Gradual Bargaining in Decentralized Asset Markets

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Background

Models of decentralized asset markets

- to explain asset/market liquidity

Two approaches

- New Monetarist approach: Assets as media of exchange
- Finance approach: Illiquid assets traded over the counter

Based on search paradigm with two core components:

1. search frictions and pairwise meetings
2. bargaining

This paper is about **bargaining**
Background: 2nd generation of models

Restricted asset holdings: $a \in \{0, 1\}$
Background: 3rd generation of models

Portfolio of divisible assets: \( a \in \mathbb{R}_+^J \)
Background: How is bargaining handled?

Bargaining with \( a \in \mathbb{R}^J_+ \) like with \( a \in \{0, 1\} \)

- Generalized Nash or Kalai solution
- Agents negotiate their portfolio all at once
Background: How is bargaining handled?

Bargaining with $a \in \mathbb{R}^J_+$ like with $a \in \{0, 1\}$

- Generalized Nash or Kalai solution
- Agents negotiate their portfolio all at once

Questions

- Is this agenda (all-at-once bargaining) restrictive?
- Is it the agenda that agents/society would choose?
- Does the agenda matter for allocations and prices?
Insights

1. **Bargaining theory**
   Extensive-form bargaining games, endogenous agenda

2. **Asset prices**
   Negotiability premia, distributions of asset returns and velocities

3. **Monetary theory**
   rate-of-return dominance, exchange rate determination, OMOs
Time, goods, agents

Time: \( t = 0, 1, 2..., \infty \)

- Each period has two stages:
  1. Decentralized market (DM) for goods and assets, with pairwise meetings and bargaining
  2. Centralized market (CM) for goods and assets

- DM good is perishable, and CM good taken as numeraire
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Agents: divided into two types, unit measure of each

1. Consumers: consume DM good and produce numeraire
2. Producers: produce DM good and consume numeraire

In DM, \( \alpha \in (0, 1] \) pairwise meetings b/w consumers and producers
- Discount factor $\beta = 1/(1 + \rho)$
- Efficient DM output: $u'(y^*) = v'(y^*)$
Assets

- Lucas trees: pay off $d \geq 0$ in the CM
  - Fiat money: $d = 0$

- Exogenous supply: $A_{t+1} = (1 + \pi)A_t$
  - if $d > 0$, $\pi = 0$

- Asset price in terms of the numeraire: $\phi_t$

- No private IOUs: no record-keeping and no commitment
Bargaining game

Game has $N$ rounds

- Asset owner has $z$ units of assets (in terms of numeraire)
- Divided into $N$ equal sizes: $z/N$
- In each round, agents negotiate sale of $z/N$ assets for some output $y$
Alternative ultimatum offer game

$N$ two-stage rounds, identity of the proposer alternates

- Stage 1: One player makes an offer
- Stage 2: Other player accepts/rejects
Intermediate Pareto frontier

- Denote $\tau \equiv nz/N$ where $n = 1, \ldots, N$
- For each $\tau$, feasibility constraint on asset sales: $p(\tau) \leq \tau$
- For each $\tau$, a Pareto frontier:

$$\max u_b(\tau) \quad \text{s.t.} \quad u_s(\tau) \geq u_s \quad \text{and} \quad p(\tau) \leq \tau$$

\[ \downarrow \]

$$H(u^b, u^s, \tau) = 0.$$
The diagram illustrates the gradual bargaining framework with the equation $H(u_b, u_s, \tau) = 0$. The axes represent $u^s$ and $u^b$, and the point $(u_0^b, u_0^s)$ is marked on the diagram.
Subgame Perfect Equilibrium

Round $N-1$: Buyer makes an offer

Round $N-2$: Seller makes an offer
Solution to alternating ultimatum offer bargaining game

Take the limit as $N$ approaches $\infty$

- SPE exists with $\{u^b(\tau), u^s(\tau)\}$ converging to solution to:

$$u^{\chi'}(\tau) = -\frac{1}{2} \frac{\partial H(u^b, u^s, \tau) / \partial \tau}{\partial H(u^b, u^s, \tau) / \partial u^{\chi}}$$

expressed in utils of player $\chi$,

Robustness: coincides with axiomatic gradual bargaining solution by O’Neill et al. (2004)

- Pareto optimality, scale invariance, symmetry, directional continuity, time consistency
Gradual bargaining path
Solution in terms of allocations

Asset price (in terms of DM goods) solves:

$$y'(\tau) = \frac{1}{2} \left( \frac{1}{\upsilon'(y)} + \frac{1}{u'(y)} \right)$$

for all $y < y^*$

Suppose $\upsilon'(y) = 1$. Asset price is:

$$\frac{1}{2} \left( 1 + \frac{1}{u'(y)} \right)$$

- Price increases with the size of the trade
Alternative Extensive Game

Round #1 Round #2 \ldots Round #n \ldots Round #N

Round game

Buyer
Seller
Yes
No
[\xi]
Buyer
Yes
No
[\xi]

[1 - \xi]
[1 - \xi]

Move to next round
Move to next round
Trade and move to next round
Trade and move to next round
Intermediate output levels, $\{y_n\}_{n=1}^N$, solve:

\[
\int_{y_{n-1}}^{y_n} \left( \frac{v'(y_n)}{u'(y_n) + v'(y_n)} u'(x) + \frac{u'(y_n)}{u'(y_n) + v'(y_n)} v'(x) \right) dx = \frac{z}{N}
\]

**Proposition:** Consumers (asset owners) prefer $N = +\infty$ to any $N < +\infty$. 
Asset negotiability

Agenda indexed by time, $\tau$

- An implicit mapping between $\tau$ and $z$

New asset characteristic: Negotiability

- $\delta > 0$ units of assets can be sold per unit of time

- What is negotiability in practice:
  - time to authenticate assets
  - time to value complex assets
  - time to execute trade and transfer ownership
    (e.g., blockchain technologies)
Making time relevant

Random time to negotiate asset sales: $\bar{\tau} \sim \text{Exp}(\lambda)$

- negotiation breakdown, proxy for discounting

Formally:

\[
\begin{align*}
\text{Asset sales} & \quad \leq \quad \text{Negotiability} \quad \times \quad \text{Time to negotiate} \\
p(y) & \quad \leq \quad \delta \quad \times \quad \bar{\tau}
\end{align*}
\]
Pricing of Lucas trees

Interest rate spread (liquid vs non-liquid):

\[
\text{spread} = \text{search} \times \text{bargaining} \times \text{negotiability} \times \text{liquidity needs}
\]

where \( \ell(y) \equiv \frac{u'(y)}{v'(y)} - 1 \)

- \( e^{-\frac{\lambda}{\delta} p(y)} \) akin to a pledgeability coefficient
  - endogenous with \( \neq \) comparative statics
- \( s \) decreases with \( Ad \) but increases with \( \delta \) and \( 1/\lambda \)
Endogenous negotiability

Consumers choose $\delta$ when a match is formed but before $\bar{\tau}$ is realized

- Cost to enhance negotiability: $\psi(\delta)$

Proposition

1. If A is not too large, an increase in A reduces $s$, but raises $\delta$.

2. If A is not too large, asset negotiability is too low for all bargaining powers.
   - a pecuniary externality
Multiple assets

\( J \) one-period lived trees, one unit of each pays off one unit of numeraire

- Fiat money: \( j = 0 \); asset \( j \) has fixed supply \( A_j, j = 1, \ldots, J \)
- Negotiability of asset \( j \) is \( \delta_j \) with \( \delta_0 \geq \delta_1 \geq \ldots \geq \delta_J \)
  - Pecking order: sell assets with high negotiability first

Asset prices:

\[
\underbrace{\text{spread} \quad s_j}_{\alpha \theta} = \underbrace{\text{search\&bargaining}}_{\lambda} \sum_{k=j+1}^{J} \int_{T_k}^{T_{k+1}} \left( \frac{\delta_j - \delta_k}{\delta_j} \right) e^{-\lambda \tau} \ell[y(\tau)] d\tau + \alpha \theta e^{-\lambda T_{J+1}} \ell[y(T_{J+1})]
\]

\( \ell \) is the liquidity premium.
OMOs: negotiability vs liquidity

In Regime 3, increase in $A_1$ (bond supply) leads to reduction in output.
Multiple fiat monies

Multiple cryptocurrencies: Bitcoins, Litecoin, Ethereum

- Confirmation times vary across currencies, modeled as different $\delta$
- 2 currencies: 0 and 1, with inflation rates $\pi_0 > \pi_1$ but with $\delta_0 > \delta_1$

Dual currency equilibrium

- For intermediate $\bar{\tau}$’s a unique eq. exists with both currencies valued
- $\frac{\partial y}{\partial \pi_0} < 0$ and $\frac{\partial y}{\partial \pi_1} > 0$
- Currency 0 appreciates vis-a-vis currency 1 as $\alpha$ or $\theta$ increases or as $\bar{\tau}$ decreases
  - because agents put more weight on negotiability
Conclusion

New approach to bargaining over portfolios in decentralized asset markets

- Axiomatic and strategic foundations
- Tractable
- Encompasses Nash and Kalai solutions for specific agendas

Insights

- Normative: gradualism desirable individually and socially
- Positive: negotiability premia, distribution of asset returns, determinacy of exchange rate, OMOs