nu \varphi_{\mu \nu} = \int_\Sigma \partial^\nu \varphi (x + y) dy_\mu \wedge dy_\nu = \int_{\partial \Sigma} \varphi (x + y) dy_\mu \text{ is hollow.} \]

\[ H_{\mu \nu \sigma}^{(-)} \] would have been hollow if \( dH^{(-)} = 0 \) but
Hollowness for cuvepoles

\[ \delta_{\xi} \Phi^{\mu\nu} = \xi^\sigma \partial_\sigma \Phi^{\mu\nu} + \xi^\mu \partial_\sigma \Phi^{\nu\sigma} - \xi^\nu \partial_\sigma \Phi^{\mu\sigma} \]

\[ \mathbf{v}_\mu := \frac{1}{2} H_{\mu\nu\sigma}^{(-)} \nu^\sigma - \frac{1}{2} \partial^\nu \varphi^{\mu\nu} \]

\[ \partial^\nu \varphi^{\mu\nu} = \int_{\Sigma} \partial^\nu \varphi(x + y) dy_\mu \wedge dy_\nu = \int_{\partial \Sigma} \varphi(x + y) dy_\mu \text{ is hollow.} \]

\[ H_{\mu\nu\sigma}^{(-)} \nu^\sigma \text{ would have been hollow if } dH^{(-)} = 0 \text{ but } \]

\[ \partial_{[\alpha} H^{(-)}_{\mu\nu\sigma]} = -\frac{3\pi i}{32} \epsilon_{\alpha\mu\nu\sigma\gamma\delta} \partial_\tau J^{[\gamma]} \delta_{[\alpha} J_{\mu]}^{\nu\sigma]}. \]
Hollowness for cuvepoles

$\xi : \Sigma \to \mathbb{R}^6$
is the deformation vector

$\delta_\xi \Phi^\mu{}_{\nu\sigma} = \xi^\sigma \partial_\sigma \Phi^\mu{}_{\nu\sigma} + \xi^\mu \partial_\sigma \Phi^\nu{}_{\sigma} - \xi^\nu \partial_\sigma \Phi^\mu{}_{\sigma}$

$\mathbf{V}_\mu := \frac{1}{2} H^{(-)}_{\mu\nu\sigma}{}^\nu{}^\sigma - \frac{1}{2} \partial^\nu \varphi_{\mu\nu}$

$\partial^\nu \varphi_{\mu\nu} = \int_\Sigma \partial^\nu \varphi(x + y) dy_\mu \wedge dy_\nu = \int_{\partial \Sigma} \varphi(x + y) dy_\mu$ is hollow.

$H^{(-)}_{\mu\nu\sigma}{}^\nu{}^\sigma$ would have been hollow if $dH^{(-)} = 0$ but

$\partial[\alpha H^{(-)}_{\mu\nu\sigma}] = -\frac{3\pi i}{32} \epsilon_{\alpha\mu\nu\sigma\gamma} \partial_\tau J[\gamma]^{\delta\tau} + \frac{3\pi}{4} \partial[\alpha J_\mu]_{\nu\sigma}$.  

Nevertheless ...
The action (again)

\[ L^{(bosonic)} = \]
\[ \frac{1}{12\pi} H_{\mu\nu\sigma} H^{\mu\nu\sigma} + \frac{1}{4\pi} \partial_\mu \varphi \partial^\mu \varphi + \partial_\mu \bar{\phi}_i \partial^\mu \phi^i + \frac{\pi}{2} \left[ \partial_\mu M^j_i \right]_{\mu\sigma} \left[ \partial_\nu M^i_j \right]^{\nu\sigma} \]

- hollow

\[ - J^\mu V_\mu + V_\mu V^\mu \bar{\phi}_i \phi^i - \frac{3\pi}{16} \left[ J^\mu \right]^{\sigma\nu} \left[ J_\mu \right]_{\sigma\nu} \]

- not hollow

\[ J_\mu := i(\bar{\phi}_i \partial_\mu \phi^i - \partial_\mu \bar{\phi}_i \phi^i) \quad U(1) \text{ current} \]
\[ M^j_i := \bar{\phi}_i \phi^j - \frac{1}{2} \delta^j_i \phi_k \phi^k \quad SU(2)_R \text{ triplet} \]
\[ V_\mu := \frac{1}{2} \left[ H^{(-)}_{\mu
u\sigma} \right]^{\nu\sigma} - \frac{1}{2} \left[ \partial^\nu \varphi \right]_{\mu\nu} \quad \text{effective gauge field defined above} \]
Hollowness (continued)

\[ \xi : \Sigma \rightarrow \mathbb{R}^6 \]

is the deformation vector

\[ \delta_{\xi} \Phi^{\mu\nu} = \xi^\sigma \partial_\sigma \Phi^{\mu\nu} + \xi^\mu \partial_\sigma \Phi^{\nu\sigma} - \xi^\nu \partial_\sigma \Phi^{\mu\sigma} \]
Hollowness (continued)

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But with a suitable field redefinition ( \( \delta_{\xi} B_{\mu\nu} = \cdots, \delta_{\xi} \phi^i = \cdots \), etc.)
Hollowness (continued)

\[ \Sigma \rightarrow \mathbb{R}^6 \]

\[ \delta \xi \Phi^{\mu \nu} = \xi^{\sigma} \partial_{\sigma} \Phi^{\mu \nu} + \xi^{\mu} \partial_{\sigma} \Phi^{\nu \sigma} - \xi^{\nu} \partial_{\sigma} \Phi^{\mu \sigma} \]

But with a suitable field redefinition ( \( \delta \xi B_{\mu \nu} = \cdots \), \( \delta \xi \phi^i = \cdots \), etc.),

\[ \delta \xi \int d^6 x \left( \cdots - J^{\mu} \Phi_{\mu} + \Phi_{\mu} \Phi_{\mu} \Phi_{i} \Phi_{i} - \frac{3\pi}{16} \left[ J_{[\mu}^{\sigma \nu] \right] J_{[\mu}^{\sigma \nu]} \right) = \]

non-hollow terms
Hollowness (continued)

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is the deformation vector

\[ \delta_{\xi} \Phi^{\mu\nu} = \xi^{\sigma} \partial_{\sigma} \Phi^{\mu\nu} + \xi^{\mu} \partial_{\sigma} \Phi^{\nu\sigma} - \xi^{\nu} \partial_{\sigma} \Phi^{\mu\sigma} \]

But with a suitable field redefinition (\( \delta_{\xi} B_{\mu\nu} = \cdots , \delta_{\xi} \phi^i = \cdots \), etc.),

\[ \delta_{\xi} \int d^6x \left( \cdots - J^\mu V_\mu + V_\mu V^\mu \bar{\phi}^i \phi^i - \frac{3\pi}{16} J[\mu]^{\sigma\nu} J[\nu]_{\sigma\mu} \right) = \left\{ \text{non-hollow terms} \right\} \]

\[ \frac{\pi i}{32} \int \epsilon_{\alpha\beta\gamma\mu\nu\sigma} J^\alpha_{\beta\gamma} \xi^\tau \partial_\tau J^\mu_{\nu\sigma} d^6x. \]
Hollowness (continued . . . )

\[ \xi : \Sigma \rightarrow \mathbb{R}^6 \]

is the deformation vector
Hollowness (continued . . .)

\[ \delta \xi \int \left( \cdots \cdots \cdots \right) d^6x = \frac{\pi i}{32} \int \varepsilon_{\alpha\beta\gamma\mu\nu\sigma} J^\alpha_{\beta\gamma} \xi^\tau \partial_\tau J^\mu_{\nu\sigma} d^6x. \]
Hollowness (continued . . .)

\[ \xi : \Sigma \rightarrow \mathbb{R}^6 \]

is the deformation vector

\[ \delta \xi \int \left( \ldots \ldots \right) d^6 x = \frac{\pi i}{32} \int \epsilon_{\alpha \beta \gamma \mu \nu \sigma} J^\alpha_{\beta \gamma} \xi^\tau \partial_\tau J^\mu_\nu \sigma d^6 x. \]

\[ \epsilon_{\alpha \beta \gamma \mu \nu \sigma} \ldots \beta \gamma \ldots \nu \sigma = 0 \quad \text{for 3-planar } \Sigma \text{ (i.e., } \Sigma \subset \mathbb{R}^3 \subset \mathbb{R}^6) \]
Hollowness (continued . . .)

\[ \xi : \Sigma \rightarrow \mathbb{R}^6 \]

is the deformation vector \( \xi \). 

\[
\delta_{\xi} \int \left( \cdots \cdots \right) d^6x = \frac{\pi i}{32} \int \epsilon_{\alpha\beta\gamma\mu\nu\sigma} J^\alpha_{\beta\gamma} \xi^\tau \partial_\tau J^\mu_{\nu\sigma} d^6x.
\]

\[ \epsilon_{\alpha\beta\gamma\mu\nu\sigma} \cdots \beta\gamma \cdots ^\nu\sigma = 0 \] for 3-planar \( \Sigma \) (i.e., \( \Sigma \subset \mathbb{R}^3 \subset \mathbb{R}^6 \))

\[ \Rightarrow \delta_{\xi} \int Ld^6x = 0 \text{ for 3-planar } \Sigma. \] (up to quartic terms)
Applications of hollowness

Insisting on **Hollowness** is useful:
Applications of hollowness

Insisting on **Hollowness** is useful:

1. SUSY alone doesn’t rule out adding a quartic term

\[
\Delta L = (#) \left( \frac{1}{2} J_\sigma \mu \nu J_\sigma \mu \nu - \partial_\sigma M_i^j \mu \nu \partial^\sigma M_j^i \mu \nu + \text{fermions} \right)
\]

not hollow
Applications of hollowness

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2. Ruling out higher order terms:
Applications of hollowness

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   not hollow

2. Ruling out higher order terms:

   hollowness requires a \( \partial \) for every \( \cdots \).

   Terms of the form \( \partial^{n-2} \cdots \partial^{n-2} \phi^n \) have dimension \( (n + 2) \);
Applications of hollowness

Insisting on **Hollowness** is useful:

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\Delta L = (#) \left( \frac{1}{2} J_\sigma^{\mu\nu} J^{\sigma\mu\nu} - \partial_\sigma M^j_i \partial^{\sigma} M^j_i + \text{fermions} \right)
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2. Ruling out higher order terms:

   hollowness requires a \( \partial \) for every \( \cdots \).

   Terms of the form \( \partial^{n-2} \cdots \partial^{n-2} \phi^n \) have dimension \( (n + 2) \);
   \( n + 2 > 6 \) for \( n > 4 \).
Applications of hollowness

Insisting on **Hollowness** is useful:

1. SUSY alone doesn’t rule out adding a quartic term

   \[ \Delta L = (\#) \left( \frac{1}{2} J_\sigma^{\mu\nu} J^\sigma_{\mu\nu} - \partial_\sigma M^i_j \partial^\sigma M^j_i + \text{fermions} \right) \]

   not hollow

2. Ruling out higher order terms:

   hollowness requires a \( \partial \) for every \( \cdots \).

   Terms of the form \( \partial^{n-2} \cdots n^{-2} \phi^n \) have dimension \( (n + 2) \);
   \( n + 2 > 6 \) for \( n > 4 \).
Is hollowness anomalous?

The field redefinition is

\[ \delta B_{\gamma\delta} = \frac{\pi i}{4} \epsilon_{\alpha\mu\nu\sigma\gamma\delta} \xi^\alpha J^\mu \]

\[ \delta \phi^i = i \epsilon \phi^i , \]

\[ \delta \bar{\phi}^i = -i \epsilon \bar{\phi}^i , \]

\[ \delta \psi^i = i \epsilon \psi^i , \]

\[ \delta \bar{\psi}^i = -i \epsilon \bar{\psi}^i , \]

\[ \epsilon := \frac{1}{2} \xi^\alpha H^{(-)}_{\alpha\mu\nu} \mu\nu . \]
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The field redefinition is

\[ \delta B_{\gamma\delta} = \frac{\pi i}{4} \epsilon_{\alpha\mu\nu\sigma} \gamma_{\delta} \left[ \xi^{\alpha} J^{\mu} \right]^{\nu\sigma} \]

\[ \delta \phi^{j} = i \epsilon \phi^{j}, \]

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\[ \delta \psi^{j} = i \epsilon \psi^{j}, \]

\[ \delta \bar{\psi}^{j} = -i \epsilon \bar{\psi}^{j}, \]

\[ \epsilon := \frac{1}{2} \left[ \xi^{\alpha} H_{\alpha \mu \nu} \right]^{\mu \nu}. \]

Anomaly \( \sim \int \epsilon d\mathbf{V} \wedge d\mathbf{V} \wedge d\mathbf{V} \) at order \( \cdots 4 \)

Reminder: \( \mathbf{V}_{\mu} := \frac{1}{2} \left[ H_{\mu \nu \sigma}^{(-)} \right]^{\nu \sigma} - \frac{1}{2} \left[ \partial^{\nu} \varphi \right]_{\mu \nu} \)
Open Questions
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Higher than quartic orders? Additional restrictions on $\Sigma$?
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Which $\Sigma$ are realized? (Probably disks.)

Perhaps technique of [Ho & Matsuo, 2008, and Ho & Huang & Matsuo, 2011],
applying Bagger-Lambert-Gustavsson theory to Poisson triple-product, can help here.

Applications to interactions induced on M5-brane by $G$-flux?

More Questions
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Can pure-spinor techniques simplify the equations?

Cf. [Cederwall & Karlsson, 2011] for BI.

Recast in projective superspace?

[Galperin & Ivanov & Ogievetsky & Sokatchev; Karlhede & Lindström & Roček]
Summary

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Thanks!
MORE DETAILS
Antiselfduality with interactions

Action: \[ S = \frac{1}{4\pi} \int dB \wedge *dB + \int (dB - *dB) \wedge K \]

Field strength: \[ H^{(\pm)} := \frac{1}{2} (dB \pm *dB) \]

EOMs: \[ 0 = d \left[ \frac{1}{2\pi} *dB + (K + *K) \right] = d \left[ \frac{1}{\pi} H^{(+)} + (K + *K) \right] \]

Corrections to free anti-selfduality condition:

\[ H^{(+)} = -\pi (K + *K) \]

Can define: \[ \widetilde{H}^{(+)} := \frac{1}{2} (dB + *dB) + \pi (K + *K) = 0 \]
Spinor notation in 6D

\[ a, b, c, d, \ldots = 1, \ldots 4 \] spinor indices

\[ \partial_{ab} = \frac{1}{2} \epsilon_{abcd} \partial^{cd} \] derivative

\[ B^b_a \] 2-form \((B^a_a = 0)\)

\[ H_{ab}^{(-)} = H_{ba}^{(-)} \] antiselfdual 3-form

\[ H^{(+)}_{ab} = H^{(+)}_{ba} \] selfdual 3-form

\[ H_{ab}^{(-)} = 2 \partial_{c(a} B_{b)c} \] \[ \iff \]

\[ H_{\mu\nu\sigma}^{(-)} = \frac{1}{2} (H_{\mu\nu\sigma} - \frac{i}{3!} \epsilon_{\mu\nu\sigma\alpha\beta\gamma} H^{\alpha\beta\gamma}) \]

\text{for } H_{\mu\nu\sigma} = 3 \partial_{[\sigma} B_{\mu\nu]} \]
SUSY transformations at 0\(^{th}\) order

\[\delta_0 \phi^i = \eta^{ai} \psi_a\]

\[\delta_0 \bar{\phi}_i = \eta^a_i \bar{\psi}_a\]

\[\delta_0 \psi_a = -2i \eta^b_i \partial_{ab} \phi^i\]

\[\delta_0 \bar{\psi}_a = -2i \eta^b_i \partial_{ab} \bar{\phi}^i\]

\[\delta_0 \varphi = \eta^a_i \chi^i_a\]

\[\delta_0 \chi^i_a = i \eta^{bi} \partial_{ab} \varphi + i \eta^{bi} H_{ab}\]

\[\delta_0 H_{ab} = \eta^c_i \partial_{ca} \chi^i_b + \eta^c_i \partial_{cb} \chi^i_a\]
Effective vector multiplet

Out of $H$ and $\chi$ we construct a vector multiplet:

$$V_{ab} := H_{c[a} \chi_{b]} + \partial_{c[a} \phi_{b]}$$

Vector field

$$\rho^{ia} := \partial^{ab} \chi_{c}^{i}$$

Gaugino field

$$F_{a}^{b} = \partial^{bc} V_{ac} - \frac{1}{4} \delta^{b}_{a} \partial^{dc} V_{dc}$$

Associated field strength

Their $0^{th}$ order SUSY transformations:

$$\delta_{0} V^{ab} = -\eta_{i}^{a} \rho^{ib} + \eta_{i}^{b} \rho^{ia} - \partial^{ab} \lambda$$

$$\delta_{0} \rho^{ia} = 2i \eta^{bi} F_{b}^{a}$$

$$\lambda := \eta_{i}^{a} \chi_{b}^{i}$$

Field-dependent gauge parameter
SUSY transformations at 1\textsuperscript{st} order

\[ \delta_1 \phi^i = -i \eta^a_j \phi^i \chi^j_b a \]

\[ \delta_1 \phi = i \eta^a_j \phi^i \chi^j_b a \]

\[ \delta_1 \psi_a = 2 \eta^b_i [H_{c[b}] c \phi^i + 2 \eta^b_i \phi^i \partial_{c[b]} \varphi_a c + i \eta^b_i \psi_a \chi^i_c c ] \]

\[ \delta_1 \bar{\psi}_a = 2 \eta^b_i [H_{c[a]} c \phi^i + 2 \eta^b_i \bar{\phi}^i \partial_{c[a]} \varphi_b c - i \eta^b_i \bar{\psi}_a \chi^i_c c ] \]

\[ \delta_1 \varphi = 0 \]
SUSY transformations at 1\textsuperscript{st} order (Continued)

\[ \delta_1 \chi_a^i = \pi i \eta^c_i \left( \overline{\psi_b \psi}(a)_c \right)^b + \pi i \eta^c_i \left( \overline{\psi_b \psi}(a)_c \right)^b \]

\[-\pi \eta^c_j \left( \frac{1}{2} \phi \partial_{bc} \phi^i + \frac{1}{2} \phi \partial_{bc} \phi^i + \frac{3}{2} \phi \partial_{bc} \phi^j + \frac{3}{2} \phi \partial_{bc} \phi^j \right)^b_a \]

\[-\pi \eta^c_j \left( \frac{1}{4} \phi \partial_{ab} \phi^j + \frac{1}{4} \phi \partial_{ab} \phi^j + \frac{3}{4} \phi \partial_{ab} \phi^i + \frac{3}{4} \phi \partial_{ab} \phi^i \right)^b_c \]

\[ \delta_1 B_a^b = -\frac{i \pi}{2} \eta_i^c \left( \psi_c \phi^i + \overline{\psi_c \phi}^i \right)^b_a + i \pi \eta_i^c \left( \psi_b \phi^i + \overline{\psi_b \phi}^i \right)^b_c \]

\[ + i \pi \eta_i^a \left( \psi_c \phi^i + \overline{\psi_c \phi}^i \right)^d_c \]

\[ - \frac{i \pi}{2} \delta_b^a \eta_i^c \left( \psi_d \phi^i + \overline{\psi_d \phi}^i \right)^a_c \]
Origin of Balanced Curvepole condition

We want \((\delta_0 + \delta_1 + \delta_2 + \cdots) \int (L_2 + L_3 + L_4 + \cdots) = 0\).

Look at the \(\phi \bar{\psi} \psi \psi\) terms in \(\delta_1L_3\)

\[
\begin{align*}
+ \eta_i^e \partial^{ab} & \phi^a_i \bar{\psi}_c \psi_d \psi_e b & - \eta_i^e \partial^{ab} & \phi^a_i \bar{\psi}_c \psi_d \psi_e a \\
+ \eta_i^e \partial^{ab} & \phi^a_i \bar{\psi}_c \psi_d \psi_e b & - \eta_i^e \partial^{ab} & \phi^a_i \bar{\psi}_c \psi_d \psi_e a \\
+ \frac{1}{2} \eta_i^e \partial^{ab} & \phi^a_i \bar{\psi}_c \psi_d \psi_e b & - \frac{1}{2} \eta_i^e \partial^{ab} & \phi^a_i \bar{\psi}_c \psi_d \psi_e a \\
+ \frac{1}{2} \eta_i^e \partial^{ab} & \phi^a_i \bar{\psi}_c \psi_d \psi_e b & - \frac{1}{2} \eta_i^e \partial^{ab} & \phi^a_i \bar{\psi}_c \psi_d \psi_e a \\
\end{align*}
\]

 Doesn’t seem to be possible to cancel with \(\delta_0L_4 + \delta_2L_2\) unless

\[
\int \[X \ b \ Y \ d \]_e = \int \[X \ e \ Y \ b \]
\]
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The bigger picture?

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Noncommutative spacetime

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**Noncommutative spacetime**  
[Connes & Douglas & Schwarz, Douglas & Hull, Seiberg & Witten, 1998]

**Dipole theories**

**Puff Field Theory**  
[Hashimoto & Jue & Kim & Ndirango & OG, 2007]
Thanks!