Procurement Design with Optimal Sequential R&D

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Introduction

- **Procurement**
  - A **buyer** seeks to procure a service/good among multiple potential **suppliers**

- **Procurement bidding**
  - The buyer elicits private provision costs
  - Pay information rent

- **We consider:** suppliers don’t endowed with information about own provision costs
  - To learn own provision cost, must go through costly R&D process
  - R&D cost
    - e.g., testing, accounting, proposals
Introduction Cont.

- To elicit information about provision cost
  - information rent, as well as R&D cost (provide incentive for them to learn provision cost)

- Whether to elicit information about provision cost?
  - R&D cost?
  - Contribution to procurement cost reduction?

- Optimal procurement mechanisms in this environment?
  - When to elicit?
Procurement Mechanism

- Suppliers
  - R&D cost: initial private information
  - Provision cost: after incur R&D cost

- Procurement Mechanisms
  - A sequential shortlisting procedure + final procurement bidding stage

- Shortlisting: inviting candidate suppliers to go through the R&D process and participate final bidding stage.
  - Seq. Shortlisting: contingent on seq. reports
Related Literature

- **Auctions with costly entry**
  - McAfee and McMillan, 1987; Engelbrecht-Wiggans, 1993; Tan, 1992; Levin and Smith, 1994; and Ye, 2004
  - Samuelson, 1985; Stegeman, 1996; Campbell, 1998; Menezes and Monteiro, 2000; Tan and Yilankaya, 2006; Cao and Tian, 2009; and Lu, 2009
  - Ye, 2007; Quint and Hendricks, 2013
  - Lu and Ye, 2013, 2016

- **Dynamic revenue-maximizing mechanism design**
  - Baron and Besanko, 1984; Courty and Li, 2000; Eso and Szentes, 2007; Mezzetti, 2007; Li and Shi, 2016
    - Two-stage sequential screening problem
  - Battaglini, 2005; Pavan, Segal, and Toikka, 2014; and Bergemann and Strack, 2015
    - Infinitehorizon Markovian environments
Related Literature Cont.

- **Information acquisition**
  - Persico, 2000; Compte and Jehiel, 2001; Bergemann et al., 2009; and Rezende, 2013
    - fix the mechanism, study information acquisition
  - Bergemann and Valimaki (2002); Shi (2012)
    - mechanism design approach and taking information acquisition into account
The Model

• A buyer needs to acquire a good/service from \( N \) potential suppliers
  - minimizing the procurement costs
  - \( \Omega = \{1, 2, \ldots, N\} \)

• Each supplier
  - R&D cost \( c_i \): initial private information
    - \( c_i \sim \Phi \) over \([c, \bar{c}]\), density function \( \varphi \)
  - provision cost \( \alpha_i \): after incur R&D cost
    - \( \alpha_i \sim F \) over \([\alpha, \bar{\alpha}]\), density function \( f \)
Assumption

- $c_i$ and $\alpha_i$ are independent
- $(c_i, \alpha_i)$ are independent across $i$
- Assumption 1: $c + \frac{\Phi(c)}{\varphi(c)}$ increases in $c$
Mechanism

- Mechanism: \((p, x, M)\)
- \(M \geq 1\) shortlisting stage with \((M + 1)th\) final bidding stage
  - Stage 1: All \(N\) potential suppliers report initial R&D costs \(c_i\)s, denote \(m_1 = (m_{1,i})\) where \(m_{1,i} = c_i\).
    - \(\forall g_1 \in 2^\Omega\), shortlisting with prob. \(p_{g_1}(m_1)\); \(\forall i\), transfer \(x_{1,i}(m_1)\)
    - If shortlisting \(g_1\), discover provision cost \(\alpha_i\) at expense of \(c_i\)
  - Stage 2: Based on additional reports from \(g_1\), denote \(m_2 = (m_{2,i})\) where \(m_{2,i} = \begin{cases} \alpha_i & i \in g_1 \\ \phi & i \notin g_1 \end{cases}\).
    - \(\forall g_2 \in 2^\Omega \setminus g_1\), shortlisting with prob. \(p_{g_2}(m_1, m_2 | g_1)\); \(\forall i\), transfer \(x_{2,i}(m_1, m_2)\)
    - If shortlisting \(g_2\), discover provision cost \(\alpha_i\) at expense of \(c_i\)
- Stage Continues
Mechanism Cont.

- Stage M: Based on additional reports from $g_{M-1}$, denote
  
  $$m_M = (m_M,i) \text{ where } m_{M,i} = \begin{cases} 
  \alpha_i & i \in g_{M-1} \\
  \phi & i \notin g_{M-1}
  \end{cases}$$

- $\forall g_M \in 2^{\Omega \setminus \bigcup_{i=1}^{M-1} g_i}$, prob. $p^{g_M}(m_1, \ldots, m_M | g_1, \ldots, g_{M-1})$; $\forall i$, transfer $x_{M,i}(m_1, \ldots, m_M)$

- If shortlisting $g_M$, discover provision cost $\alpha_i$ at expense of $c_i$

- Stage M+1: $m_{M+1,i} = \begin{cases} 
  \alpha_i & i \in g_M \\
  \phi & i \notin g_M
  \end{cases}$

- outcome $g = \{g_1, g_2, \ldots, g_M\}$, $G_g = \bigcup_{i=1}^{M} g_M$

- $\forall i \in G_g$, $p_{G_g}^{i}(m_1, \ldots, m_{M+1})$; $\forall i$, transfer $x_{M+1,i}(m_1, \ldots, m_{M+1})$

- Focus on $M \geq N$, and observable $\alpha_i$ first

- can be relaxed
Objective Function

\[ TC = E_c E_\alpha \left[ \sum_{g} \Pr(g|c,\alpha) \left( \sum_{i \in G_g \cup \{0\}} p_i^g(c, m_2^\alpha, \ldots, m_{M+1}^\alpha) \alpha_i \right) \right] + E_c E_\alpha \left[ \sum_{G \in 2^\Omega} \Pr(G|c,\alpha) \sum_{i \in G} \left( c_i + \frac{\Phi(c_i)}{\varphi(c_i)} \right) \right] + \sum_{i} U_i(\bar{c}). \]

Lemma

For any \( \{ \Pr(G|c,\alpha), \forall G \in 2^\Omega, c, \alpha \} \), the principal should set \( U_i(\bar{c}) = 0 \) and adopt an efficient procurement in the final stage.

\[ TC = E_c E_\alpha \left\{ \sum_{G \in 2^\Omega} \Pr(G|c,\alpha) \left( \min\{ \{ \alpha_i \}_{i \in G}, \alpha_0 \} + \sum_{i \in G} \left( c_i + \frac{\Phi(c_i)}{\varphi(c_i)} \right) \right) \right\}. \]  

Optimal \( \{ \Pr(G|c,\alpha), \forall G \in 2^\Omega, c, \alpha \} \)?
Optimal Shortlisting

Lemma

*Shortlisting one agent at a stage until the last being shortlisted yields weakly lower procurement cost than other rules.*

- Any \( \{ \Pr(G|c, \alpha), \forall G \in 2^\Omega, c, \alpha \} \) derived from a general shortlisting procedure can be duplicated by this rule

Lemma

*At the optimum, at each stage there is no loss of generality for the principal to either shortlists an agent with probability 1 or stop shortlisting.*

- Compare TCs

Lemma

*At any stage, the principal should shortlist the one who has the lowest virtue cost \( c_i + \frac{\Phi(c_i)}{\varphi(c_i)} \) among the remaining players or stop shortlisting.*
Optimal Shortlisting Rule

Theorem

(i) $\forall t_0 \in \{1, \ldots, N\}$ and $G^{t_0-1}$, let

$$i_{t_0}^* \in \arg\min_{i \in \Omega \setminus G^{t_0-1}} \left[ c_i + \frac{\Phi(c_i)}{\varphi(c_i)} \right]$$

and

$$\delta_{t_0} = \min \{ \{ \alpha_i \}_{i \in G^{t_0-1}, \alpha_0} - E_{\alpha_i_{t_0}} \min \{ \alpha_{i_{t_0}}^*, \{ \alpha_i \}_{i \in G^{t_0-1}, \alpha_0} \} \}.$$ If

$$\delta_{t_0} \geq \left( c_{i_{t_0}^*} + \frac{\Phi(c_{i_{t_0}^*})}{\varphi(c_{i_{t_0}^*})} \right),$$

shortlisting $i_{t_0}^*$ with probability 1 is optimal; if

$$\delta_{t_0} < \left( c_{i_{t_0}^*} + \frac{\Phi(c_{i_{t_0}^*})}{\varphi(c_{i_{t_0}^*})} \right),$$

the shortlisting process stops. (ii) The last stage procurement is efficient.
Optimal M

  - $M > N$: optimal shortlisting procedure lasts $N$ stages at most
  - $M < N$: a shortlisting procedure $(p, x, M)$ with $M < N$ can be duplicated by $(p, x, M)$ with $M = N$
    - no shortlisting prob., no transfer
Private Provision costs

**Theorem**

*Under Assumption 1, the shortlisting rule and the final provider allocation rule of Theorem 1 are truthfully implementable when both R&D and provision costs are private information of the agents.*
Private Provision costs

- Report on provision cost $\alpha_{i_k}$
- At a shortlisting stage $k + 1$, agent $i_k$, i.c. requires that
  \[
  \pi_{i_k}(\alpha_{i_k}, \alpha_{i_k}; c, (\alpha_j)_{j=1}^{k-1}) - \pi_{i_k}(\alpha_{i_k}, \hat{\alpha}_{i_k}; c, (\alpha_j)_{j=1}^{k-1}) \geq 0
  \]
- By the envelop theorem
  \[
  \pi_{i_k}(\alpha_{i_k}, \alpha_{i_k} | c, (\alpha_j)_{j=1}^{k-1})
  = \int_{\alpha_i} E(\alpha_{i_{k+1}}, \ldots, \alpha_{i_N}) \sum_{\forall h=k}^M \Pr^*(g_{k,h} | c, \alpha_{-i_k}, y) p_{i_k}^* G_{g_{k,h}, i_k}(c, \alpha_{-i_k}, \hat{\alpha}_{i_k}) dy,
  \]
  \[
  \text{where } g_{k,h} = (g_1 = \{i_1\}, g_2 = \{i_2\}, \ldots, g_{k-1} = \{i_{k-1}\}, \ldots, g_h = \{i_h\}, g_{h+1} = \emptyset, \ldots, g_M = \emptyset), h \geq k \geq 1 \text{ is a sequence of the shortlisted suppliers}
  \]
  \[
  \sum_{\forall h=k}^M \Pr^*(g_{k,h} | c, \alpha_{-i_k}, \hat{\alpha}_{i_k}) p_{i_k}^* G_{g_{k,h}, i_k}(c, \alpha_{-i_k}, \hat{\alpha}_{i_k}) \text{ decreases with } \hat{\alpha}_{i_k}
  \]
  \[
  \text{implies i.c}
  \]
Private Provision costs

- Initial reporting stage: we have to examine a bidder’s incentive to report his R&D cost $c_i$
  - A miss-reported R&D cost would change the shortlisting sequence
- However, when the agent is called on to report his provision cost $\alpha_i$, the incentive does not depend on his R&D cost
- When turn to the incentive compatibility at the first stage, it is WLOG to consider a candidate must report his provision cost truthfully
  - When called on, $i$ must report his provision cost truthfully regardless of his first stage report.
Private Provision costs

- Initial reporting stage: i.e. requires $U_i(c_i, c_i) \geq U_i(c_i, \hat{c}_i)$
- By setting $U_i(\bar{c}, \bar{c}) = 0$,

$$U_i(c_i, c_i) = \mathbb{E}_{c_{-i}} \int_{c_i}^{\bar{c}} \mathbb{E}_\alpha \left\{ \sum_{\forall h \text{ s.t. } k \leq h \leq M, \hat{c}_i \leq c_{ih}} \Pr^*(g_k, h | c_{-i}, y, \alpha) \right\} dy$$

- The transfer $x_i(c_i, c_{-i})$ is constructed following the envelope condition
Concluding Remarks

- Optimal Sequential Shortlisting in Procurement Bidding
  - suppliers doesn’t know provision costs after costly R&D process
- Rank the candidate and Compare cost and contribution
- Shortlisting one by one
  - Better coordinate entry
  - Batter control information
- Whether to observe provision cost after discovery doesn’t matter
  - Transfer
Thanks!