Topological approach to modeling spatial cognition

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How does brain represent space?
Cognitive map concept

Cognitive map – an internalized representation of the environment, that enables spatial navigation and spatial planning

E. Tolman, 1947
Cognitive map concept

Cognitive map – an internalized representation of the environment, that enables spatial navigation and spatial planning

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Where is it located?
Hippocampus and space coding

(D) Rat with hippocampus lesioned

First trial

Hidden platform

After 10 trials

First trial

After 10 trials
Hippocampus and space coding

First trial

Hidden platform

After 10 trials

(D) Rat with hippocampus lesioned

First trial

After 10 trials
How a cognitive map is produced by the hippocampal network?
Cognitive representation of space emerges from spiking activity
What is the mechanism?
Properties of the cognitive map

1. What information is represented in the cognitive map?
   - Distances? Other metrics? Directions?
   - Locations? Spatial order?

2. How this information is read out and processed by the downstream networks?
In vivo recordings of neuronal activity
How to proceed?
Alexandrov-Čech theorem
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Alexandrov-Čech theorem: $H_*(X) = H_*(\mathcal{N})$
Alexandrov-Čech theorem suggests how spiking information could be integrated.

$H_\ast(X) = H_\ast(N)$

Does it also suggest how hippocampus works?
Topological map
Topological map

Geometric map
Topological map

Geometric invariance

Dabaghian et al, eLife 2014
2D versus 1D spatial frame

Vs. = Correlation, 2D

Vs. = Correlation, 1D

Correlation decay

1D

2D
\[ \mathcal{E} = \bigcup_{i} U_{i} \]
\[ \mathcal{E} = \bigcup_i U_i \]
Čech’s simplicial complex from spikes

\[ \mathcal{N} \]
Spikes $\rightarrow$ Space reconstruction
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Temporal nerve simplicial complex
Topological information unfolds over time

Accumulation of topological information

Accumulation of spikes
Topology of $\mathcal{T}$ represents topology of $\mathcal{E}$

Temporal pattern of neuronal co-firing

Temporal nerve complex

Rat’s environment
Timelines of the topological loops

Persistent loops, persistent Betti numbers: $b_0 = 1$, $b_1 = 1$, $b_{n > 1} = 0$
Individual cell’s firing rates: \( f_1, f_2, \ldots, f_N \)
Individual place field sizes: \( s_1, s_2, \ldots, s_N \)

\[ \begin{align*}
\text{2N parameters} & \quad \rightarrow \\
P_f (f, \sigma_F) & \quad P_s (s, \sigma_s)
\end{align*} \]

1. mean firing rate, \( \bar{f} \)
2. mean place field size, \( \bar{s} \)
3. number of cells, \( N \)
Testing numerically simulated place cell ensembles

Each point \((s, f, N)\) represents a place cell ensemble.
Which place cell ensembles produce reliable maps?

Each point \((s, f, N)\) represents a place cell ensemble.

Different neural ensembles acquire information with different efficiencies, depending on firing rate, place field sizes, and size of cell population: the most competent ensembles form the **Learning Region**.
Parameters recorded in healthy animals fall into the learning region.

Different neural ensembles acquire information with different efficiencies, depending on firing rate, place field sizes, and size of cell population: the most competent ensembles form the Learning Region.
Spike trains + Brain waves
Spike modulation by the brain waves is essential for successful space coding.
Temporal pattern of neuronal co-firing

How can this structure be implemented in the brain?

$$\sigma_3 = [c_1, c_2, c_4, c_5]$$
Temporal pattern of neuronal co-firing

Temporal nerve complex

\[ \sigma_2 = [c_2, c_4, c_5, c_7] \]
Temporal pattern of neuronal co-firing

The pool of coactive place cell combinations is huge, but the number of readout neuron is limited.

\[ \# \text{coactive combinations} \sim \binom{\# \text{cells}}{\# \text{coactive cells}} \]

\[ \sigma_1 = [c_1,c_3,c_5,c_7] \]
1. High dimensionality $\bar{D} \sim 20$
2. Low ignition rate $f_\sigma$
3. Irregularity
4. etc.

$\#	ext{coactive combinations} \sim \left( \begin{array}{c} \#\text{cells} \\ \#\text{coactive cells} \end{array} \right)$
1. Cell assemblies correspond to maximal simplexes $\sigma \in \mathcal{T}_{CA}$, high ignition rate $f_\sigma$, low $\dim(\sigma)$

2. $N_{\text{cell assemblies}} = N_{\text{readout neurons}} \approx N_{\text{place cells}}$, hence $N_{\text{max simplexes}} \approx N_{\text{vertexes}}$

3. Maximize “contiguity of simplexes,” $\xi_{\sigma_i} = \frac{\dim(\sigma_i \cap \sigma_{i+1})}{\sqrt{\dim(\sigma_i) \cdot \dim(\sigma_{i+1})}}$

4. $\mathcal{T}_{CA}$ should correctly represent the topology of the environment, $H_*(\mathcal{T}_{CA}) = H_*(\mathcal{T}) = H_*(\mathcal{E})$

5. Learning times, $T_{\text{min}}$, should be reasonable
Cover

Place cell coactivity

Temporal nerve complex $\mathcal{T}$

Pairwise coactivity

Coactivity graph $\mathcal{G}$

Clique coactivity complex $\mathcal{T}(\mathcal{G})$

Links with high activation rate

$f_{ij} > \theta$
Pairwise coactivity

Coactivity graph $G$

Clique coactivity complex $\mathcal{T}$

$f_{ij} > \theta$
Select the most active *combinations* of place cells
Navigation in cell assembly complex

1. # Cell Assemblies $\approx$ # place cells, $N_{\text{max}} \approx N_{\text{vtx}}$

2. Mean contiguity $\xi = \left\langle \frac{\dim(\sigma_i \cap \sigma_{i+1})}{\sqrt{\dim(\sigma_i)\dim(\sigma_{i+1})}} \right\rangle \approx 0.78$

3. 

4. Correct topology, $H_*(\mathcal{T}_{CA}) = H_*(\mathcal{T}) = H_*(\mathcal{E})$

5. Learning times $T_{\text{min}}$ are the approximately the same
Rewiring cell assembly network

1. Cell assemblies are unstable
2. Cognitive maps are stable

How can that work?
Transient (finite time) cell assemblies

\[ \tau = \langle t_i \rangle = 9.1 \text{ secs} \]

\[ \langle n_i \rangle = 1.5 \]

\[ \langle T_i \rangle = 14.57 \text{ secs} \]
Transient (flickering) cell assembly complex \( \mathcal{F}(t) \)

\[
C(t) = \langle \mathcal{F}(t+1), \mathcal{F}(t) \rangle
\]

\[
D(t) = \langle \mathcal{F}(1), \mathcal{F}(t) \rangle
\]

\[
d_{ij} = \langle \mathcal{F}(t_i), \mathcal{F}(t_j) \rangle
\]
Rewiring cell assembly network encodes a stable map

Betti numbers

\[ T_{\text{min}} \]

\[ \tau = \langle t \rangle = 9.1 \text{ secs} \]

\[ \langle n \rangle = 1.5 \]

\[ \langle T \rangle = 14.57 \text{ secs} \]
Rewiring cell assembly network encodes a stable map

- Betti numbers

- \( T_{\text{min}} \)

- Learning time, minutes

- Number of simplexes \( \times 10^2 \)

- Time, minutes

- Mean lifetime, \( t_i \) secs

- Appearance, \( n_i \)

- Total lifetime, \( T_i \) secs

- \( \tau = \langle t \rangle = 9.1 \text{ secs} \)

- \( \langle n \rangle = 1.5 \)

- \( \langle T \rangle = 14.57 \text{ secs} \)
Rewiring cell assembly network encodes a stable map
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