Coordinating on Networks

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Motivating examples

Games with complements, binary actions and incomplete information:

1. Cryptocurrency adoption
   - the more others adopt, the more viable as medium of exchange,
   - value (exchange rates) of currency in the future unknown.

2. Crime
   - the more neighbors partake in crime, (i) the more help you have,
   - (ii) the less likely you will be caught,
   - presence / strength of police force unknown.

3. Immigration / refugee policy
   - bordering countries’ open-door-policies imply less refugee flow to your country,
   - state of economy / outcome of war unknown.
Who coordinates with whom? *Non-coordination*
Who coordinates with whom? *Non-coordination*
Who coordinates with whom? *Non-coordination*
Who coordinates with whom? Non-coordination
Who coordinates with whom? Non-coordination
Who coordinates with whom? Coordination sets

[Diagram of a network with interconnected nodes]
Who coordinates with whom?  *Coordination sets*
Who coordinates with whom?  *Coordination sets*
Who coordinates with whom? Coordination sets
Who coordinates with whom? *Common coordination*
Who coordinates with whom? Common coordination
Global games: review


<table>
<thead>
<tr>
<th></th>
<th>Invest</th>
<th>NotInvest</th>
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<tbody>
<tr>
<td>Invest</td>
<td>$\theta, \theta$</td>
<td>$\theta - 1, 0$</td>
</tr>
<tr>
<td>NotInvest</td>
<td>$0, \theta - 1$</td>
<td>$0, 0$</td>
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- If $\theta > 1$: *Invest* dominant strategy
- If $\theta \in [0, 1]$: two symmetric Nash.
- If $\theta < 0$: *NotInvest* dominant strategy

\[
\begin{array}{c|cc}
 & \text{Invest} & \text{NotInvest} \\
\hline
\text{Invest} & \theta, \theta & \theta - 1, 0 \\
\text{NotInvest} & 0, \theta - 1 & 0, 0 \\
\end{array}
\]

- Assume each \( i = 1, 2 \) observe \( s_i = \theta + \epsilon_i, \epsilon_i \sim N(0, \sigma) \),
- \( \theta \) uniform on \( \mathbb{R} \).
- Cutoff strategy of \( j \neq i \):

\[
\begin{cases} 
\text{Invest} & \text{if } s_j \geq s^+ \\
\text{NotInvest} & \text{if } s_j < s^+ 
\end{cases}
\]

\( \Rightarrow \) probability \( j \rightarrow \text{NotInvest} \) given \( s_i \) is \( \Phi(\frac{s^+ - s_i}{\sqrt{2\sigma}}) \) (stand. norm. cdf \( \Phi \)).
Global games: review


Given \( s_i, s^\dagger \), value to Invest is:

\[
(1 - \Phi\left(\frac{s^\dagger - s_i}{\sqrt{2}\sigma}\right))s_i + \Phi\left(\frac{s^\dagger - s_i}{\sqrt{2}\sigma}\right)(s_i - 1) = s_i - \Phi\left(\frac{s^\dagger - s_i}{\sqrt{2}\sigma}\right)
\]

\[
\text{Decreasing } \sigma \uparrow
\]
Summary of results

- Coordination sets characterize limit-equilibrium properties:
  - solve for unique set of limiting coordination sets,
  - easy to obtain common coordination: e.g., trees, bipartite graphs.

- Contagion contained within coordination sets:
  - all agents within sets respond uniformly to targeted adoption subsidy,
  - strategic affect falls discontinuously across coordination sets.

- Welfare and policy implications:
  - in the limit, optimal policies reduce to targeting coordination sets,
  - externalities wedge between adoption vs. welfare policies.
Global Games (Morris and Shin 2006 for review)


Network Games

- Ballester et al. (2006), Galeotti et al. (2010), Bramoullé et al. (2014), many more.

Model setup (I)

- Connected network $G = (N, E)$, nodes $i \in N$, edges $(i, j) \in E$.
- Assume undirected graph: $(i, j) \in E$ iff $(j, i) \in E$.
- $N_i := \{j : (i, j) \in E\}; \ d_i := |N_i|; \ d_i(S) := |S \cap N_i|, \ S \subseteq N$.
- $i$’s action $a_i$, binary action space $\{0, 1\}$: adopt ($a_i = 1$) and not adopt ($a_i = 0$) technology.
- Payoff-relevant state $\theta \in \Theta$, interval support $\Theta \subseteq \mathbb{R}$.
- Payoff from adopting ($a_i = 1$):
  \[
  u_i(a_{-i}|\theta) = v_i + \sigma(\theta) + \phi \sum_{j \in N_i} a_j, 
  \]
  $v_i \in \mathbb{R}$, $\sigma : \Theta \mapsto \mathbb{R}$ is $C^1$ and strictly increasing, $\phi > 0$.
- Payoff from not adopting ($a_i = 0$): zero.
Existence of dominance regions

For each $i$, assume $v_i$, $\sigma$ and $\phi$ are s.t. $\exists \theta_i, \bar{\theta}_i \in \Theta$:

- $v_i + \sigma(\theta) + \phi d_i < 0$ when $\theta < \theta_i$, and
- $v_i + \sigma(\theta) > 0$ when $\theta > \bar{\theta}_i$.

Information structure (perturbed game $G(\nu)$)

- $\theta$ is observed with noise by all agents.
- Signal $s_i = \theta + \nu \epsilon_i$,
  - $\nu > 0$, $\epsilon_i$ i.i.d., pdf $f$, cdf $F_i$, with support $[-1, 1]$. 
Existence of dominance regions

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Information structure (perturbed game $G(\nu)$)

- $\theta$ is observed with noise by all agents.
- Signal $s_i = \theta + \nu \epsilon_i$,
  $\nu > 0$, $\epsilon_i$ i.i.d., pdf $f$, cdf $F_i$, with support $[-1, 1]$. 
Conditional expectations

- Cutoff strategies: \( \pi^+_i(s_i) := \begin{cases} 1 & \text{if } s_i \geq s^+_i \\ 0 & \text{if } s_i < s^+_i \end{cases} \).
  Lower cutoff \( \Rightarrow \) more adoption.

- Expected payoff to \( i \) adopting conditional on \( s_i \):
  \[
  U_i(\pi^+_i \mid s_i) := \mathbb{E}_\theta \left[ \mathbb{E}_{s-i} \left[ u_i(a_i \mid \theta) \mid \pi^+_i, \theta \right] \mid s_i \right] \\
  = \nu_i + \mathbb{E}_\theta [\sigma(\theta) \mid s_i] + \phi \sum_{j \in N_i} \mathbb{E}_\theta \left[ \mathbb{E}_{s_j} \left[ \pi^+_j(s_j) \mid \theta \right] \mid s_i \right].
  \]
Equilibrium in the noiseless limit $\nu \to 0$

$$U_i(\pi^*_i|s^*_i) = 0, \quad \forall i \in N$$

define a Bayesian Nash Equilibrium of perturbed game $G(\nu)$.

**Lemma 1**

*Bayesian Nash Equilibrium* $\pi^*$ of $G(\nu)$ in cutoff strategies exists.

Frankel et al. (2006) Theorem 1, in this setting:

**Proposition 1**

There exists a unique strategy profile $\tilde{\pi}$, which is in cutoff strategies, such that any $\pi$ surviving iterative elimination of strictly dominated strategies in $G(\nu)$ satisfies $\lim_{\nu \to 0} \pi = \tilde{\pi}$.

- $\tilde{\pi}$ defined by limit cutoffs $\theta^*$;
- limiting probability weightings $w^*_{ij} := \lim_{\nu \to 0} \mathbb{E}_{s_j}[\pi^*_j(s_j)|s_i = s^*_i]$. 
Define feasible weighting functions:

\[ \mathcal{W} = \{ \mathbf{w} = (w_{ij}, (i, j) \in E) | w_{ij} \geq 0, w_{ji} \geq 0, w_{ij} + w_{ji} = 1; \forall (i, j) \in E \} , \]

Recall \( w_{ij}^* := \lim_{\nu \to 0} \mathbb{E}_{s_j} [\pi_j^*(s_j)|s_i = s_i^*] \).

Lemma (Limit-equilibrium weights)

For each \( ij \in E \),

\[ w_{ij}^* + w_{ji}^* = 1. \]

Moreover, if \( \theta_i^* < \theta_j^* \), then

\[ w_{ij}^* = 0, \text{ and } w_{ji}^* = 1. \]

Note that this gives \( \mathbf{w}^* \in \mathcal{W} \).
Limit equilibrium

- Define affine mapping (with image $\Phi(\mathcal{W})$):

$$\Phi_i(w) := v_i + \phi \sum_{j \in N_i} w_{ij}, \quad \forall i \in \mathcal{N}.$$ 

Theorem (Limit equilibrium)

For any $v$, $\phi$, and network $G$, the equilibrium limit cutoffs $\theta^*$ are given by:

$$\sigma(\theta^*_i) + q^*_i = 0, \quad \forall i,$$

where $q^* = (q^*_1, \cdots, q^*_n)$ is the unique solution to:

$$q^* = \arg\min_{z \in \Phi(\mathcal{W})} ||z||.$$
Define affine mapping (with image \( \Phi(\mathcal{W}) \)):

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\Phi_i(w) := v_i + \phi \sum_{j \in N_i} w_{ij}, \quad \forall i \in N.
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**Theorem (Limit equilibrium)**

For any \( v, \phi, \) and network \( \mathcal{G} \), the equilibrium limit cutoffs \( \theta^* \) are given by:

\[
\sigma(\theta_i^*) + q_i^* = 0, \quad \forall i,
\]

where \( q^* = (q_1^*, \ldots, q_n^*) \) is the unique solution to:

\[
q^* = \arg\min_{z \in \Phi(\mathcal{W})} ||z||.
\]

Define \( T := \sum_i v_i + \phi|E| \). For dyad, \( T = v_1 + v_2 + \phi \).
Limit equilibrium solution: dyad

\[ \frac{T}{2} \]

\( (v_1, v_2) \)

\[ q^* \]

\[ v_1 - v_2 < -\phi; \]

\[ T = v_1 + v_2 + \phi. \]
Limit equilibrium solution: dyad

\[ q_2 = q_1 (v_1, v_2) / \Phi(W) \]

\[ T = v_1 + v_2 + \phi. \]

\[ \phi \geq v_1 - v_2 \geq -\phi; \]

\[ q^* = (v_1, v_2). \]

\[ T = v_1 + v_2 + \phi. \]
Limit equilibrium solution: dyad

\[
q_2 = \frac{T}{2}, \quad q_1 = \frac{T}{2}
\]

\[
q^* = (v_1, v_2)
\]

\[
\phi; \quad T = v_1 + v_2 + \phi.
\]
Limit partition $\mathcal{C}^*$

For $S \subseteq N$, denote $E_S \subseteq E$ corresponding to subgraph $G_S := (S, E_S)$ of $G$ restricted to vertices $S$.

**Definition (limit partition)**

The limit equilibrium $\vec{\pi}$ maps to an ordered partition $\mathcal{C}^* := (C_1^*, \ldots, C_m^*)$ of $N$ satisfying:

1. For each $C_m^*$, all members have the same cutoff $\theta_m^*$ (equiv., same $q_m^*$).
2. For each $C_m^*$, subgraph $G_{C_m^*}$ is connected.
3. For each $C_m^* \neq C_{m}'$ such that $\theta_m^* = \theta_{m}'$, $C_m^*$ and $C_m'$ are not directly connects in $G$.

For each $C_m^* \in \mathcal{C}^*$, denote $\bar{C}_m^* := \bigcup_{m' < m} C_m'$, and $\bar{C}_m^* := \bigcup_{m' > m} C_m'$.

Denote $i$’s coordination set by $m(i)$. 
For any $S, S' \subseteq N, S \cap S' = \emptyset$, denote:

\[ v(S) := \sum_{i \in S} v_i, \]
\[ e(S, S') := \sum_{i \in S} d_i(S'), \]
\[ e(S) := \frac{1}{2} \sum_{i \in S} d_i(S). \]

**Proposition (Coordination-set cutoffs)**

Conditional on $C^*_m$:

\[ \theta^*_m = \sigma^{-1} \left( - \frac{v(C^*_m) + \phi(e(C^*_m, C^*_m) + e(C^*_m))}{|C^*_m|} \right). \]
Characterizations & Comparative Statics: Homogenous Values

Assume: $v_i = v$ for each $i \in N$. 
Corollary (Limit partition homogeneity)

Under homogeneous valuations, $C^*$ is independent of $v$ and of $\phi$. Moreover, $q^* = v1_n + \phi \hat{q}^*$, where $\hat{q}^* := q^*$ under $v = 0$ and $\phi = 1$. 
Corollary (Limit partition homogeneity)

Under homogeneous valuations, $C^*$ is independent of $v$ and of $\phi$. Moreover, $\mathbf{q}^* = v\mathbf{1}_n + \phi \hat{\mathbf{q}}^*$, where $\hat{\mathbf{q}}^* := \mathbf{q}^*$ under $v = 0$ and $\phi = 1$.

- If we scale (i) common certain value $v$, (ii) slope of $\sigma$, (iii) fundamental uncertainty of $\theta$, and/or (iv) value to neighbors’ adoption, who coordinates with whom does not change.
Corollary (Limit partition homogeneity)

Under homogeneous valuations, $C^*$ is independent of $v$ and of $\phi$.
Moreover, $q^* = v\mathbf{1}_n + \phi \hat{q}^*$, where $\hat{q}^* := q^*$ under $v = 0$ and $\phi = 1$.

- If we scale (i) common certain value $v$, (ii) slope of $\sigma$, (iii) fundamental uncertainty of $\theta$, and/or (iv) value to neighbors’ adoption, who coordinates with whom does not change.
- Without loss, set $v = 0$ and $\phi = 1$ in the following examples...
Example 1

Why periphery’s cutoff not below center’s cutoff:

\[ d_{ip}(\{ip\}) = 0 < d_c(\{1p, 2p, 3p\}) + d_c(\{c\}) = 3 + 0, \]

Why center’s cutoff not below periphery’s cutoff:

\[ d_c(\{c\}) = 0 < d_{ip}(\{c\}) + d_{ip}(\{ip\}) = 1 + 0. \]
Triad-core-periphery network
Quad-core-periphery network

Why core nodes have strictly lower cutoff than periphery:

\[
\frac{e(C_m^*, \emptyset) + e(C_m^*)}{4} = \frac{0 + 6}{4} > e(\{jp\}, C_m^*) + e(\{jp\}) = 1 + 0, \\
\]

for \( C_m^* = \{1c, 2c, 3c, 4c\} \Rightarrow C_1^* = \{1c, \ldots, 4c\}, \ C_j^* = \{jp\}, \ j = 1, \ldots, 4. \)
Example 4

Large core-periphery network
Example 4

Large core-periphery network
Example 4

Large core-periphery network
Example 4

Large core-periphery network
Example 4

Large core-periphery network

Limit partition: $C_1^* = \{ic : i = 1, \ldots, 6\}, \; C_2^* = \{r\}, \; C_3^* = \{1q, 2q\}, \; \{C_4^*, \ldots, C_7^*\} = \{\{1p\} \ldots \{4p\}\}$. 
We can characterize when a single coordination set on $\mathcal{G}$ obtains:

**Proposition (Single coordination set)**

*Under homogeneous valuations, a single coordination set exists (i.e. $C^* = \{C_1\}$) if and only if $\mathcal{G}$ is balanced, in the sense that for every nonempty $S \subset N$,*

\[
\frac{e(S)}{|S|} \leq \frac{e(N)}{|N|}.
\]

*When this condition is satisfied, the common cutoff in the network is* $\theta_1^* = \sigma^{-1}(-\nu - \phi \frac{e(N)}{|N|})$. 

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Single coordination set: examples

- **Regular-bipartite network**: $B_1$ and $B_2$, with $B_1 \cup B_2 = N$ and of sizes $n_s := |B_s|$ and degrees $d_s := d_i$, $i \in B_s$, for sides $s = 1, 2$.

---

**Proposition 2 (Single coordination set: examples)**

*Under homogeneous valuations, there exists a single coordination set if $G$:*  
1. is a regular network, or  
2. is a tree network, or  
3. is a regular-bipartite network, or  
4. has a unique cycle, or  
5. has at most four nodes.*
Examples: real-world networks

Figure: Banerjee et al. (2013) network 1
Examples: real-world networks

\[ q_1^* = 3 \]

\[ q_2^* = 2.5 \]

\[ q_3^* = 2.5 \]

\[ q_4^* = 2 \]

\[ q_5^* = 1.33 \]

\[ q_6^* = 1 \]

\[ 2^q \times 4 = 2 \]

\[ 2^q \times 2 = 2.5 \]

\[ 2^q \times 3 = 2.5 \]

\[ 2^q \times 5 = 1.33 \]

\[ 2^q \times 6 = 1 \]

Figure: Banerjee et al. (2013) network 2
Examples: real-world networks

$q_1^* = 1$

Figure: Add Health friendship network
Comparative statics: linkage

- $G_{+ij}$: supergraph of $G$ including link $(i, j)$.
- $C_{+ij}^*$: limit partition under $G_{+ij}$.
- $q_{m,+ij}^*$ corresponds to $C_m^*$ under network $G_{+ij}$.

**Proposition (Linkage)**

Take $i, j$ with $m(i) \geq m(j)$, $(i, j) \not\in E$, such that $C_{+ij}^* = C^*$. If:

1. $\theta_{m(i)}^* > \theta_{m(j)}^*$, then:
   
   $$q_{m(i),+ij}^* - q_{m(i)}^* = \phi \frac{1}{|C_m^*(i)|},$$
   
   and
   
   $$q_{m(j),+ij}^* - q_{m(j)}^* = 0;$$

2. $m(i) = m(j) =: m$, then:
   
   $$q_{m,+ij}^* - q_{m}^* = \phi \frac{1}{|C_m^*(i)|}.$$
Comparative statics: heterogeneous values

Arbitrary \( (v_i). \)
Heterogeneous values: example

\[ u_i(a_{-i} | \theta) = v_i - 3 \frac{1 - \theta}{\theta} + \sum_{j \in N_i} a_j. \]

Exercise: vary \( v_{1p} \), hold \( v_i = 1 \) for other agents.
Heterogeneous values: example

\[ u_i(a_{-i} | \theta) = v_i - 3 \frac{1-\theta}{\theta} + \sum_{j \in N_i} a_j ; \quad \nu = 0.05. \]
Comparative statics: sticky coordination

For each $i \in C_m^*$ denote:

$$
\hat{v}_i^* := \arg\max\{v_i : \theta_i^* = \theta_j^*, j \in C_m^* \setminus \{i\}; v_{-i}\},
$$

$$
\check{v}_i^* := \arg\min\{v_i : \theta_i^* = \theta_j^*, j \in C_m^* \setminus \{i\}; v_{-i}\}.
$$

**Proposition (Sticky coordination)**

Take coordination set $C_m^* \in C^*$ with $|C_m^*| > 1$. Then for each $i \in C_m^*$:

$$
\hat{v}_i^* - \check{v}_i^* \geq \phi d_i(C_m^*).
$$

When $C^*$ is constant for $v_i \in (\hat{v}_i^*, \check{v}_i^*)$, then:

$$
\hat{v}_i^* - \check{v}_i^* = \frac{|C_m^*|}{|C_m^*| - 1} \phi d_i(C_m^*).
$$
Comparative statics: local contagion

**Proposition (Local contagion)**

In the limit, the mapping $q^*(v)$ is piecewise linear, Lipschitz continuous, and monotone. Generically, $\frac{\partial q^*}{\partial v}$ exists. Generically, when $i, j \in C_m$ and $k \notin C_m$, then:

$$\frac{\partial q_m^*}{\partial v_i} = \frac{1}{|C_m^*|},$$

and

$$\frac{\partial q_k^*}{\partial v_i} = 0.$$

Note: $\frac{\partial \theta_m^*}{\partial v_i} = \frac{\partial q_m^*}{\partial v_i} \frac{-1}{\sigma'(\theta_j^*)}$, by $\sigma(\theta_j^*) + q_j^* = 0$. 
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*In the limit, the mapping* \( q^*(v) \) *is piecewise linear, Lipschitz continuous, and monotone. Generically,* \( \frac{\partial q^*}{\partial v} \) *exists. Generically, when* \( i, j \in C_m \) *and* \( k \not\in C_m \), *then:*

\[
\frac{\partial q_m^*}{\partial v_i} = \frac{1}{|C_m^*|}, \quad \text{and} \quad \frac{\partial q_k^*}{\partial v_i} = 0.
\]

- **Note:** \( \frac{\partial \theta_m^*}{\partial v_i} = \frac{\partial q_m^*}{\partial v_i} \frac{-1}{\sigma'(\theta_j^*)} \), by \( \sigma(\theta_j^*) + q_j^* = 0 \).

- **Contained contagion:** perturbations spread within coordination sets, evenly so, but discontinuously drops to zero across coordination sets.
Welfare and Policy Implications
Benchmarks

Benchmark 1 (adoption maximization):

\[ ma^*_i := \sum_j \frac{\partial}{\partial v_i} E_{s_j} [\pi^*_j] . \]

Benchmark 2 (welfare maximization):

\[ mw^*_i := \sum_j \frac{\partial}{\partial v_i} E_{s_j} \left[ U_j(\pi_{-j}^*|s_j) \right] . \]
Proposition (Policy impact)

Denote $H$ the marginal cdf of $\theta$. For each $C_m^* \in C^*$ and $i \in C_m^*$:

1. $\lim_{\nu \to 0} m a_i^* = \frac{H'(\theta_m^*)}{\sigma'(\theta_m^*)}$

2. $\lim_{\nu \to 0} m w_i^* = (1 - H(\theta_m^*)) + \phi \left( \frac{e(C_m^*, C_m^*) + e(C_m^*)}{|C_m^*|} \right) \frac{H'(\theta_m^*)}{\sigma'(\theta_m^*)}$. 

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Denote $H$ the marginal cdf of $\theta$. For each $C_m^* \in C^*$ and $i \in C_m^*$:

1. \[ \lim_{\nu \to 0} ma_i^* = \frac{H'(\theta_m^*)}{\sigma'(\theta_m^*)}, \]

2. \[ \lim_{\nu \to 0} mw_i^* = (1 - H(\theta_m^*)) + \phi \left( \frac{e(C_m^*, C_m^*) + e(C_m^*)}{|C_m^*|} \right) \frac{H'(\theta_m^*)}{\sigma'(\theta_m^*)}. \]

- Optimal policy problems reduce to targeting a coordination set.
- Wedge between $ma^*$ and $mw^*$: subsidies to high-cutoff coordination sets impose positive externalities on low-cutoff coordination sets; low-cutoffs unresponsive (contained contagion).
Variation: miscoordination costs

Under heterogeneous valuations, set $v_i$ to $v_i - \phi d_i$ to give:

$$u_i(a_{-i}|\theta) = v_i + \sigma(\theta) - \phi \sum_{j \in N_i} (1 - a_j).$$

- **Interpretation:** homogeneous values under miscoordination costs.
- If $v_i = v$ for each $i$, common coordination persists within trees, regular-bipartite networks, networks with a unique cycle, and networks with at most four nodes.
2015: more than a million migrants and refugees crossed into Europe.

Some European countries such as Germany and Sweden were positively inclined towards these migrants, whereas other countries such as Poland and Hungary took strong stances against regularizing them.
Miscoordination in immigration policy

- $a_i = 0$: anti-immigration (i.e. “isolationist”) stance.
- $a_i = 1$: inclusive policy.
- $v_i + \sigma(\theta)$: value of taking an inclusive policy (political support).
- Inflow of immigrants into country $i$: $f + \tau \sum_{j \in N_i} (1 - a_j)$;
- $\tau > 0$: overflow of migrants into $i$ when $j \in N_i$ takes anti-immigration stance.

$$u_i(a_{-i} | \theta) = v_i + \sigma(\theta) - c \left( f + \tau \sum_{j \in N_i} (1 - a_j) \right)$$

$$= v_i - cf + \sigma(\theta) - c \tau \phi \sum_{j \in N_i} (1 - a_j).$$
Miscooordination in immigration policy

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- $a_i = 1$: inclusive policy.
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- Inflow of immigrants into country $i$: $f + \tau \sum_{j \in N_i} (1 - a_j)$; 
- $\tau > 0$: overflow of migrants into $i$ when $j \in N_i$ takes anti-immigration stance.

$$u_i(a_{-i} | \theta) = \nu_i + \sigma(\theta) - c \left( f + \tau \sum_{j \in N_i} (1 - a_j) \right)$$

$$= \nu_i - cf + \sigma(\theta) - c \tau \sum_{j \in N_i} (1 - a_j).$$

- $\nu_i \approx \nu$: countries with many borders take high $\theta_i^*$ (isolationism).
- EU should have a common immigration policy: avoid micoordination costs due to excessive immigrant flows to pro-immigration countries.
Technical Contributions:

- Provide solution to limit cutoffs for general networks, incorporating multiple coordination sets in a global-game setting.
- Characterize (↔) network conditions for common coordination.
Conclusion

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- Characterize (↔) network conditions for common coordination.

Equilibrium characterizations unique to network-games literature: coordinated adoption cutoffs in noiseless limit.

- Homogeneous values: stratified coordination across network cliques/peripheries.
- Heterogeneous values: “sticky” coordination amongst interconnected agents.
- *Local contagion*: strategic spillovers contained within coordination sets.

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Conclusion

Common coordination not hard to obtain:

- Homogenous values: regular networks, trees, regular-bipartite networks, networks with a unique cycle, and networks with at most four nodes.
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Comparative statics:

- Quantify effect of linkage on equilibrium cutoffs,
- Quantify marginal affect of adoption subsidization on equilibrium cutoffs.
Common coordination not hard to obtain:
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Comparative statics:
- Quantify effect of linkage on equilibrium cutoffs,
- Quantify marginal affect of adoption subsidization on equilibrium cutoffs.

Welfare implications:
- Optimal policy problems reduce to targeting a coordination set.
- Planner aiming to maximize adoption designs intervention to yield large strategic effects.
- Planner aiming to maximize welfare also accounts for direct (ex-ante) externalities on neighbors with strictly lower cutoffs.