Weakly Homogeneous Structures

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ABSTRACT

We continue the investigation from [1] on the notion of weakly ultrahomogeneous structures and their effective categoricity. It was shown that any computable ultrahomogeneous structure is Δ^0_2 categorical. A structure $\mathcal{A}$ is said to be *weakly ultrahomogeneous* if there is a finite (*exceptional*) set of elements $a_1, \ldots, a_n$ such that $\mathcal{A}$ becomes ultrahomogeneous when constants representing these elements are added to the language. Characterizations were obtained for weakly ultrahomogeneous linear orderings, equivalence structures, injection structures and trees, and these were compared with characterizations of the computably categorical and Δ^0_2 categorical structures. Index sets can be used to determine the complexity of the notions of ultrahomegenous and weakly ultrahomogeneous for various families of structures. We report on recent work on weakly ultrahomogeneous Boolean algebras and Abelian p-groups.

References

[1] Adams, F. and D. Cenzer, D. *Computability and Categoricity of Weakly Ultrahomogeneous Structures*, Computability (2017).

The automorphisms of the lattice of x-computably enumerable vector spaces

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ABSTRACT

We will present some recent developments in the study of the automorphisms of the lattice of computably enumerable vector spaces, and of its quotient lattice modulo finite dimension. Computably enumerable vector spaces were first studied by Metakides, Nerode, Downey, Remmel, and others. The investigation of the lattice automorphism orbits led to various notions of maximality for vector spaces. We will discuss recent progress on the problem of finding orbits for spaces over Q. This is joint work with Dimitrov. We also investigate, for a given Turing degree x, the automorphism group of the lattice of x-computably enumerable vector spaces. We establish the equivalence of the embedding relation for these automorphism groups with the order relation for the corresponding Turing degrees. This is joint work with Dimitrov and Morozov.

Some Computability Theory of Finitely Generated Structures

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ABSTRACT

Every countable structure has a sentence of infinitary logic, called a Scott sentence, which describes it up to isomorphism among countable structures. We can characterize the complexity of a structure by the complexity of the simplest description of that structure. A finitely generated structure always has a Σ_3^0 description. We show that there is a finitely generated group which has no simpler description.

The proof of this leads us to talk about notions of universality for finitely generated structures. Finitely generated groups are universal, but finitely generated fields are not. By this, we mean that for every finitely generated structure, there is a finitely generated group which has the same computability-theoretic properties; but the same is not true for finitely generated fields. We apply the results of this investigation to pseudo Scott sentences.

The Reverse Mathematics of Model Theory and First-Order Principles

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ABSTRACT

I will discuss some results in the computability theory and reverse mathematics of basic model-theoretic notions, and their connections with first-order principles, drawing in particular from my recent paper *Induction, bounding, weak combinatorial principles, and the Homogeneous Model Theorem* with Lange and Shore.

Computability and model-theoretic aspects of families of sets and its generalizations

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ABSTRACT

In the talk I will discuss different approaches to the study of computability of families of sets. The most of these aproaches are based on a presentation of a family as a special alebraic structure. This allows to carry model-theoretic notions and properties to the familes and their generalizations.

Roots of polynomials in fields of generalized power series

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ABSTRACT

We consider roots of polynomials in fields of formal power series. Newton [1] and Puiseux [2], [3] showed that if K is algebraically closed of characteristic 0, then the field of Puiseux series with coefficients in K is also algebraically closed. Maclane [6] extended the Newton-Puiseux Theorem to Hahn fields. Our goal is to measure the recursion-theoretic complexity of the root-taking process in these fields. Puiseux series have length at most ω. In this setting, we already have results bounding the complexity. Hahn series have varying ordinal length, and complexity seems to go up with length. The first two authors have results bounding the lengths of roots of polynomials, in terms of the lengths of the coefficients [4], [5]. The work on complexity is ongoing.

References

[1] I. Newton, "Letter to Oldenburg dated 1676 Oct 24", *The Correspondence of Isaac Newton II*, 1960, Cambridge University Press, pp. 126-127.

[2] V. A. Puiseux, "Recherches sur les fonctions algébriques", *J. Math. Pures Appl.*, vol. 15(1850), pp. 365-480.

[3] V. A. Puiseux, "Nouvelles recherches sur les fonctions algébriques", *J. Math. Pures Appl.*, vol. 16(1851), pp. 228-240.

[4] J. F. Knight and K. Lange, "Lengths of developments in $K((G))$", pre-print.

[5] J. F. Knight and K. Lange, "Truncation-closed subfields of a Hahn field", pre-print.

[6] S. MacLane, "The universality of formal power series fields", *Bull. Amer. Math. Soc.*, vol. 45(1939), pp. 888-890.

An optimal description of computably categorical torsion abelian groups

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ABSTRACT

The index set of c.c. torsion abelian groups is Π^0_4-complete. The result solves one of the only two missing cases in a 50yo problem of Malcev who was the first to ask for a classification of c.c. abelian groups. Both the result and its proof are somewhat counter-intuitive. As a corollary, the index set of c.c. profinite commutative groups is also Π^0_4-complete.
(This is a joint work with Ng.)

Ordered abelian groups, generalized series and integer parts

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ABSTRACT

D'Aquino, Knight and Lange [1] initiated an investigation of the complexity of integer parts within ordered real closed fields, and Knight and Lange [3] characterized the complexity of several of the natural algebraic objects associated to real closed fields. One can investigate similar questions and constructions in the context of ordered abelian groups where the underlying algebraic structure is simpler. In this talk, we will revisit results from Downey and Solomon [2], emphasizing how they serve as analogs for ordered abelian groups of Knight and Lange's work on real closed fields. We will describe recent preliminary results of Lange and Solomon characterizing the complexity of an analog of integer parts for ordered abelian groups.

References

[1] P. D'Aquino, J. Knight and K. Lange, *Limit computable integer parts*, Archive for Mathematical Logic **50**, 2011, 681-695.

[2] R. Downey and R. Solomon, *Reverse mathematics, archimedian classes and Hahn's theorem*, in Reverse Mathematics 2001 (ed. S.G. Simpson), AK Peters, 2001, 147-163.

[3] J. Knight and K. Lange, *Complexity of structures associated with real closed fields*, Proceedings of the London Mathematical Society, Third Series, **107**, 2013, 177-197.