Stable Matching-Based Selection in Evolutionary Multiobjective Optimization

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Introduction

◦ Multiobjective optimization problem (MOP)

  ◦ minimize/maximize \( F(x) = (f_1(x), f_2(x), ..., f_m(x))^T \)

  ◦ subject to \( x \in \Omega \)
Real World MOPs

- Multiobjective traveling salesman problem (TSP)

A salesman need to travel from the origin city to a couple of cities to meet customers. Each city need to be visited exactly once. There are different ways to travel from each city to another, each of which takes different travelling times and different costs. The sequence to visit each city could be arbitrary.

- The goal of the planning is to minimize the travel time and travel cost
Real World MOPs

- Multiobjective traveling salesman problem (TSP)
  - Different visiting sequences result in significance differences in traveling time and cost
  - Even if the visiting sequence is fixed, different transportation may result in different objectives

![Graph showing traveling time vs. cost with bus and plane as examples of low and high cost respectively.](Image)
Real World MOPs

- Antenna design

There is growing interest for small antennas that concurrently have higher functionality and operability. Designers are interested in antennas with higher gain and larger bandwidth.
Real World MOPs

- Antenna design
  - E.g., [1]
Pareto Dominance

- Pareto Dominance
- \( \min F(x) = (f_1(x), f_2(x))^T \)

Solution 1 dominates solution 2
Pareto Dominance

- Pareto Dominance
  - minimize $F(x) = (f_1(x), f_2(x))^T$

Solution 1 and solution 2 cannot be compared. They are *nondominated* with each other.
Pareto Dominance

- Pareto Dominance
  - minimize $F(x) = (f_1(x), f_2(x))^T$

Pareto font (PF) consists of the objective vectors of all Pareto optimal solutions.
Selection

- EA framework

Start

Population initialization

Offspring reproduction

Selection

Terminate?

End

A set of solutions are randomly sampled in the search space

Produce a set of offspring solutions using reproduction operators

Promising solutions are selected to survive in the next iteration
Selection in EMO

- Two main requirements
  - Convergence
    - Obtained solutions are as close to the PF as possible
  - Diversity
    - Obtained solutions are widely distributed along the PF
Selection in EMO

- Two main requirements
  - Convergence
    - Obtained solutions are as close to the PF as possible
  - Diversity
    - Obtained solutions are widely distributed along the PF
MOEA/D

- Multiobjective evolutionary algorithm based on decomposition [2]
  - Decomposes an MOP into a number of single-objective optimization problems and optimizes them simultaneously
    - Weighted sum, Tchebycheff and penalty-based intersection approaches
  - E.g. Tchebycheff approach

\[
\begin{align*}
\text{minimize} & \quad g^{tch}(x|w, z^*) = \max_{1 \leq i \leq m} \left\{ \frac{|f_i(x) - z^*_i|}{w_i} \right\} \\
\text{subject to} & \quad x \in \Omega.
\end{align*}
\]
Selection in MOEA/D

- Select a solution for each subproblem to minimize the objective of the subproblem \(\rightarrow\) Convergence
- Subproblems are constructed using a set of evenly distributed weight vectors \(\rightarrow\) Diversity

**Problem:**
During the optimization, the currently elite solutions of some relatively easier subproblems might also be good candidates for the others. In this case, these elite solutions can easily take over all these subproblems. MOEA/D may fail to maintain population diversity.
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- Introduction
- **Stable Matching-Based Selection**
- Stable Matching-Based Selection with Incomplete Lists
- Adaptive Stable Matching-Based Selection
- Conclusion
Stable marriage problem (SMP) [2] is a problem of finding a stable matching between two sets of matching agents, given that each matching agent has a preference list over all matching agents in the other set.

It is an early work of Nobel Prize Winner Lloyd S. Shapley.
Stable Marriage Problem

Not a stable matching.

$m_2$ prefers $w_3$ to $w_1$, meanwhile, $w_3$ prefers $m_2$ to $m_3$. Therefore, $m_2$ and $w_3$ may divorce and marry to each other.

This unstable situation should be avoided in a stable matching solutions.
Stable Marriage Problem

A stable matching solution.

Fig. 1. Simple example of SMP, where the matching \{(m_1, w_3), (m_2, w_1), (m_3, w_2)\} is stable but the matching \{(m_1, w_2), (m_2, w_1), (m_3, w_3)\} is unstable.
Stable Marriage Problem

- Stable matching algorithm proposed in [3]

```
Algorithm 3: Stable matching
1 Set all men and women to be unmatched;
2 while some man m is still unmatched do
3     w ← first woman on m’s preference list;
4     if w is unmatched then
5         Match m and w as a matching pair;
6     else
7         if w prefers m to its its current partner m* then
8             Set m* to be unmatched;
9             Remove w from m*’s preference list;
10            Match m and w as a matching pair;
11         else
12             Remove w from m’s preference list;
13         end
14     end
15 end
```

Note: this is a men-oriented algorithm, where the number of men should be no more than the number of women.
Stable Marriage Problem

- SMP is a one-one matching problem

- Matching problem can be generalized into many different problems
  - College admission problem/hospitals-residents problem (many-to-one matching)
  - Stable roommate problem (only one set of matching agents)
Stable Matching-Based Selection

- Model the selection in MOEA/D as an SMP
  - Subproblems and solutions are treated as two sets of matching agents
    - Each subproblem has a preference list over all solutions
    - Each solution has a preference list over all subproblems

- Preference value of $p$ on $x$
  - $\Delta p(p, x) = g(x| w, z^*)$
    - Objective function of subproblem $p$ given solution $x$
    - Subproblem prefers solution that can minimize its objective

- Preference value of $x$ on $p$
  - $\Delta x(x, p) = \bar{F}(x) - \frac{w^T \bar{F}(x)}{w^T w} w$
    - Distance between the objective vector of solution $x$ and weight vector of subproblem $p$
    - Solution prefers subproblem closer in objective space
Stable Matching-Based Selection

- Model the selection in MOEA/D as an SMP
  - The selection is achieved by computing a stable matching between subproblems and solutions using proposed algorithm in [3]
  - Since the preferences are defined considering convergence and diversity
  - A stable matching to the modeled SMP is regarded as a balance between convergence and diversity
Stable Matching-Based Selection


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<th>MOEA/D-DE</th>
<th>MOEA/D-DRA</th>
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Wilcoxon’s rank sum test at a 0.05 significance level is performed between MOEA/D-STM and each of the other competing algorithms. 5 and 6 denote that the performance of the corresponding algorithm is significantly worse than or better than that of MOEA/D-STM, respectively. The best mean is highlighted in boldface with gray background. UP1 to UP7 have two objectives and UP8 to UP10 have three objectives.
Stable Matching-Based Selection

- **Problem**
  - A stable matching solution cannot always ensure that a subproblem select a solution close to itself
  - Experiments in [5] show that for MOP test instances, MOEA/D-STM cannot maintain diverse solutions over the entire PF

![Final result of MOEA/D-STM on MOP1](image-url)
Stable Matching-Based Selection

E.g.

Reason:
Some part of PF can be located easily, while other part is not. All subproblems prefer solutions closer to the PF.

Outcome:
Stable matching cannot ensure that the selected solutions are matched to their closest subproblems. Population diversity may be destroyed.
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    - Stable Matching with Incomplete Preference Lists
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    - Many-one stable matching-based selection
- Adaptive Stable Matching-Based Selection
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Stable Matching with Incomplete Lists

- Special case for a matching problem
  
  Some men or women refuse to be matched to some candidates. They just remove these candidates in their preference lists. A matching will never to made to these rejected candidates.

- Stable matching with incomplete lists (STM-IC) [6]
  
  - A matching agent does not need to have a complete preference list over all matching agents in the other sets
  - A matching agent can only be matched with one of the matching agents present on its preference list
Stable Matching with Incomplete Lists

- If we only keep a few most preferred subproblems on a solution’s preference list, then
  - A solution is only allowed to be matched with its one of most preferred subproblems
  - We can ensure that the selected solutions are matched to its closest subproblems after stable matching
Stable Matching with Incomplete Lists

E.g.

\[ \Psi_p = \begin{bmatrix}
  1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 10 & 9 \\
  1 & 3 & 2 & 4 & 5 & 6 & 7 & 8 & 10 & 9 \\
  10 & 1 & 3 & 2 & 4 & 6 & 5 & 7 & 8 \\
  10 & 1 & 3 & 2 & 4 & 9 & 6 & 5 & 7 & 8 \\
\end{bmatrix} \]

The length of solution’s preference list is reduced to be 2. Only the two favorite subproblems remained.

Population diverse is ensured.
Stable Matching with Incomplete Lists

**Algorithm 3: STMIC\((P, S, \Psi_P, \Psi_X, R)\)**

**Input:**
- subproblem set \(P\), solution set \(S\)
- preference matrices \(\Psi_P\) and \(\Psi_X\)
- length of solution's preference list set \(R\)

**Output:** stable matching set \(M\)

1. \(P_u \leftarrow P\), \(M \leftarrow \emptyset\);
2. for \(i \leftarrow 1\) to \(|S|\) do
3.   Keep the first \(x_i\)'s complete preference list and remove the remainders;
4. while \(P_u \neq \emptyset\) do
5.   \(p \leftarrow\) Randomly select a subproblem from \(P_u\);
6.   if \(p\)'s preference list \(\neq \emptyset\) then
7.     \(x \leftarrow\) First solution on \(p\)'s preference list;
8.     Remove \(x\) from \(p\)'s preference list;
9.     if \(p\) is on \(x\)'s preference list then
10.    \[M \leftarrow DAP(p, x, P_u, M, \Psi_P, \Psi_X)\];
11. else
12.    \(P_u \leftarrow P_u \setminus p\);
13. Return \(M\);

**Algorithm 2: DAP\((p, x, P_u, M, \Psi_P, \Psi_X)\)**

**Input:**
- current subproblem \(p\) and solution \(x\)
- unmatched subproblem set \(P_u\)
- current stable matching set \(M\)
- sets of preference lists \(\Psi_P\) and \(\Psi_X\)

**Output:** stable matching set \(M\)

1. if \(x \notin M\) then
2.   \(M \leftarrow M \cup (p, x);\) \hspace{1cm} // match \(p\) and \(x\)
3.   \(P_u \leftarrow P_u \setminus p;\)
4. else
5.   \(p' \leftarrow M(x);\) \hspace{1cm} // current partner of \(x\)
6.   if \(x\) prefers \(p\) to \(p'\) then
7.     \[M \leftarrow M \cup (p, x) \setminus (p', x);\]
8.     \(P_u \leftarrow P_u \cup p' \setminus p;\)
9. else
10.    \(P_u \leftarrow P_u \setminus p;\)
11. Return \(M\);

Note: an SMP with incomplete lists cannot guarantee to match a stable solution for all subproblems.
Stable Matching with Incomplete Lists

◦ A stable matching to an SMP with incomplete lists cannot guarantee to match a stable solution for all subproblems

◦ Two versions of stable matching-based selection mechanisms with incomplete preference lists are developed in [9]
  ◦ Two-level one-one matching-based selection
  ◦ Many-one matching-based selection
Two-Level One-One Matching-Based Selection

- **First level (SMP-IC)**
  - The preference list of a solution is set to a limited length \( r \)
  - Only the \( r \) most preferred subproblems remain in solution’s preference list

- **Second level (original SMP)**
  - Since the stable matching of an SMP-IC might not match a stable solution for all subproblems
  - The remaining subproblems and solutions are matched using complete preference lists
Two-Level One-One Matching-Based Selection

Algorithm 4: SelectionOOSTM2L($P, S$)

Input: subproblem set $P$ and solution set $S$
Output: solution set $S$

/* First-level stable matching */
1. Compute $\Psi_P$ and $\Psi_X$ for $P$ and $S$;
2. $R \leftarrow$ set the length of each solution’s preference list;
3. $M \leftarrow$ STMIC($P, S, \Psi_P, \Psi_X, R$);
   /* Second-level stable matching */
4. $(P_m, S_m) \leftarrow M$;
5. $P_u \leftarrow P \setminus P_m$;
6. $S_u \leftarrow S \setminus S_m$;
7. Compute $\Psi'_P$ and $\Psi'_X$ for $P_u$ and $S_u$;
8. $M' \leftarrow$ STM($P_u, S_u, \Psi'_P, \Psi'_X$);
   /* Combine the stable matching pairs */
9. $M \leftarrow M \cup M'$;
10. Return $M$;
Two-Level One-One Matching-Based Selection

- MOEA/D-STM2L [7]
  - $r$ is set to be 8 for UF test instances and 4 for MOP test instances
  - Achieves competitive performance on MOP test instances
  - Inherits the main ability of MOEA/D-STM on UF test instances

<table>
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Many-One Matching-Based Selection

◦ The selection process is modeled as a many-one matching problem with a common quota [8] (college admission problem)
  ◦ Each solution can only match with one subproblem
  ◦ Each subproblem can match with more than one solutions without a limit
  ◦ The common quota is set to be equal to $N$ (the number of subproblems)
    ◦ At most $N$ solutions can be matched in a stable matching

◦ In such a way, the unmatched subproblems caused by the incomplete preference lists give the matching opportunities to other subproblems that have already been matched with a solution but still have openings.
Many-One Matching-Based Selection

- Stable condition
  - A matching to a many-one matching problem with a common quota is called stable if there does not exist any pair of subproblem $p$ and solution $x$, where:
    - $p$ and $x$ are acceptable to each other but not matched together;
    - $x$ is unmatched or prefers $p$ to its assigned subproblem;
    - the common quota is not met or $p$ prefers $x$ to at least one of its assigned solutions.
Many-One Matching-Based Selection

- Proposed many-one matching procedure
  - Main loop
    - Step 1:
      - Randomly select an unmatched solution \( x \); (line 7)
      - Match \( x \) with its current favorite subproblem \( p \) according to its incomplete preference list; (line 9-11)
      - Remove \( p \) from \( x \)’s preference list; (line 10)
    - Step 2:
      - If the number of current matching pairs \( |M| > N \), find a substitute subproblem \( p' \) and adjust its matching pairs by releasing the matching relationship with its least preferred solution \( x' \). (line 12-18)
Many-One Matching-Based Selection

- Proposed many-one matching procedure
  - The substitute subproblem $p'$ is selected according to the following criteria
    1. Choose the subproblems that have the largest number of matched solutions to form $\bar{P}$. Its underlying motivation is to reduce the chance for overly exploiting a particular subproblem; (line 13)
    2. If $|\bar{P}| > 1$, the subproblems in $\bar{P}$, whose least preferred solution holds the worst rank on that subproblem’s preference list, are used to reconstruct $\bar{P}$; (line 14)
    3. A substitute subproblem $p'$ is randomly chosen from $\bar{P}$. (line 15)
Many-One Matching-Based Selection

Algorithm 5: SelectionMOSTM(P, S)

Input: subproblem set P and solution set S
Output: stable matching set M
1 Compute $\Psi_P$ and $\Psi_X$ for P and S;
2 $R \leftarrow$ Set the length of each solution's preference list;
3 $S_u \leftarrow S$, $M \leftarrow \emptyset$;
4 for $i \leftarrow 1$ to $Q$ do
5   Keep the first $r^i$ subproblems on $x^i$'s complete
6   preference list and remove the remainders;
7 while $S_u \neq \emptyset$ do
8   $x \leftarrow$ Randomly select a solution from $S_u$;
9   if $x$'s preference list $\neq \emptyset$ then
10      $p \leftarrow$ First subproblem on $x$'s preference list;
11      Remove $p$ from $x$'s preference list;
12      $M \leftarrow M \cup (p, x)$;
13      if $|M| > N$ then
14         $\overline{P} \leftarrow \arg\max_{p \in P} |M(p)|$; \hfill // $|M(p)|$ is the
15         \hspace*{2cm} cardinality of $M(p)$
16         $\overline{P} \leftarrow \arg\max_{p \in \overline{P}} \max_{x \in M(p)} \text{rank}(p, x)$;
17         \hfill // rank$(p, x)$ is the rank of $x$ on
18         $P$'s preference list
19         $p' \leftarrow$ Randomly select a subproblem from $\overline{P}$;
20         $x' \leftarrow \arg\max_{x \in M(p') \setminus \{x\}} \text{rank}(p', x)$;
21         $M \leftarrow M \setminus (p', x')$;
22      else
23         $S_u \leftarrow S_u \cup x'$;
24   $S_u \leftarrow S_u \setminus x$;
25 return $M$;
Stable Matching-Based Selection with Incomplete Preference lists

- E.g., $r = 2$

\[
\Psi_p = \begin{bmatrix}
1 & 2 & 8 & 3 & 9 & 4 & 5 & 6 & 7 & 10 \\
1 & 3 & 2 & 4 & 5 & 9 & 8 & 6 & 7 & 10 \\
1 & 3 & 2 & 4 & 6 & 5 & 7 & 9 & 8 & 10 \\
10 & 1 & 3 & 2 & 4 & 6 & 5 & 7 & 9 & 8 \\
10 & 1 & 3 & 2 & 4 & 6 & 5 & 7 & 9 & 8 \\
\end{bmatrix}
\]

\[
\Psi_x = \begin{bmatrix}
1 & 2 \\
1 & 2 \\
2 & 1 \\
2 & 1 \\
3 & 2 \\
1 & 2 \\
1 & 2 \\
5 & 4 \\
\end{bmatrix}
\]

Note: selection results of two-level one-one matching and many-one matching might not be the same.
Stable Matching-Based Selection with Incomplete Preference lists

Steps
- $x^1$ proposes a matching request to $p^1$ ($|M|=1$)
- $x^2$ proposes a matching request to $p^1$ ($|M|=2$)
- $x^3$ proposes a matching request to $p^1$ ($|M|=3$)
- $x^4$ proposes a matching request to $p^2$ ($|M|=4$)
- $x^5$ proposes a matching request to $p^2$ ($|M|=5$)
Stable Matching-Based Selection with Incomplete Preference lists

- **Steps**
  - $x^6$ proposes a matching request to $p^2$ (|M|=6)
  - |M|>5, find a substitute problem $p'$
    - Find subproblems with most matched solutions $\bar{P} = \{p^1, p^2\}$
    - Find subproblems in $\bar{P}$, whose least preferred solution holds the worst rank on that subproblem's preference list, to reconstruct $\bar{P} = \{p^2\}$
    - Randomly select a subproblem from $\bar{P}$ to be $p' = p^2$
  - Release the matching relationship of $p^2$ with its least preferred solution $x^6$ (|M|=5)
Stable Matching-Based Selection with Incomplete Preference lists

Steps
- $x^7$ proposes a matching request to $p^3$ ($|M|=6$)
- Release the matching relationship of $p^1$ with its least preferred solution $x^3$
- $x^8$ proposes a matching request to $p^1$ ($|M|=6$)
- Release the matching relationship of $p^1$ with its least preferred solution $x^8$
Stable Matching-Based Selection with Incomplete Preference lists

Steps

- $x^9$ proposes a matching request to $p^1$ ($|M|=6$)
- Release the matching relationship of $p^1$ with its least preferred solution $x^9$
- $x^{10}$ proposes a matching request to $p^5$ ($|M|=6$)
- Release the matching relationship of $p^2$ with its least preferred solution $x^5$
Stable Matching-Based Selection with Incomplete Preference lists

◦ Steps
  ◦ $x^3$ proposes a matching request to $p^2$ (|M|=6)
  ◦ Release the matching relationship of $p^2$ with its least preferred solution $x^4$
  ◦ $x^4$ proposes a matching request to $p^1$ (|M|=6)
  ◦ Release the matching relationship of $p^1$ with its least preferred solution $x^4$
  ◦ $x^5$ proposes a matching request to $p^1$ (|M|=6)
  ◦ Release the matching relationship of $p^1$ with its least preferred solution $x^5$

$$
\Psi_p = \begin{bmatrix}
1 & 2 & 8 & 3 & 9 & 4 & 5 & 6 & 7 & 10 \\
1 & 3 & 2 & 4 & 5 & 9 & 8 & 6 & 7 & 10 \\
1 & 3 & 2 & 4 & 6 & 5 & 9 & 8 & 10 \\
10 & 1 & 3 & 2 & 4 & 6 & 5 & 7 & 9 & 8 \\
10 & 1 & 3 & 2 & 4 & 6 & 5 & 7 & 9 & 8
\end{bmatrix}
$$

$$
\Psi_x = \begin{bmatrix}
1 & 2 \\
1 & 2 \\
1 & 2 \\
1 & 2 \\
5 & 4
\end{bmatrix}
$$
Stable Matching-Based Selection with Incomplete Preference lists

- **Steps**
  - $x^6$ proposes a matching request to $p^3$ ($|M|=6$)
  - Release the matching relationship of $p^6$ with its least preferred solution $x^7$
  - $x^7$ proposes a matching request to $p^2$ ($|M|=6$)
  - Release the matching relationship of $p^2$ with its least preferred solution $x^7$
  - $x^8$ proposes a matching request to $p^2$ ($|M|=6$)
  - Release the matching relationship of $p^2$ with its least preferred solution $x^8$
  - $x^9$ proposes a matching request to $p^2$ ($|M|=6$)
  - Release the matching relationship of $p^2$ with its least preferred solution $x^9$
Contents

◦ Introduction
◦ Stable Matching-Based selection
◦ Stable Matching-Based Selection with Incomplete Lists
◦ Adaptive Stable Matching-Based Selection
◦ Conclusion
Adaptive Stable Matching-Based Selection

- Parameter sensitivity studies of $r$
  - For some test instances, performance is very sensitive to the setting of $r$
    - If $r$ is too large, the advantage of incomplete lists degenerates
    - If $r$ is too small, diversity is strengthened while the convergence is scarified
  - $r$ is a problem dependent parameter

IGD performance of MOEA/D-STM2L on MOP test suite with different $r$ settings [6]
Adaptive Stable Matching-Based Selection

- E.g.

Different settings of $r$ affect the selection result. Larger $r$, better convergence; Smaller $r$, better diversity.

Figure 1: Example of selection results using stable matching and two-level stable matching with different $r$. 
Adaptive Stable Matching-Based Selection

Example

- $p^3$ and $p^4$ select solutions far away because $x^2$ and $x^4$ contributes smaller objectives than solutions around them, e.g., $p^4$ and $x^9$ will be unstable as long as $x^2$ and $x^4$ are available.
- We can assume that it is the better/competitive solutions near the PF that have a side effect on the diversity.
- A straightforward way to overcome it is to reduce the preference list of a competitive solution to avoid it being matched with a subproblem far away.

Subproblems’ preference lists

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<tr>
<th>Subproblems</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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<td>9</td>
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<td>9</td>
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<td>7</td>
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Solutions’ preference lists

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<td>5</td>
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<td>Solution 2</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>5</td>
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<tr>
<td>Solution 3</td>
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<td>3</td>
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<td>1</td>
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<td>4</td>
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<td>5</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Stable matching based selection result

- $x^1$, $x^2$, $x^3$, $x^4$, and $x^5$ are selected.
- $x^6$ and $x^9$ are not selected.
Adaptive Stable Matching-Based Selection

○ Concept of the local competitiveness
  ○ At first, all solutions are associated with their closest subproblems. For each subproblem, choose the one which offers the best objective value, as its representative solution.

  ○ Given a neighborhood (defined as \( x \)'s \( \ell \) closest subproblems),
    ○ \( x \) is defined as a **locally competitive solution** if there exists a representative solution \( x(p) \) in this neighborhood dominated by \( x \)
    ○ \( \exists p \in \text{Neighbor}(x): x(p) < x \)
    ○ Otherwise, solution \( x \) is denoted as a **locally noncompetitive solution**
Adaptive Stable Matching-Based Selection

- **Adaptive Mechanism**
  - If solution $\mathbf{x}$ is locally competitive and better than a representative solution $\mathbf{x}(p)$ in $\mathbf{x}$’s neighborhood, then $\mathbf{x}$ will be preferred by the subproblem $p$. When $\mathbf{x}$ locates far from $p$, the diversity may be affected.
  - To avoid $p$ appearing on $\mathbf{x}$’s preference list
    - The length of preference list $r$ of solution $\mathbf{x}$ is set to be the **maximum neighborhood size** $\ell$ that keeps $\mathbf{x}$ locally noncompetitive
    - In such way, solution $\mathbf{x}$ will not be matched with a subproblem whose representative solution is worse than $\mathbf{x}$
Example of determining $r$

Step 1: determine representative solutions

- Solutions are associated to their closest subproblems
- Each subproblem select the best solution from its associated solutions
- Representative solutions:
  - $p^1: x^1$
  - $p^2: x^4$
  - $p^3: x^7$
  - $p^4$ does not have a representative solutions
  - $p^5: x^{10}$
Example of determining $r$

Step 2: determine $r$ for each solution as the *maximum neighborhood size* that keeps the solution locally noncompetitive

- $x^1$:
  - When $\ell = 1$, the neighborhood is $p^1$, the included representative solution is $x^1$. $x^1$ does not dominate $x^1$, so $x^1$ is locally noncompetitive.
  - When $\ell = 2$, the neighborhood is $\{p^1, p^2\}$, the included representative solution are $\{x^1, x^4\}$. $x^1$ dominates $x^4$, so $x^1$ becomes locally competitive.
  - Set $r^1 = 1$

- Similarly, set $r^2 = r^3 = 1$
Example of determining $r$

- **$x^4$:**
  - When $\ell = 1$, the neighborhood is $p^2$, the included representative solution is $x^4$. $x^4$ does not dominate $x^4$, so $x^4$ is locally noncompetitive.
  - When $\ell = 2$, the neighborhood is $\{p^2, p^1\}$, the included representative solution are $\{x^4, x^1\}$. $x^4$ does not dominate any solution in $\{x^4, x^1\}$, so $x^4$ is locally noncompetitive.
  - When $\ell = 3$, the neighborhood is $\{p^2, p^1, p^3\}$, the included representative solution are $\{x^1, x^4, x^7\}$. $x^4$ dominates $x^7$, so $x^4$ becomes locally competitive.
  - Set $r^4 = 2$

- Similarly, set $r^5 = 2$
Adaptive Stable Matching-Based Selection

- Example of determining $r$
  - $x^6$:
    - When $\ell = 1$, the neighborhood is $p^2$, the included representative solution is $x^4$. $x^6$ does not dominate $x^4$, so $x^6$ is locally noncompetitive.
    - When $\ell = 2$, the neighborhood is $\{p^2, p^3\}$, the included representative solution are $\{x^4, x^7\}$. $x^6$ dominates $x^7$, so $x^6$ becomes locally competitive.
  - Set $r^6 = 1$
Example of determining $r$

- $x^7$:
  - When $\ell = 1$, the neighborhood is $p^3$, the included representative solution is $x^7$. $x^7$ does not dominate $x^7$, so $x^7$ is locally noncompetitive.
  - When $\ell = 2$, the neighborhood is $\{p^3, p^2\}$, the included representative solution are $\{x^7, x^4\}$. $x^7$ does not dominate any solution in $\{x^7, x^4\}$, so $x^7$ is locally noncompetitive.
  - When $\ell = 3$, the neighborhood is $\{p^3, p^2, p^4\}$, the included representative solution are $\{x^7, x^4\}$. $x^7$ does not dominate any solution in $\{x^7, x^4\}$, so $x^7$ is locally noncompetitive.
  - When $\ell = 4$, the neighborhood is $\{p^3, p^2, p^4, p^1\}$, the included representative solution are $\{x^7, x^4, x^1\}$. $x^7$ does not dominate any solution in $\{x^7, x^4, x^1\}$, so $x^7$ is locally noncompetitive.
  - When $\ell = 5$, the neighborhood is $\{p^3, p^2, p^4, p^1, p^5\}$, the included representative solution are $\{x^7, x^4, x^1, x^{10}\}$. $x^7$ does not dominate any solution in $\{x^7, x^4, x^1, x^{10}\}$, so $x^7$ is locally noncompetitive.
  - Set $r^7 = 5$
  - Similarly, set $r^8 = r^9 = r^{10} = 5$
Adaptive Stable Matching-Based Selection

○ Adaptive Mechanism

○ Step 1:

○ Determine representative solutions; (line 1-8)

○ Step 2:

○ Gradually determine the maximum noncompetitive neighborhood size of each solution $x$ and assign it to $r$; (line 9-16)

○ Note that the search of $r$ is conducted within $[m, \ell_{max}]$

```
Algorithm 6: AdaptiveSetR(P, S, \Psi_X)

Input:
- subproblem set $P$ and solution set $S$
- solution preference matrix $\Psi_X$

Output: length of solution’s preference list set $R$

1 for $i \leftarrow 1$ to $Q$ do
2 $\Phi[i] \leftarrow \Psi_X[i][1]$;
3 for $j \leftarrow 1$ to $N$ do
4 $\chi \leftarrow \{i | \Phi[i] = j, i \in 1, 2, ..., Q\}$;
5 if $\chi = \emptyset$ then
6 $\varphi[j] \leftarrow -1$;
7 else
8 $\varphi[j] \leftarrow \arg \min_{i \in \chi} g^{tch}(x^i | \lambda^j, z^*)$;
9 for $i \leftarrow 1$ to $Q$ do
10 $r^i \leftarrow m$;
11 for $\ell \leftarrow m + 1$ to $\ell_{max}$ do
12 $t \leftarrow \varphi[\Psi_X[i][\ell]]$;
13 if $t \neq -1$ then
14 if $x^i \prec x^t$ then
15 break;
16 $r^i \leftarrow \ell$;
17 return $R$;
```
Adaptive Stable Matching-Based Selection

Algorithm 7: MOEA/D-AOOSTM/AMOSTM

Input: algorithm parameters
Output: final population $S$
1. Initialize the population $S$, a set of weight vectors $W$ and their neighborhood structure $B$;
2. $M \leftarrow$ Random one-one matching between $P$ and $S$;
3. $neval \leftarrow 0$, $iteration \leftarrow 0$;
4. while Stopping criterion is not satisfied do
5.  Select the current active subproblems to form $I$;
6.  for each $i \in I$ do
7.     if $\text{uniform}(0, 1) < \delta$ and $|E| \geq T$ then
8.        $E \leftarrow \{M(p) | p \in B(i)\}$;
9.     else
10.        $E \leftarrow S$;
11.  Randomly select mating solutions from $E$ and generate an offspring $\overline{x}$, $S \leftarrow S \cup \overline{x}$;
12.  Evaluate $F(\overline{x})$, $neval++$;
13.  $M \leftarrow \text{SelectionOOSTM2L/MOSTM}(P, S)$;
14.  $S \leftarrow \{M(p) | p \in P\}$;
15.  $iteration++$;
16.  if $\text{mod}(iteration, 30) = 0$ then
17.        Update the utility of each subproblem;
18. return $S$;
Adaptive Stable Matching-Based Selection

The trajectories of adaptively determined $r$ values of the solutions selected by subproblems $p^1$, $p^{34}$, $p^{67}$ and $p^{100}$ during the run on MOP1.
Adaptive Stable Matching-Based Selection

Fig. 1: Final solution sets with best IGD metric values found by 9 MOEAs on MOP1.
Adaptive Stable Matching-Based Selection

![Graphs showing final solution sets with best IGD metric values found by 9 MOEAs on MOP2.](image-url)
Adaptive Stable Matching-Based Selection

Fig. 3: Final solution sets with best IGD metric values found by 9 MOEAs on MOP3.
Adaptive Stable Matching-Based Selection

Fig. 4: Final solution sets with best IGD metric values found by 9 MOEAs on MOP4.
Adaptive Stable Matching-Based Selection

Fig. 5: Final solution sets with best IGD metric values found by 9 MOEAs on MOP5.
Adaptive Stable Matching-Based Selection

Fig. 6: Final solution sets with best IGD metric values found by 9 MOEAs on MOP6.
Adaptive Stable Matching-Based Selection

Fig. 7: Final solution sets with best IGD metric values found by 9 MOEAs on MOP7.
Adaptive Stable Matching-Based Selection

### TABLE III: HV Results on 3-Objective WFG Test Instances.

<table>
<thead>
<tr>
<th>Problem</th>
<th>HV</th>
<th>DRA</th>
<th>MOEA/DD</th>
<th>PICEA-g</th>
<th>NSGA-III</th>
<th>HypE</th>
<th>AOOSTM</th>
<th>AMOSTM</th>
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<td>Mean 0.845</td>
<td>0.808</td>
<td>0.773</td>
<td>0.535</td>
<td>0.325</td>
<td>1.157</td>
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<td></td>
<td>Std 6.300E-2</td>
<td>7.173E-2</td>
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<td></td>
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<td>4 ↓</td>
<td>5 ↓</td>
<td>6 ↓</td>
<td>7 ↓</td>
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<td>1.513</td>
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Note: According to Wilcoxon’s rank sum test, +, − and ≈ indicate that the corresponding EMO algorithm is significantly better than, worse than or similar to MOEA/D-AOOSTM, while ↑, ↓ and || indicate that the corresponding EMO algorithm is significantly better than, worse than or similar to MOEA/D-AMOSTM.
### Adaptive Stable Matching-Based Selection

#### TABLE IV: HV Results on 5-Objective WFG Test Instances.

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<th>HypE</th>
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According to Wilcoxon’s rank sum test, +, − and ≈ indicate that the corresponding EMO algorithm is significantly better than, worse than or similar to MOEA/D-AOSTM, while ↑, ↓ and || indicate that the corresponding EMO algorithm is significantly better than, worse than or similar to MOEA/D-AMOSTM.
### Adaptive Stable Matching-Based Selection

**TABLE V: HV Results on 8-Objective WFG Test Instances.**

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According to Wilcoxon’s rank sum test, +, − and ∼ indicate that the corresponding EMO algorithm is significantly better than, worse than or similar to MOEA/D-AOSTM, while ↑, ↓ and || indicate that the corresponding EMO algorithm is significantly better than, worse than or similar to MOEA/D-AMOSTM.
Adaptive Stable Matching-Based Selection

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</table>

According to Wilcoxon’s rank sum test, +, − and ≈ indicate that the corresponding EMO algorithm is significantly better than, worse than or similar to MOEA/D-AOSTM, while ↑, ↓ and || indicate that the corresponding EMO algorithm is significantly better than, worse than or similar to MOEA/D-AMOSTM.
Contents

- Introduction
- Stable Matching-Based Selection
- Stable Matching-Based Selection with Incomplete Lists
- Adaptive Stable Matching-Based Selection
- Conclusion
Conclusion

○ Conclusions

○ The stable matching-based selection mechanism paves a new avenue to address the balance between convergence and diversity from the perspective of achieving the equilibrium between the preferences of subproblems and solutions.

○ With the help partial preference information, a two-level one-one stable matching-based selection mechanism and a many-one stable matching-based selection mechanism are proposed to address the diversity issue of original one-one stable matching.

○ An adaptive mechanism is proposed to dynamically control the length of the preference list of each solution according to the local competitiveness information.
Conclusion

◦ Future work
    ◦ Allow elite solutions to be matched with more than one subproblems
        ◦ Improve convergence
    ◦ Stable matching may be used for mating selection
        ◦ Preferences between solutions need to be well defined
References


References


Thank you