Aspects of Computation 2017

EFFICIENT ALGORITHMS FOR HARD PROBLEMS ON STRUCTURED ELECTORATES

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IIT Gandhinagar

THE STANDARD VOTING SETUP

and some problems that we will encounter.

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SINGLE PEAKED PREFERENCES

definition, recognition, strategy-proofness, elicitation

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SINGLE CROSSING PREFERENCES

definition, recognition and Condorcet winners

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CONCLUDING REMARKS

nearly structured preferences, dichotomous preferences

THE STANDARD VOTING SETUP

and some problems that we will encounter.

The Handbook of Computational Social Choice,
Brandt, Conitzer, Endriss, Lang and Procaccia; 2016

A typical voting scenario involves
a set of alternatives, and
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Voters have preferences over the alternatives,
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Voters have preferences over the alternatives,
which can be modelled in various ways.

In this talk, we'll think of preferences as
linear orders over the rankings.

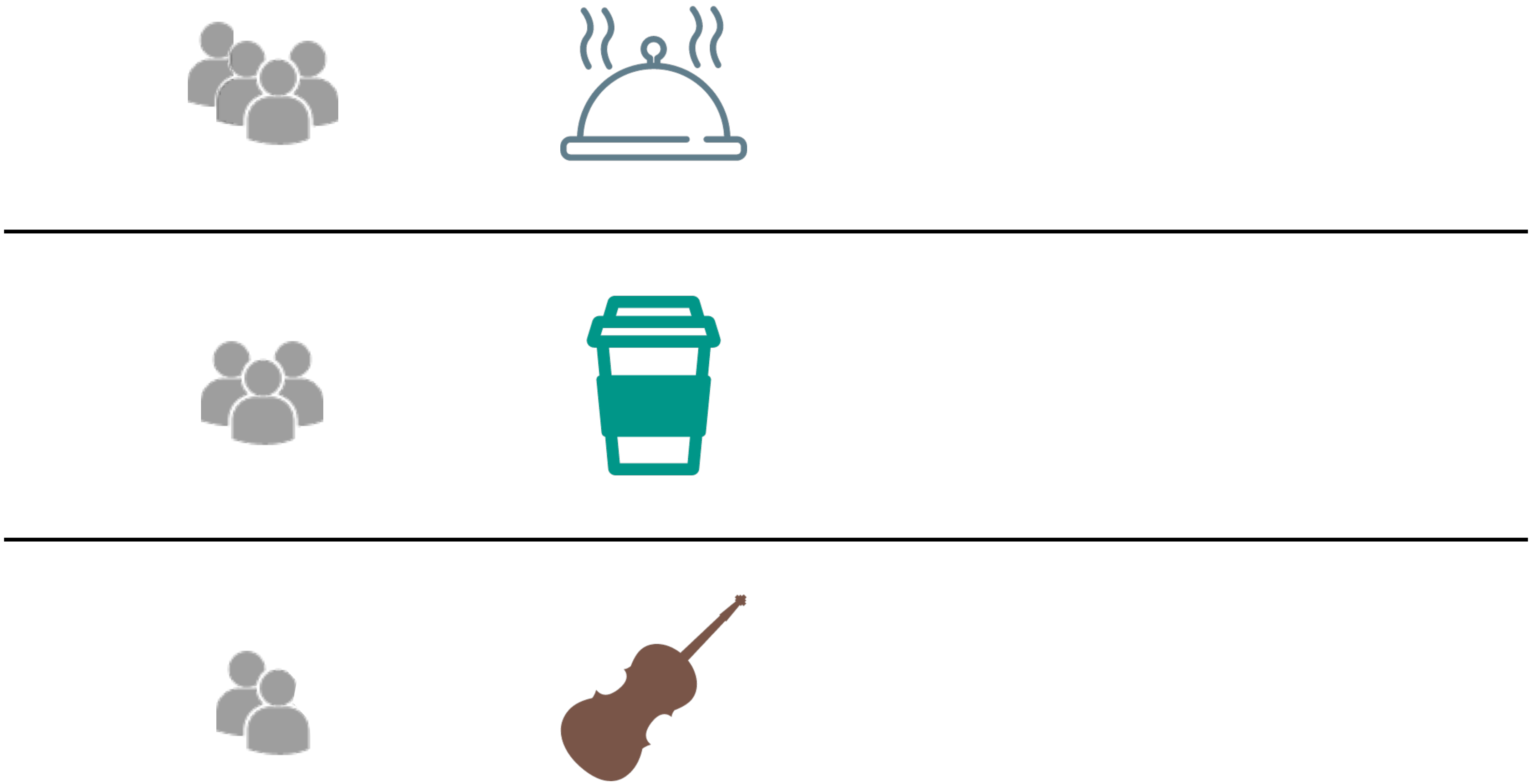
A group of friends have an evening to spend.







(Plurality)




(Plurality)

We say that a voter (or a group of voters) can manipulate if they can obtain a more desirable outcome by misreporting their preferences.

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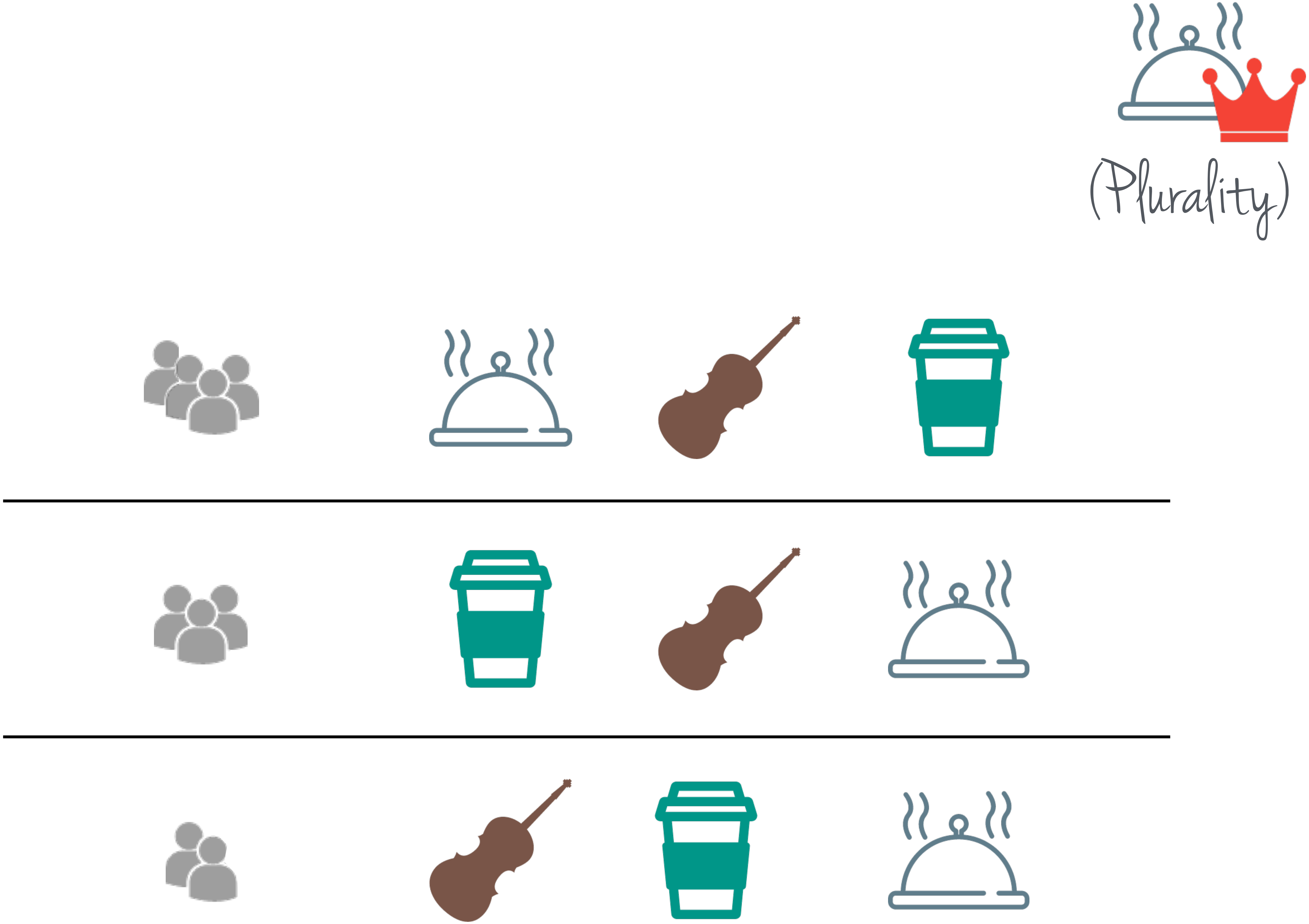
We'll see that plurality is vulnerable to this behaviour.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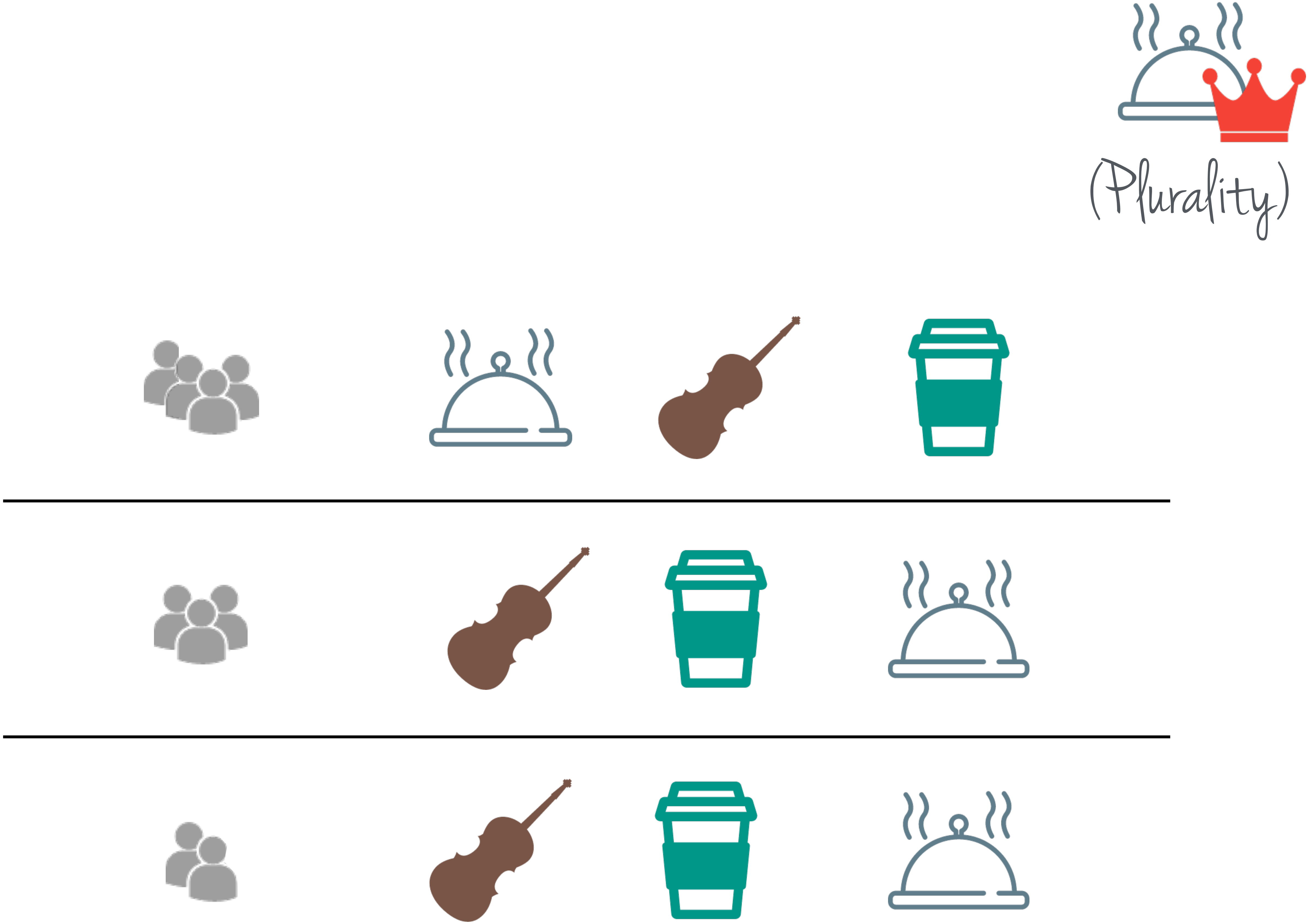
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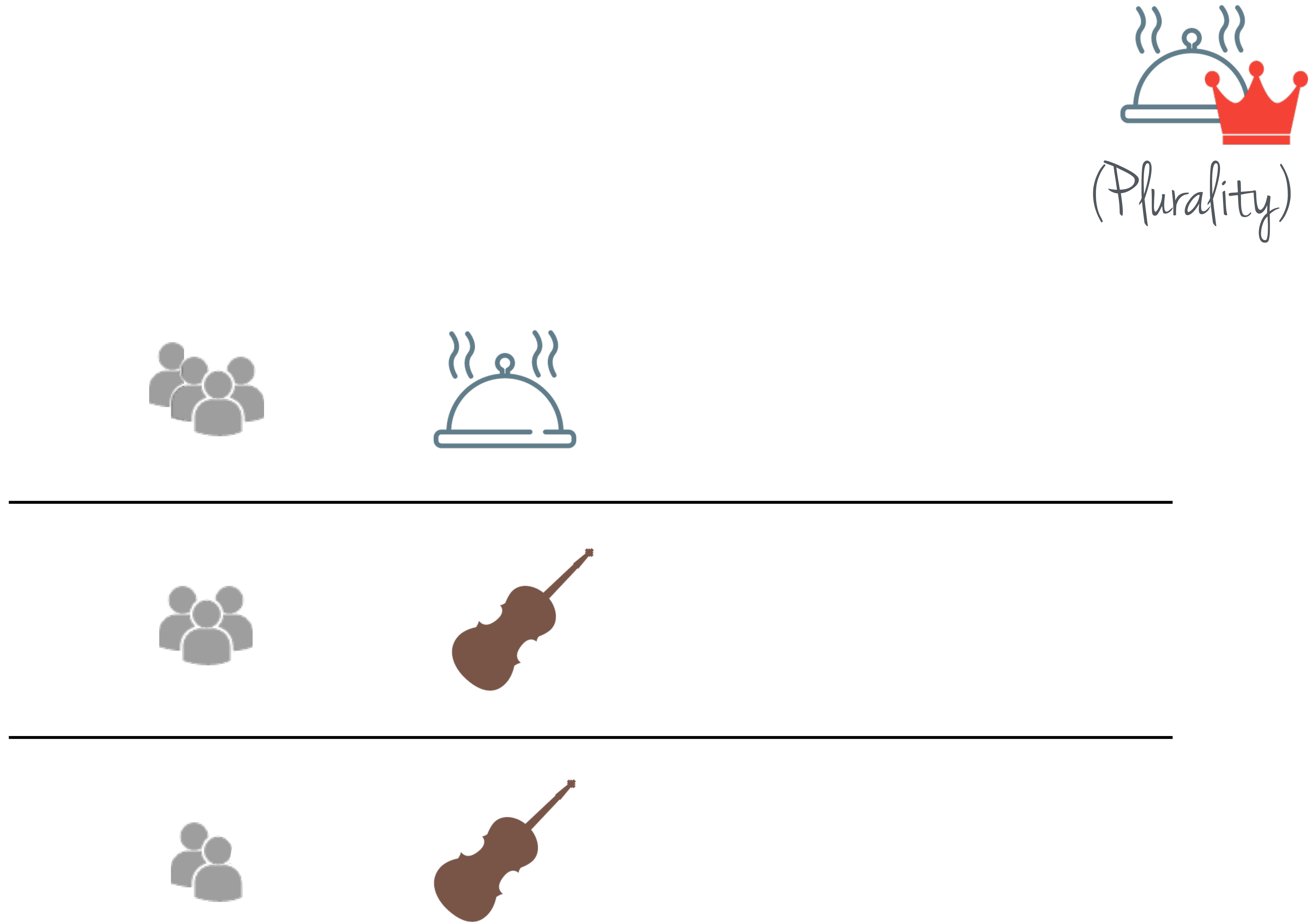
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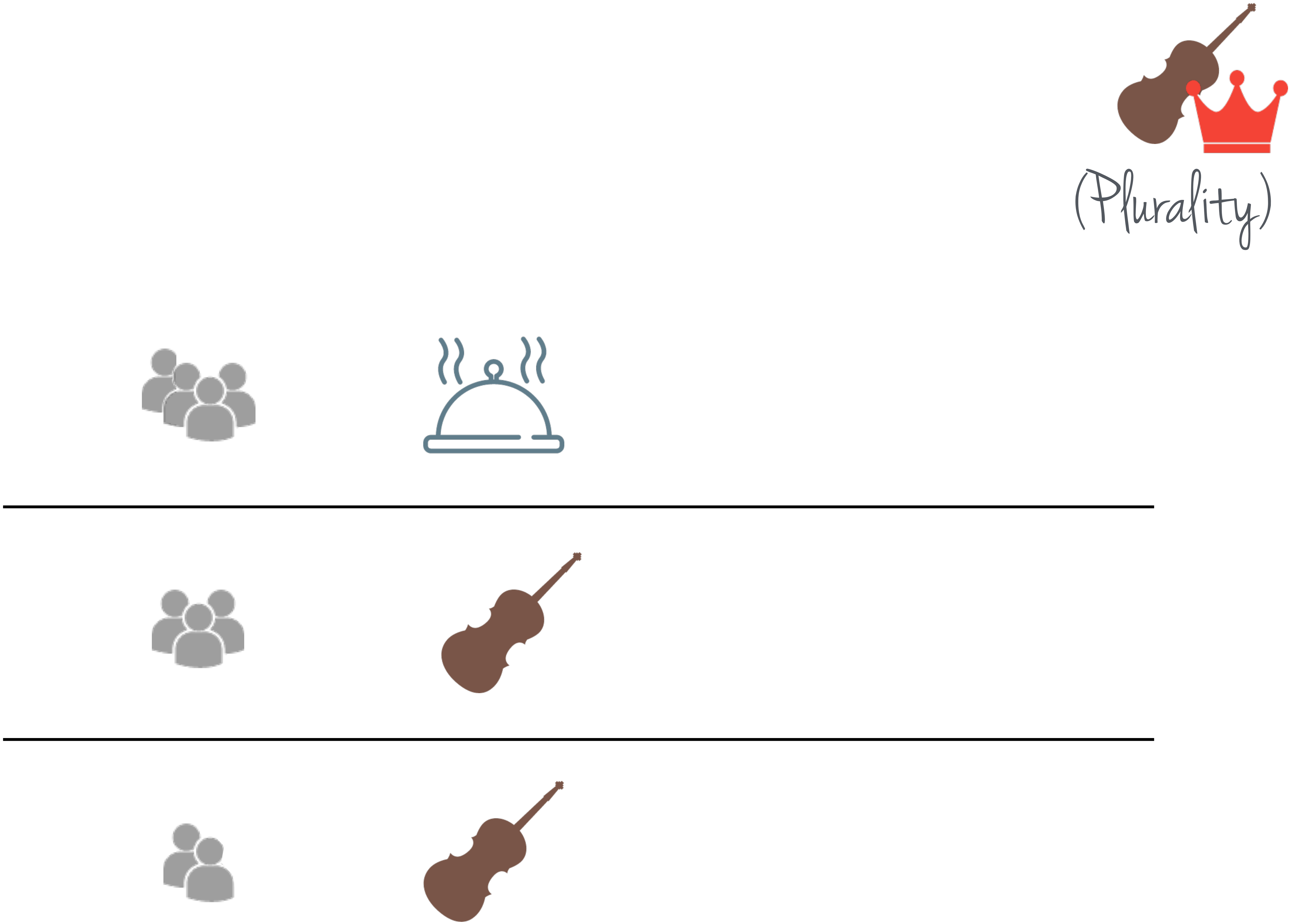
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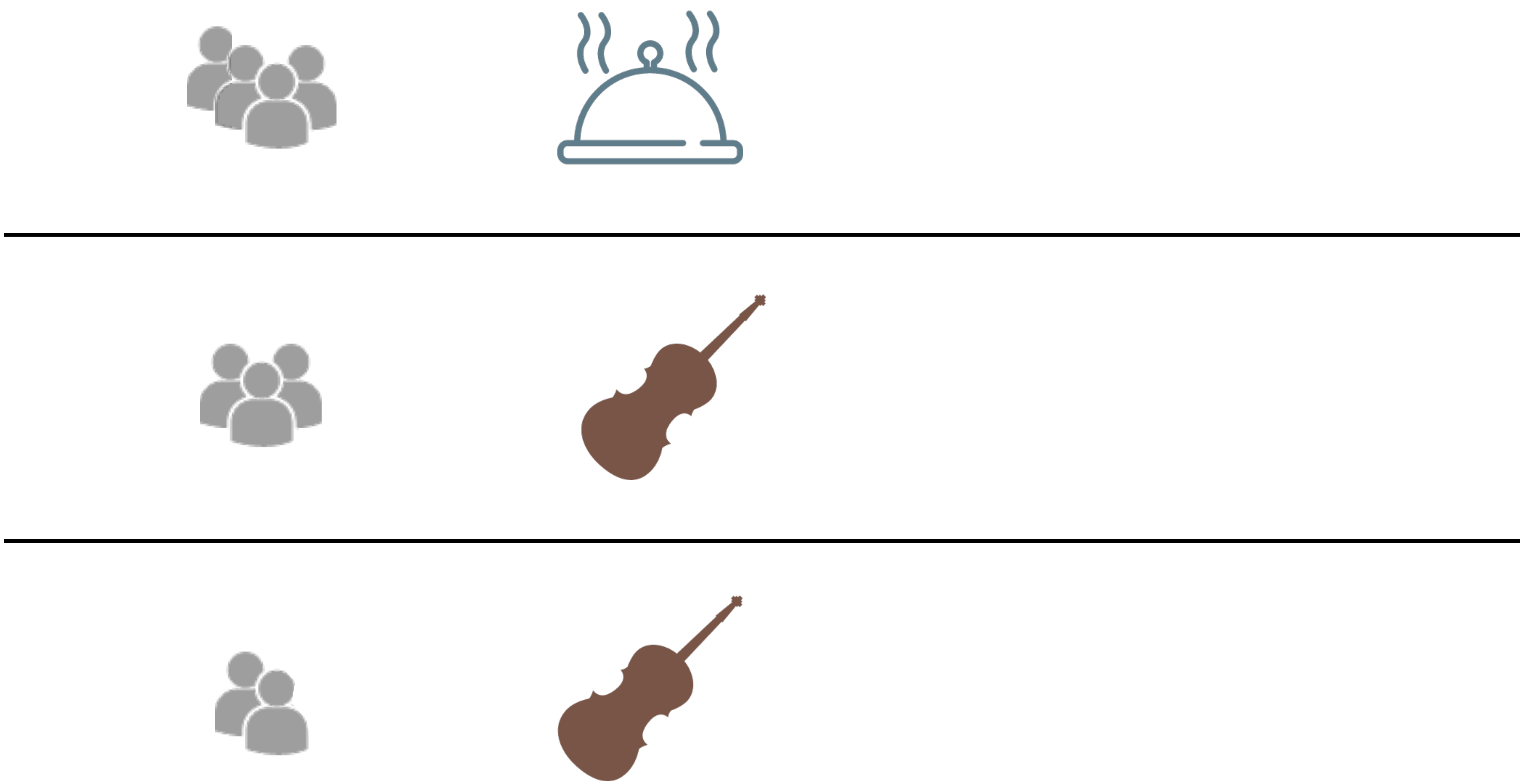
Manipulation

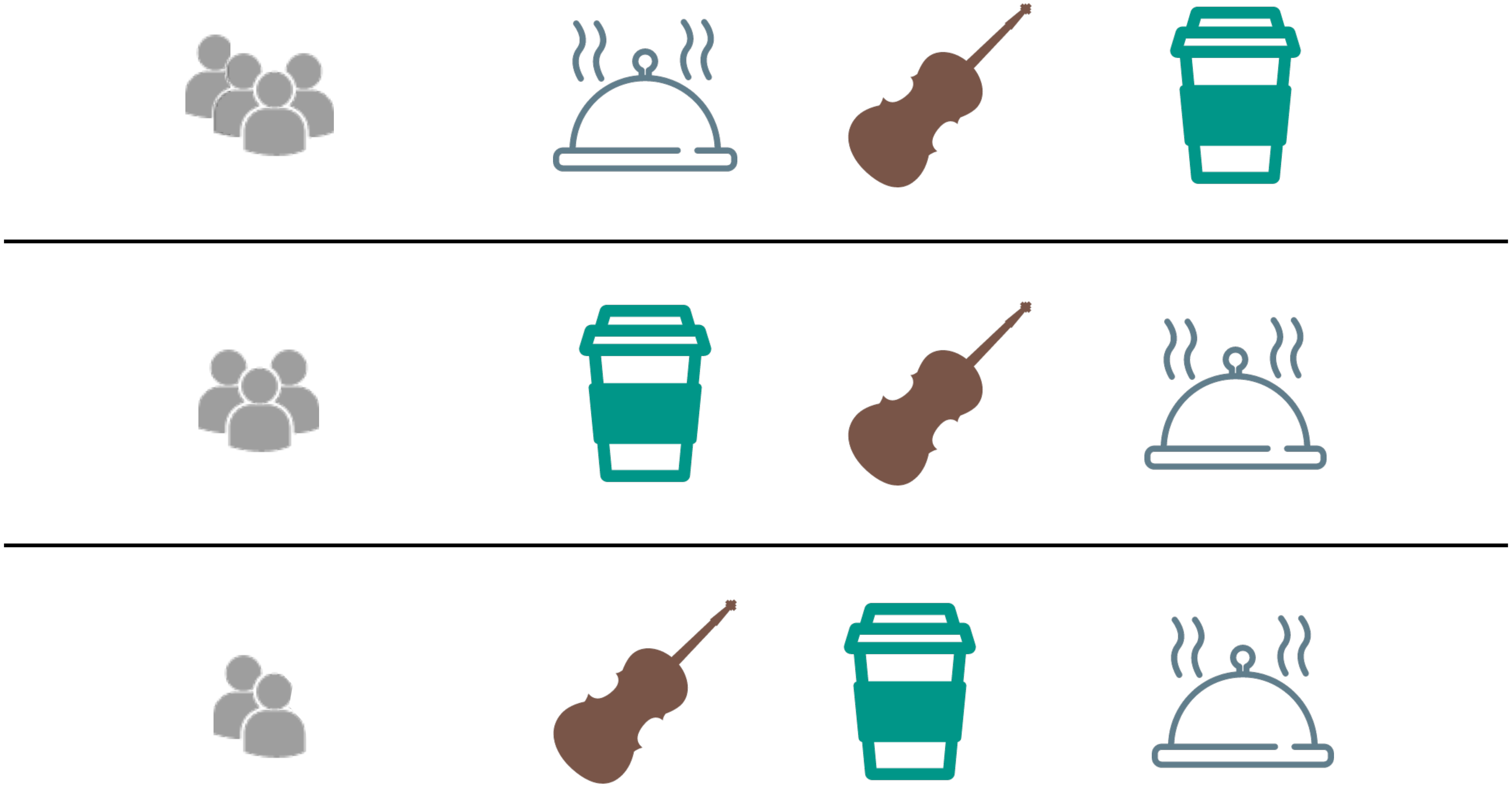
(Plurality)



Borda, 1770

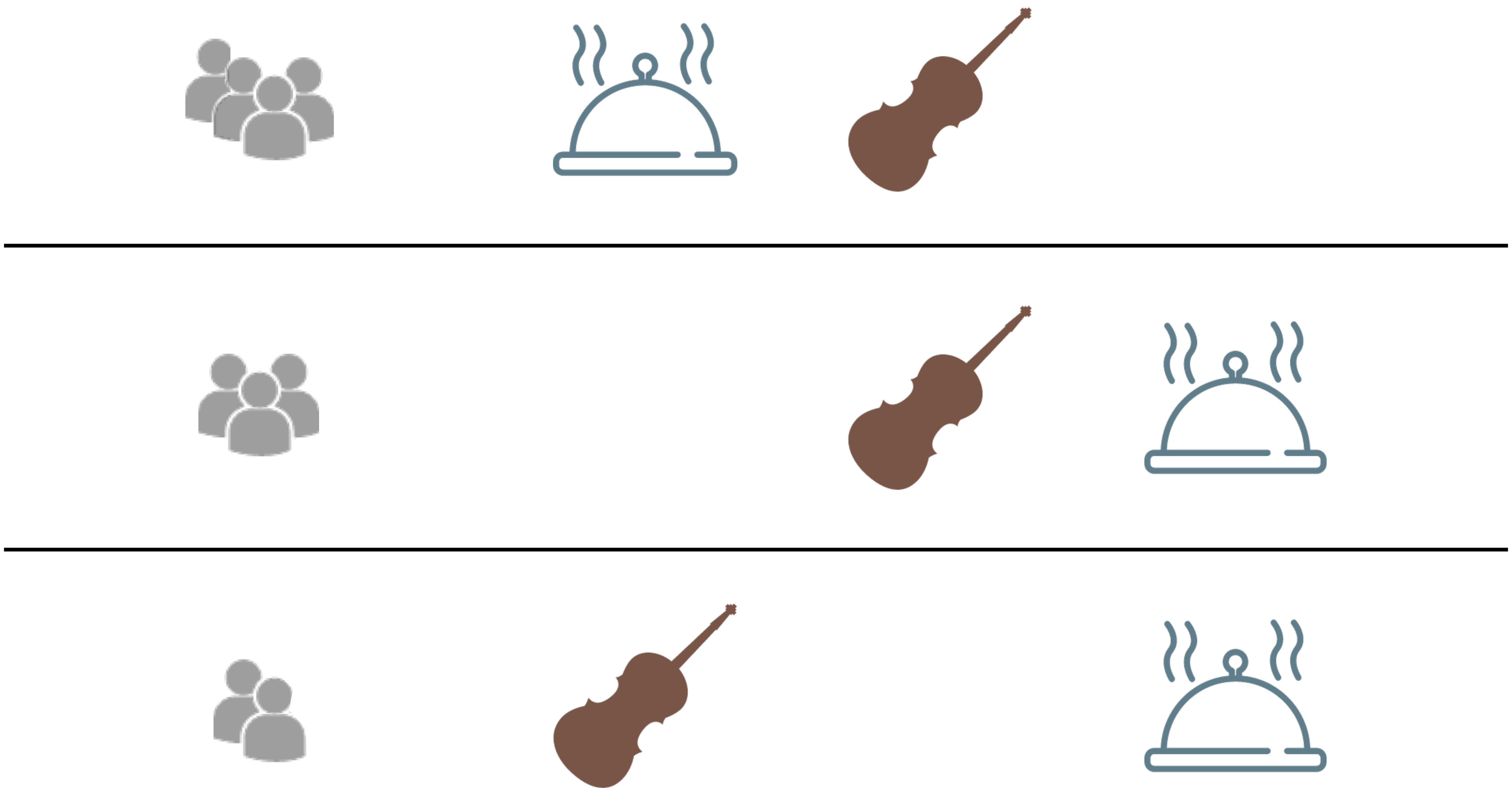
This scheme is intended only for honest men.

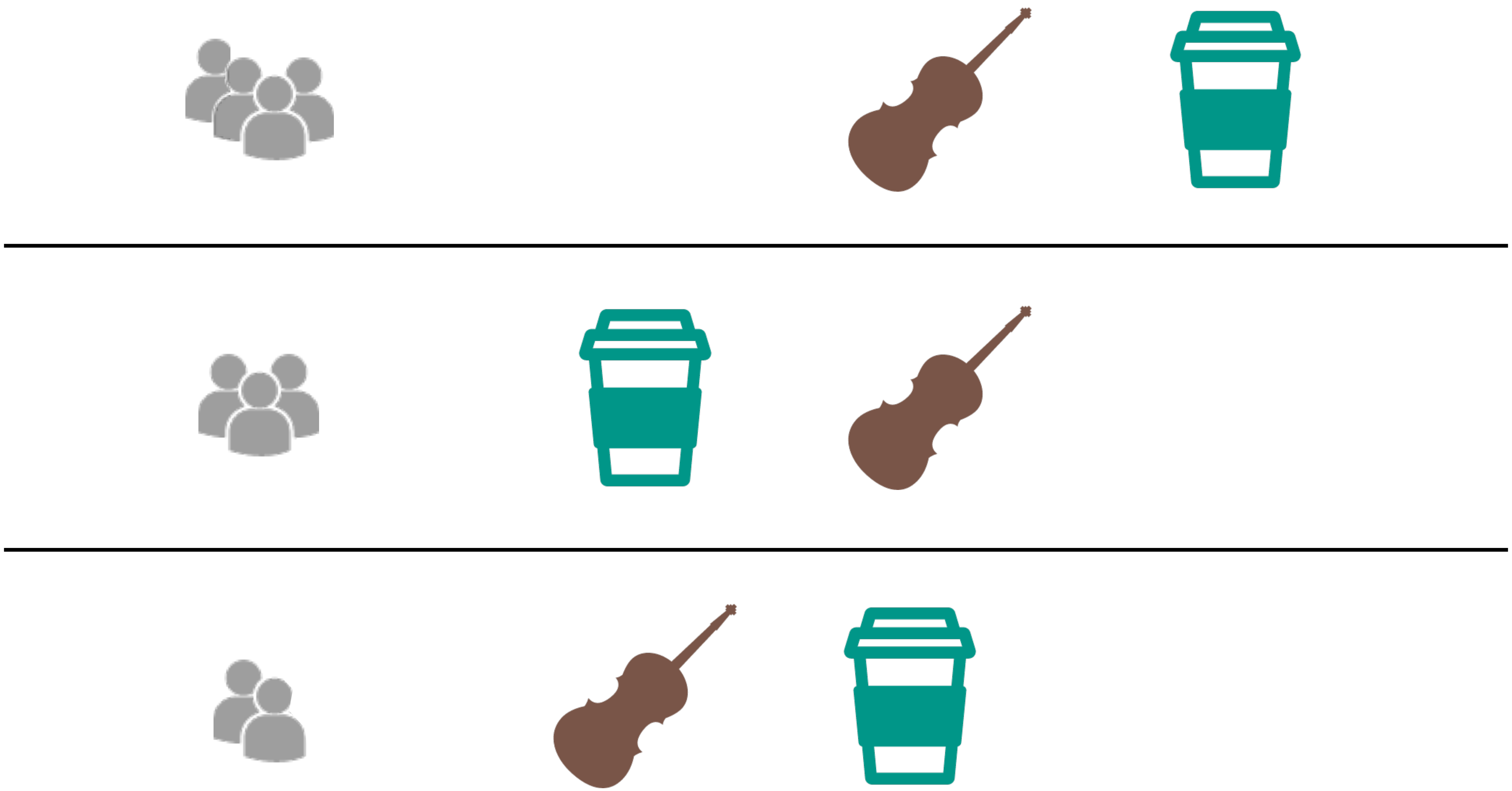
Elimination over multiple rounds



(STV)

Pairwise Elections





An alternative that beats all the others in pairwise comparisons.

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(Condorcet)

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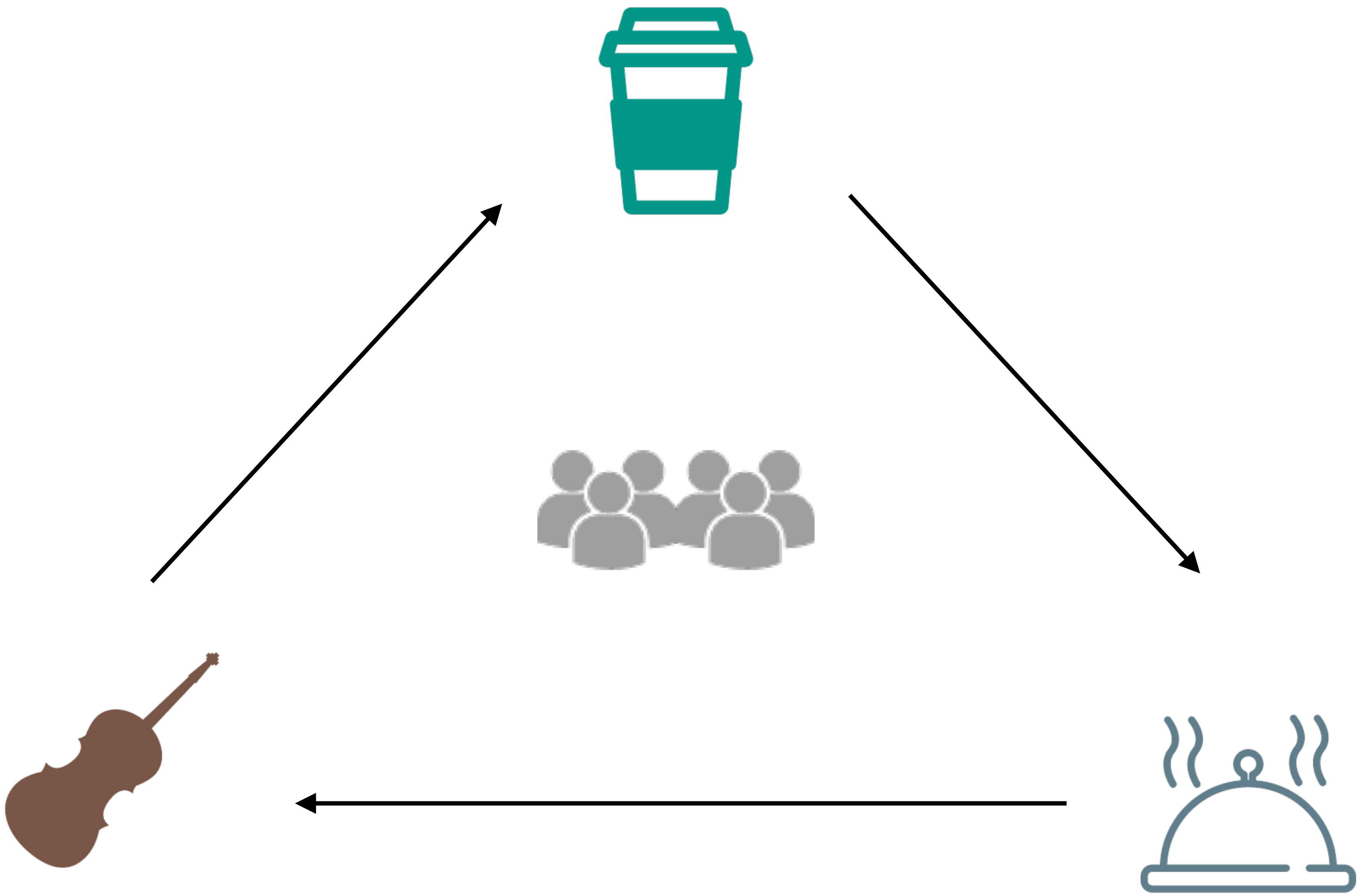

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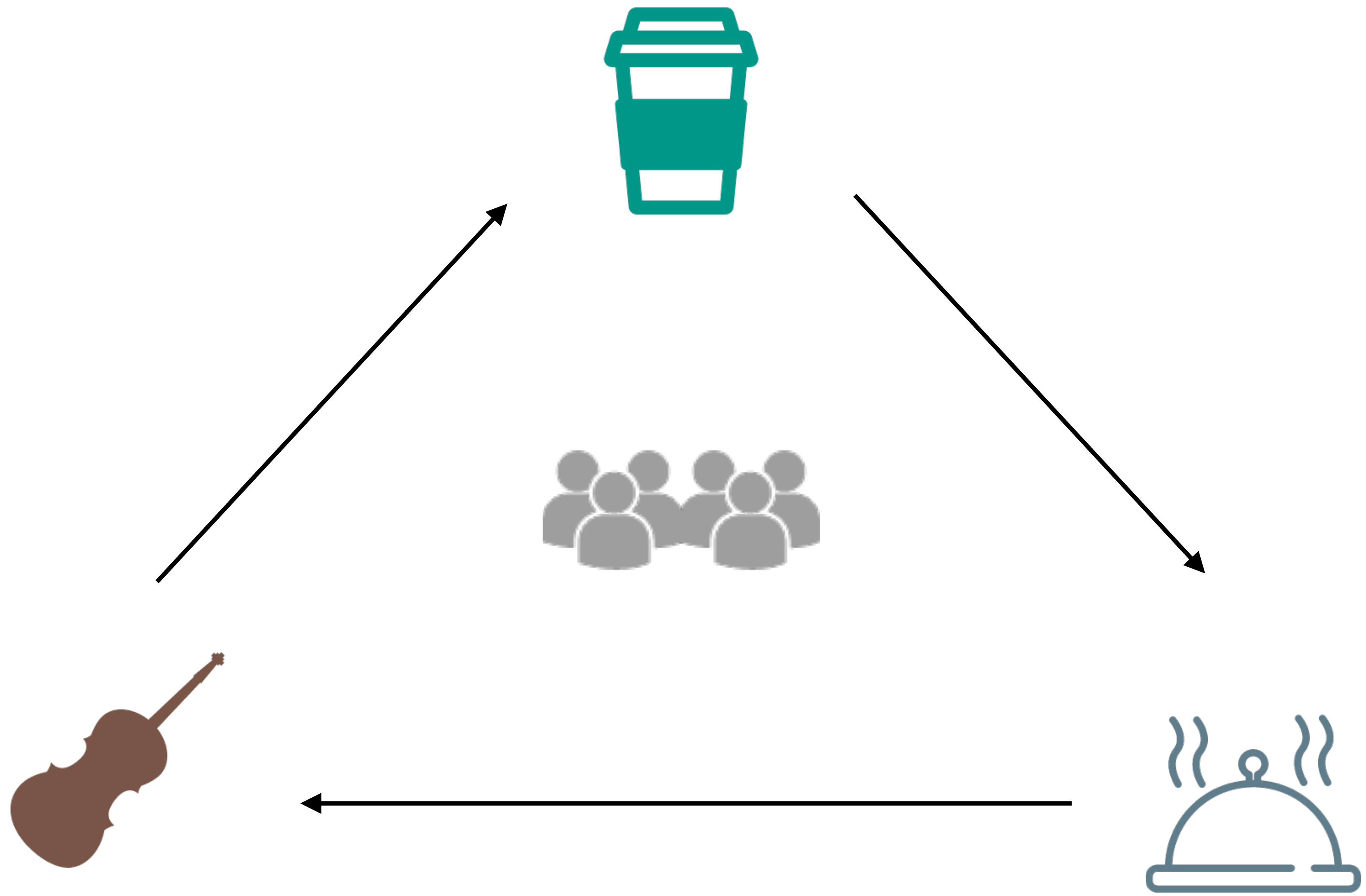
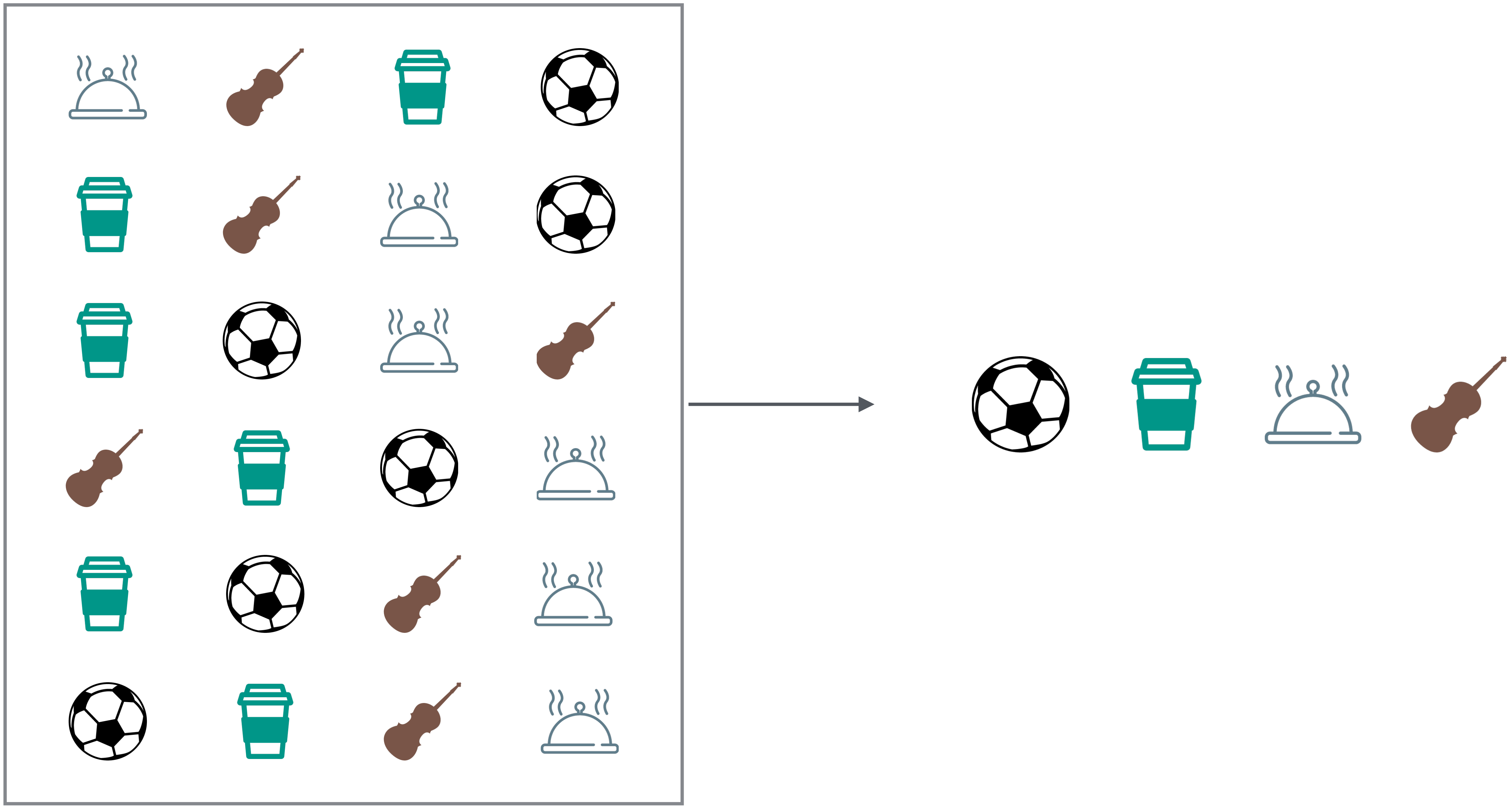


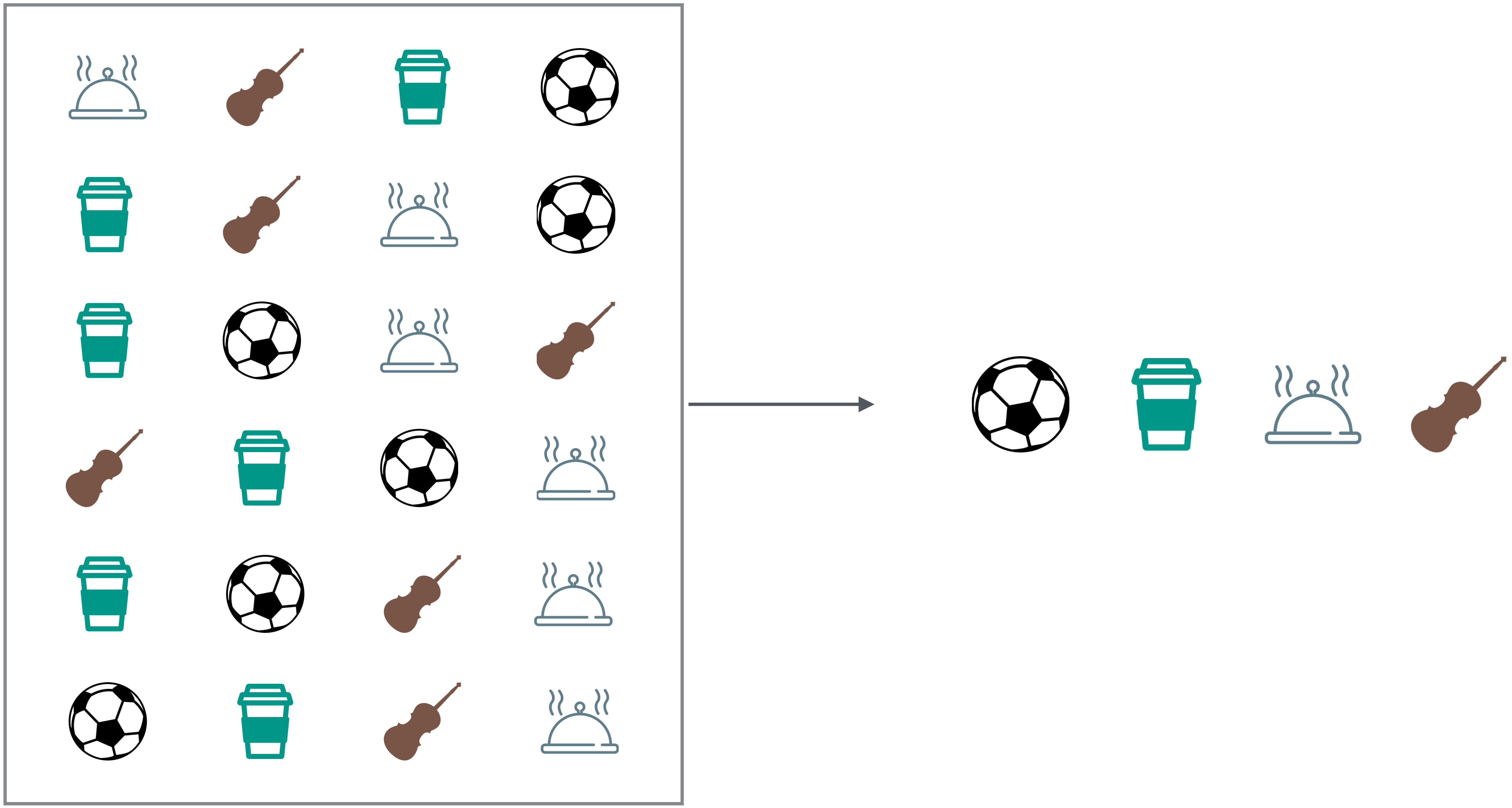
may not exist!

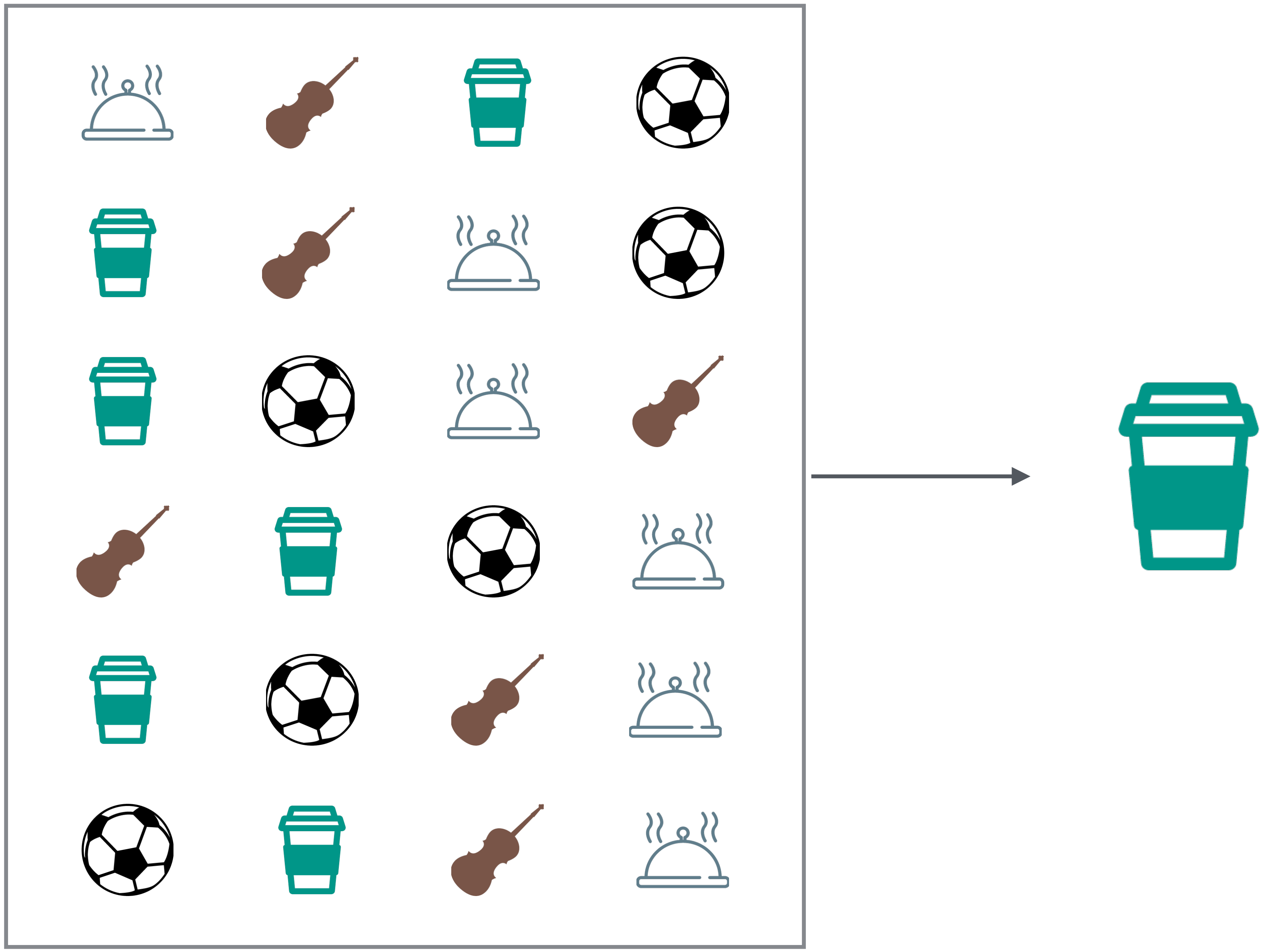
Desirable properties of voting rules.

We'll now pause to formally define
social choice functions and
social welfare functions.

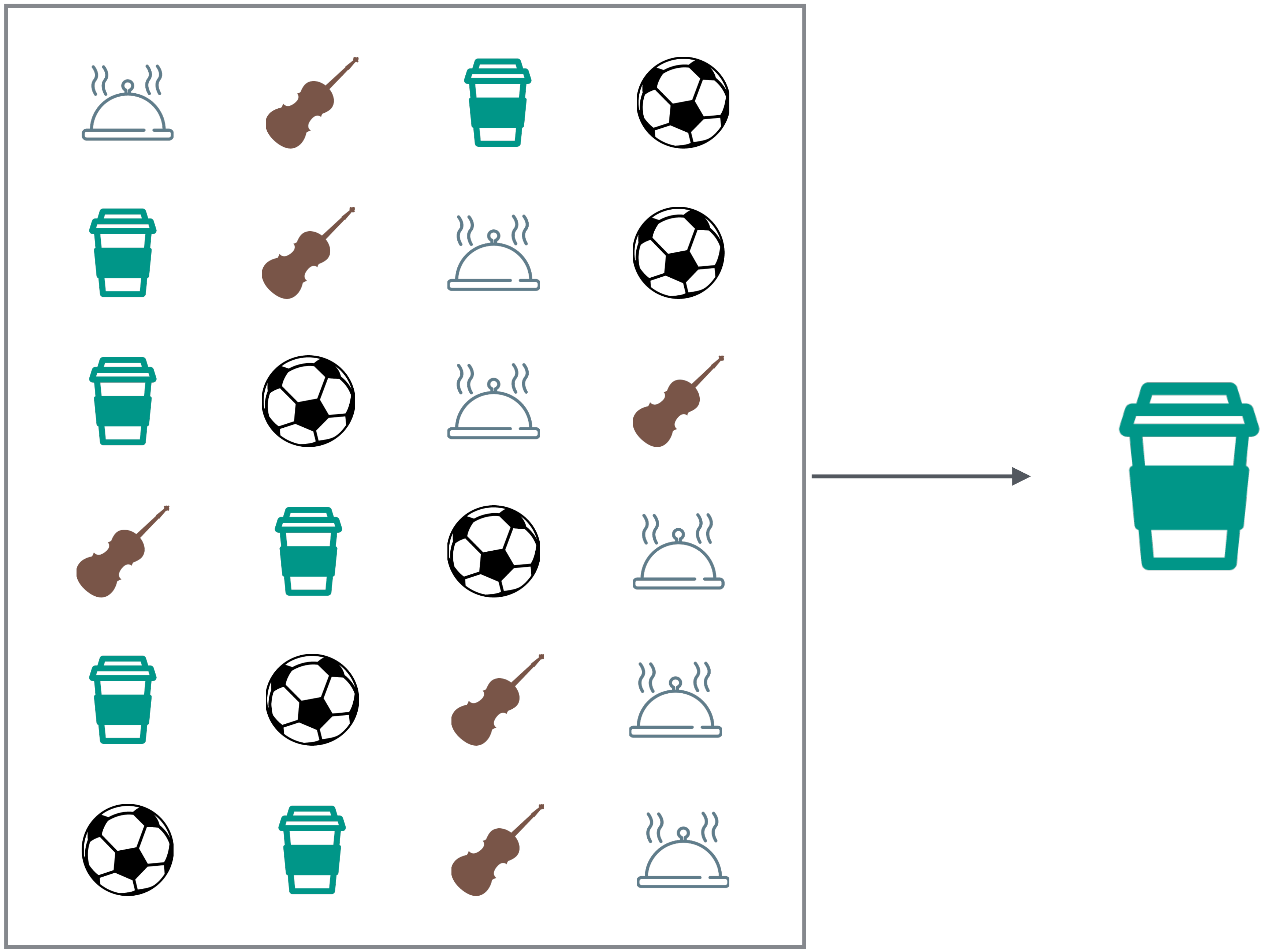


Social Welfare Functions (SWF)





Social Choice Functions (SCF)



Unanimity

Unanimity

Unanimity

If everyone prefers A to B then
the consensus ranking also prefers
A over B.

Independence of Irrelevant Alternatives

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Suppose a SWF prefers A over B in
the consensus ranking for a profile P.

Let Q be a profile that is the same as P when
projected on the candidates A and B.

Then the SWF must prefer A over B in the
consensus ranking that it determines for Q as well.

Arrow (1949)

Arrow (1949)

When voters have three or more alternatives,
any social welfare function which respects
unanimity and
independence of irrelevant alternatives
is a dictatorship.

Gibbard-Satterthwaite (1973/75)

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Any **strategy- proof** SCF where at least three candidates have some chance of being selected must be a **dictatorship.**

Some Popular Workarounds

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Domain Restrictions

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Randomised Voting Rules

Some Popular Workarounds

Domain Restrictions

Randomised Voting Rules

Computational Hardness

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SINGLE PEAKED PREFERENCES

Definition

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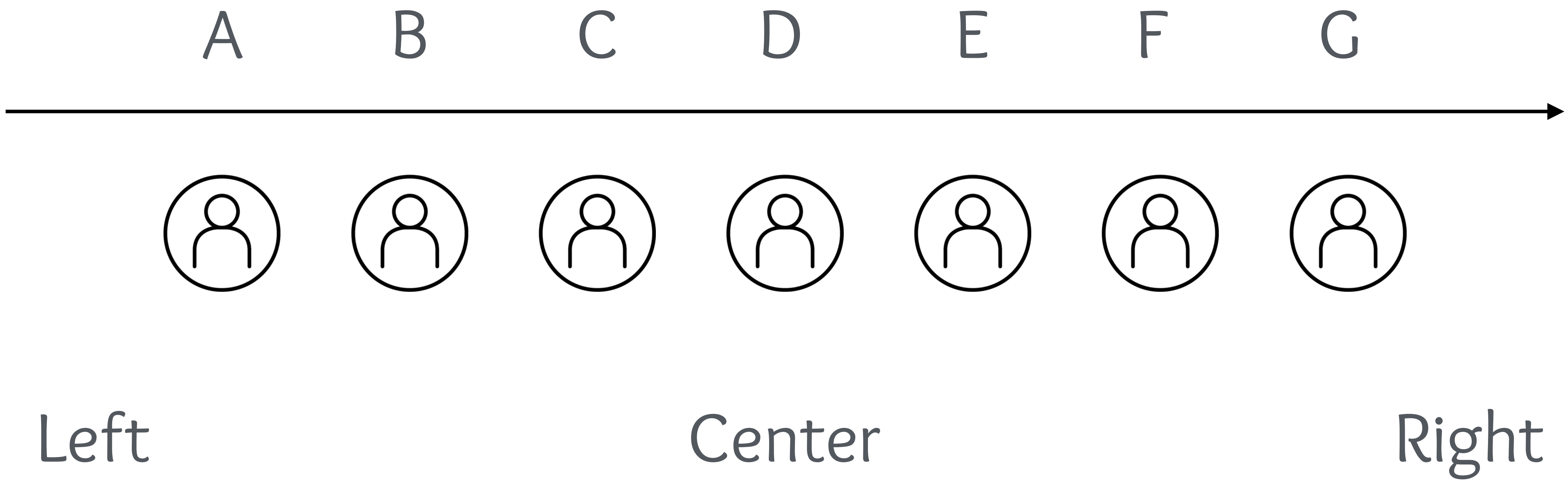
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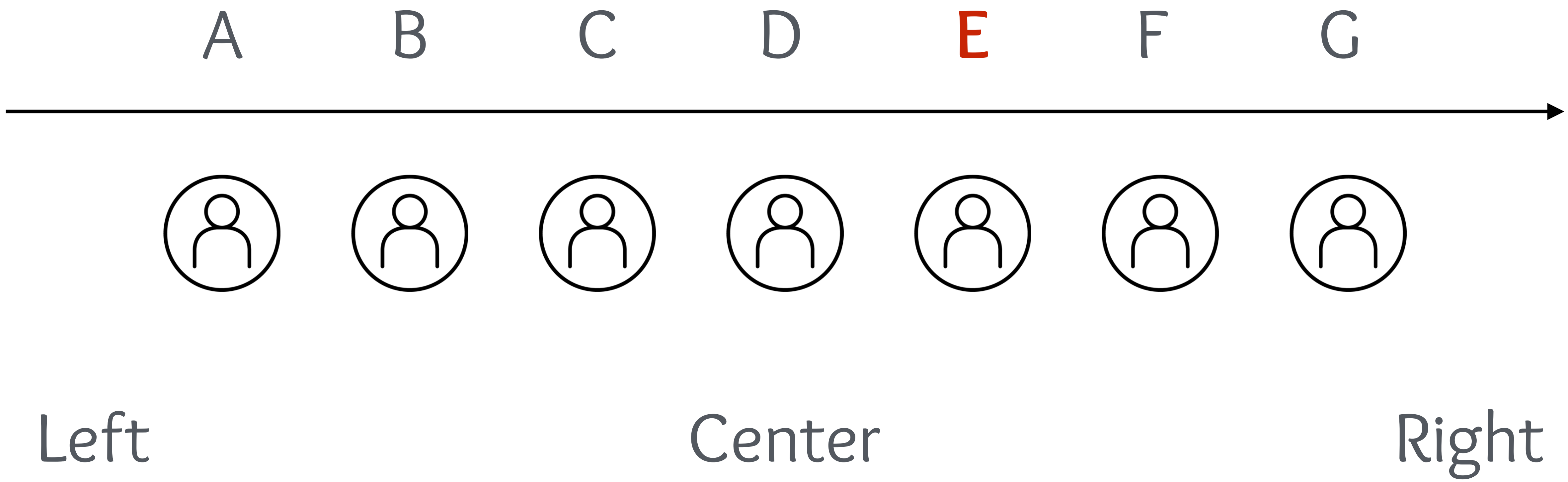
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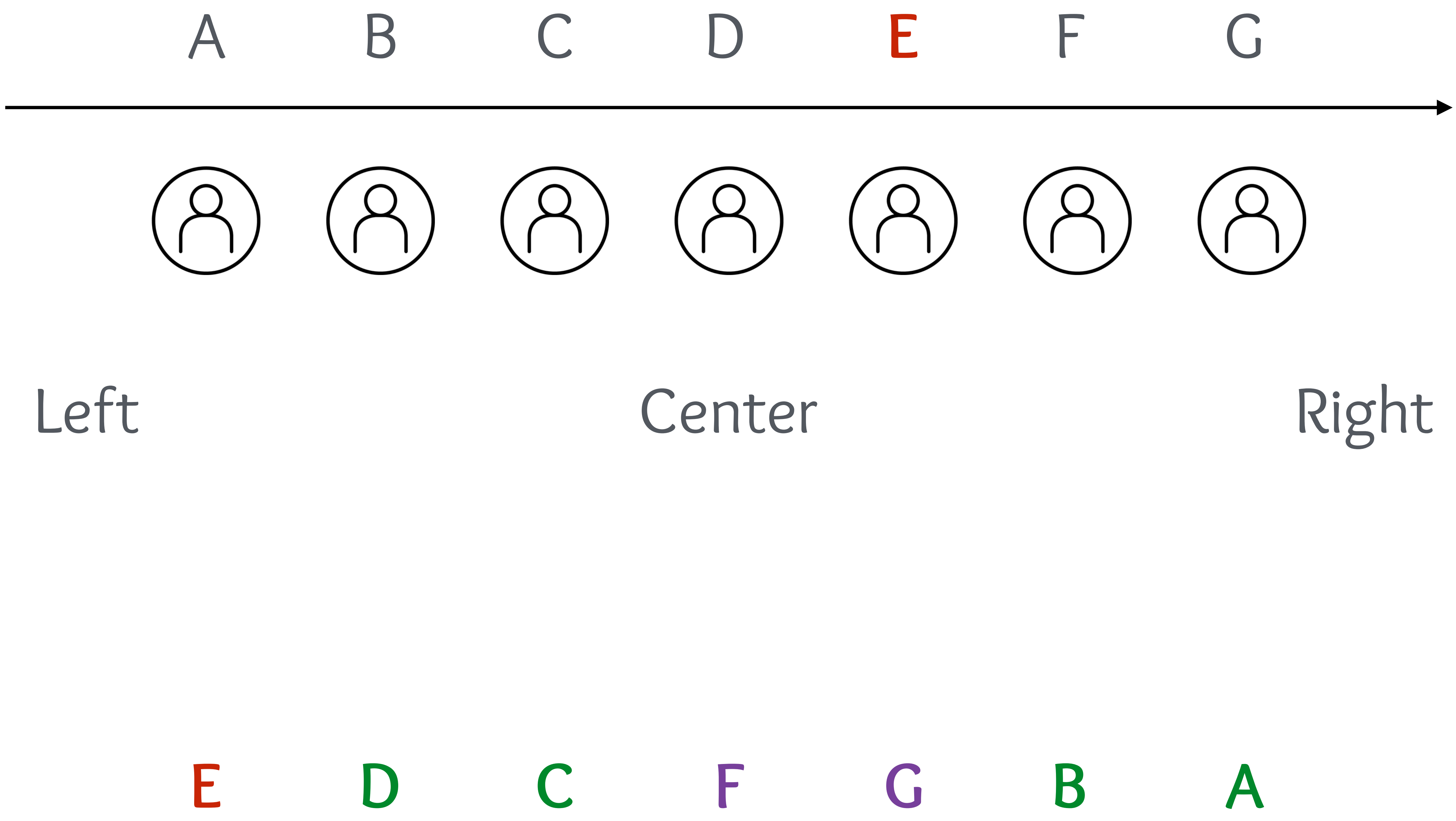
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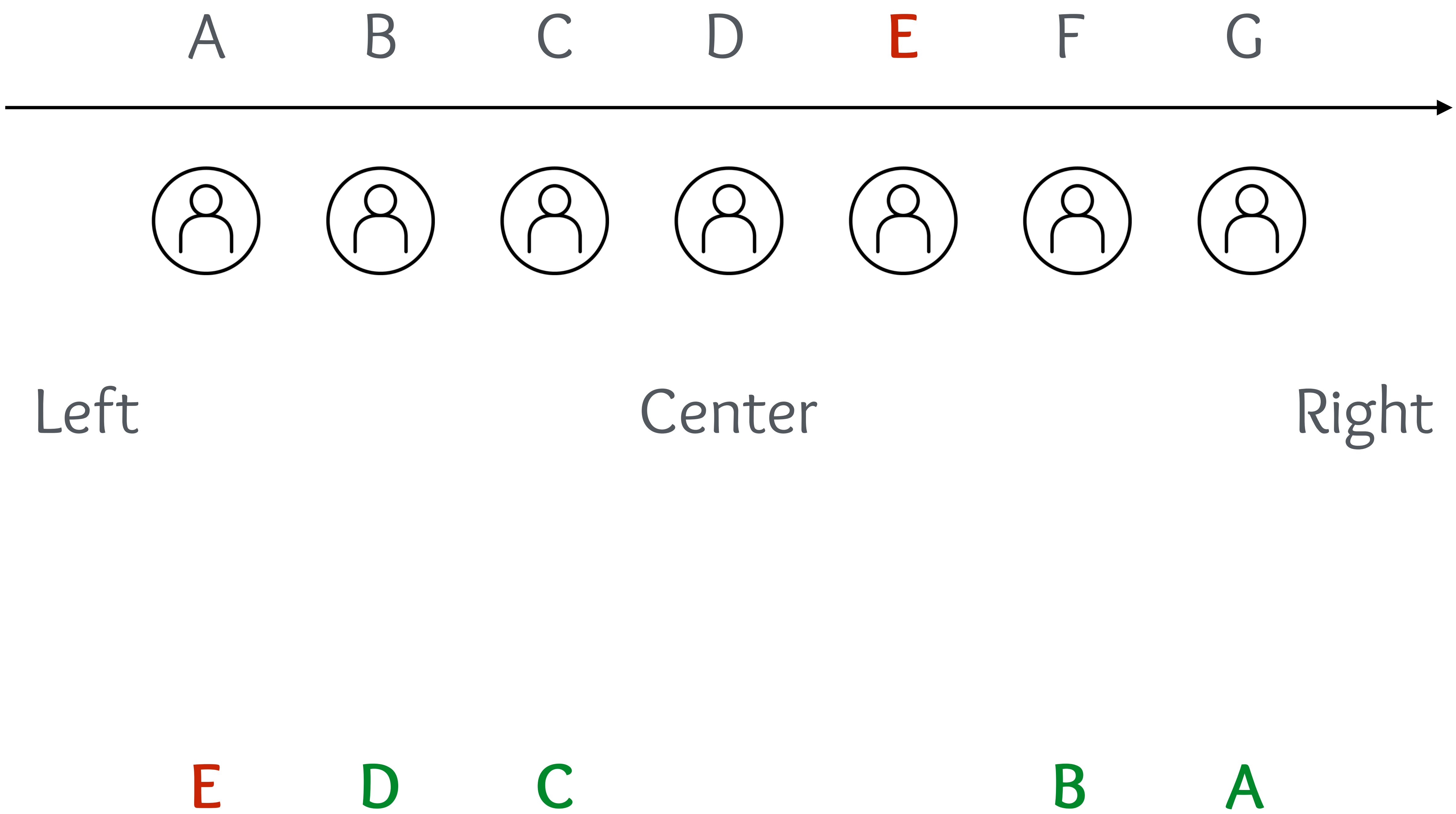

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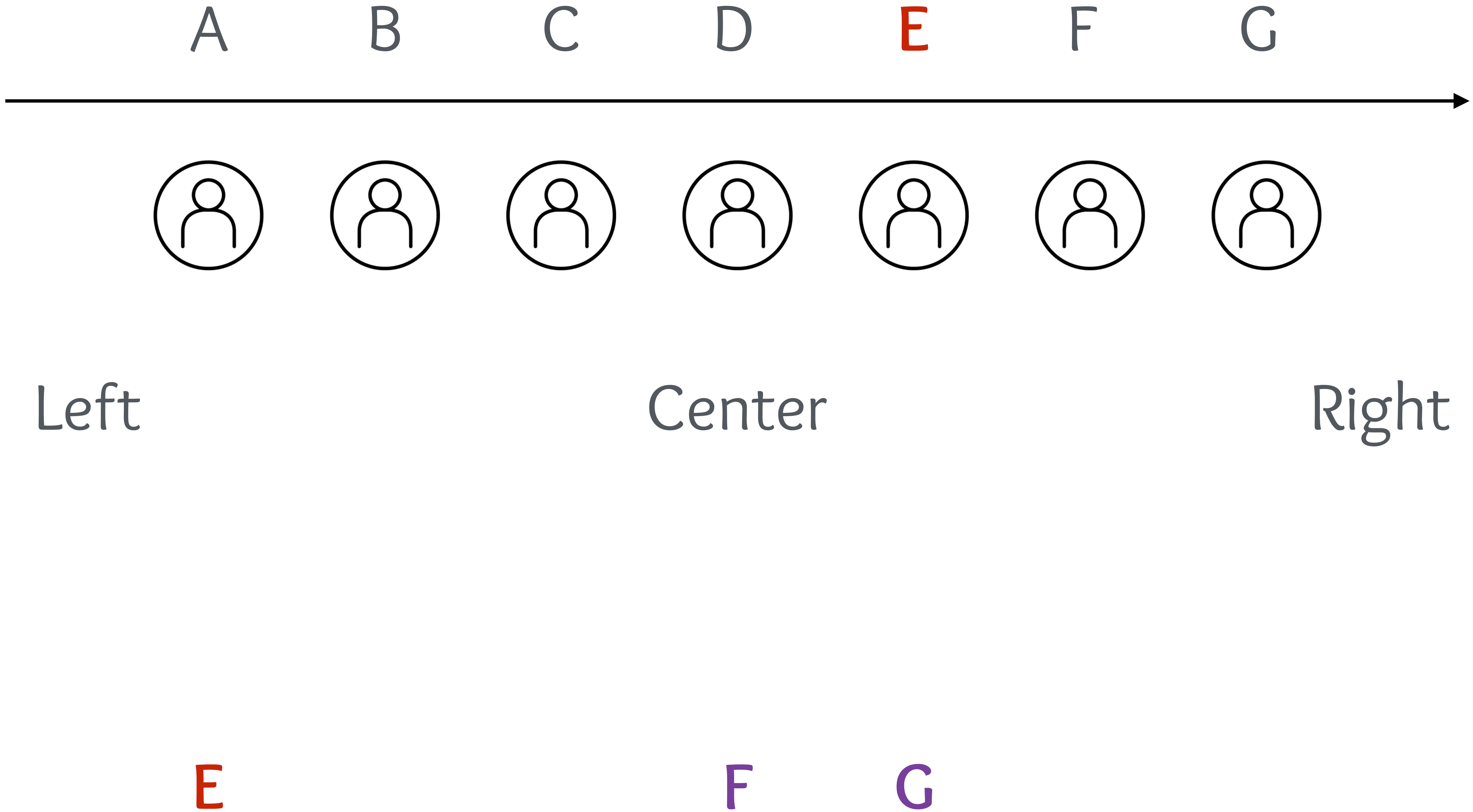
E D C
B A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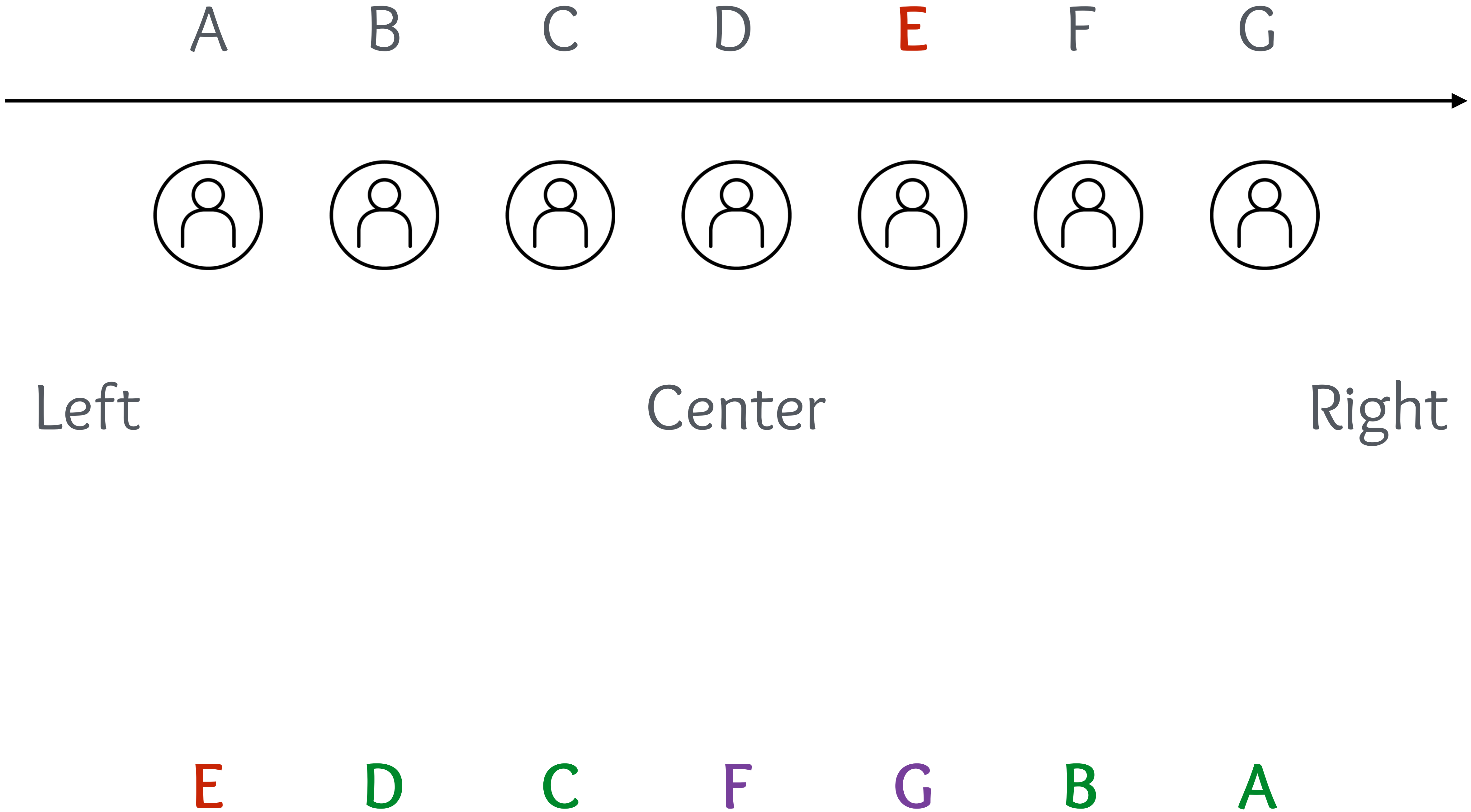
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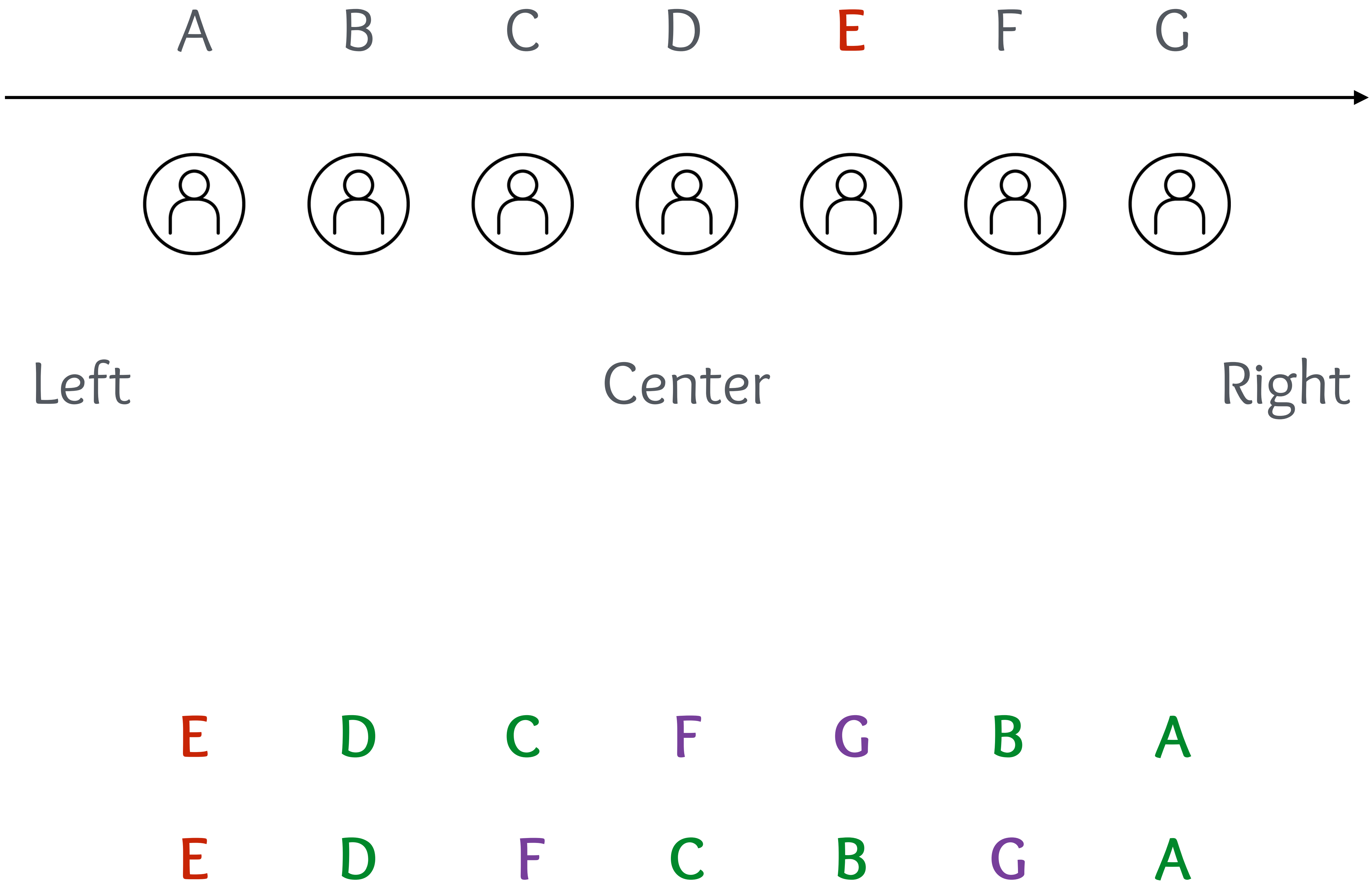
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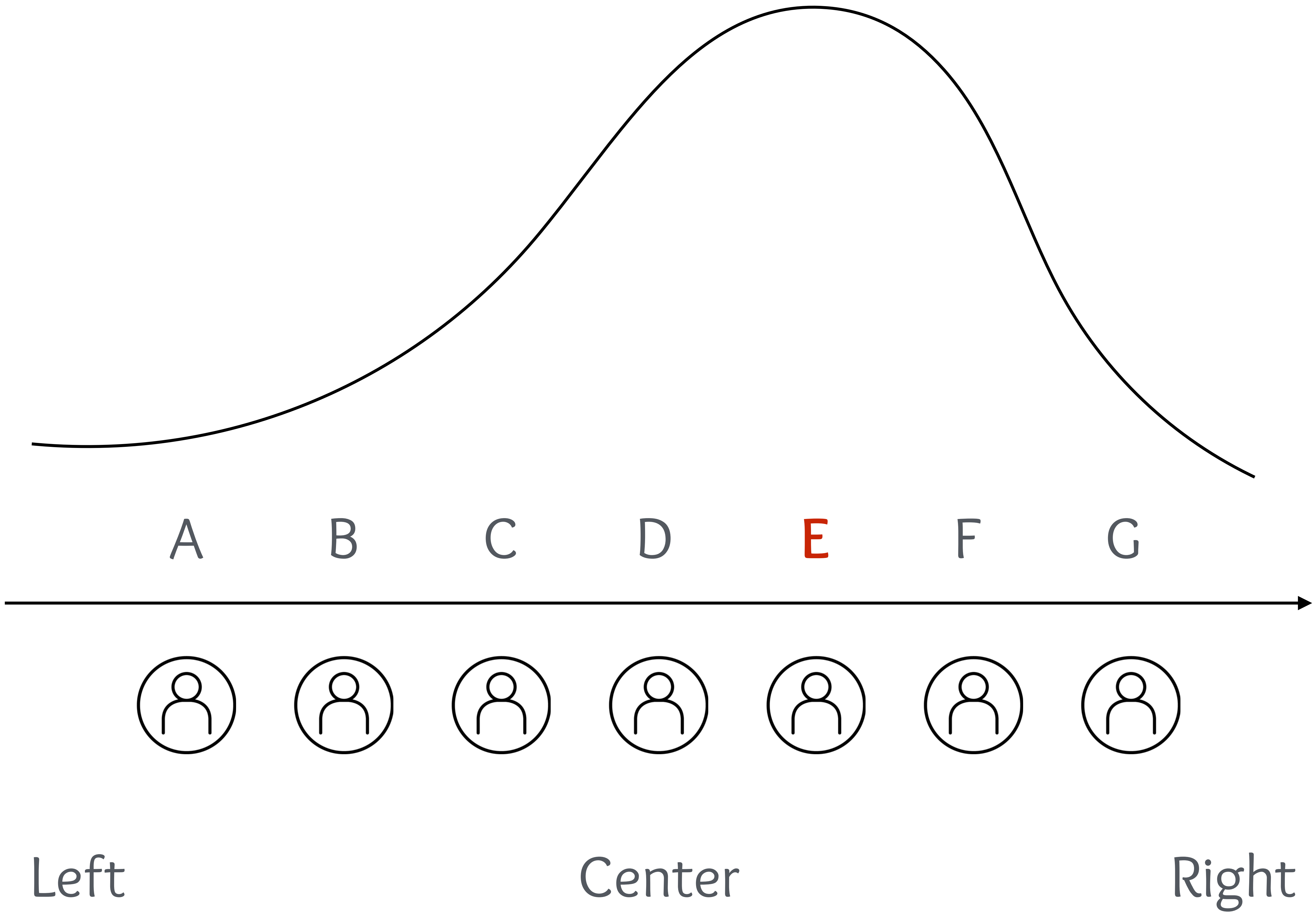
E D F C B G A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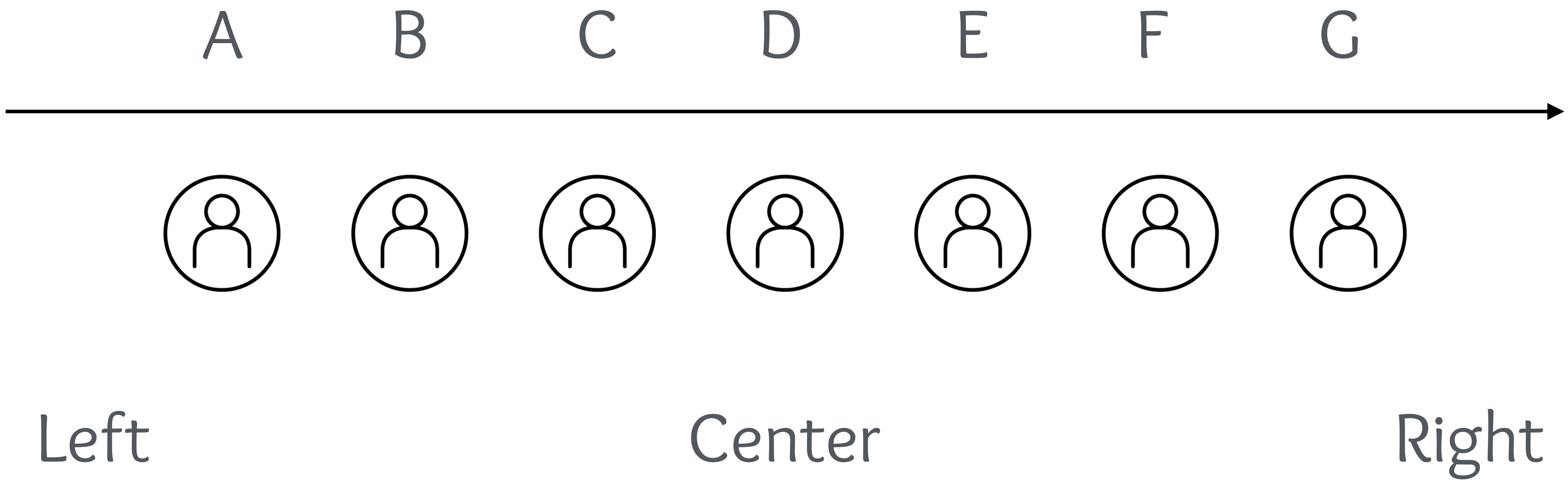
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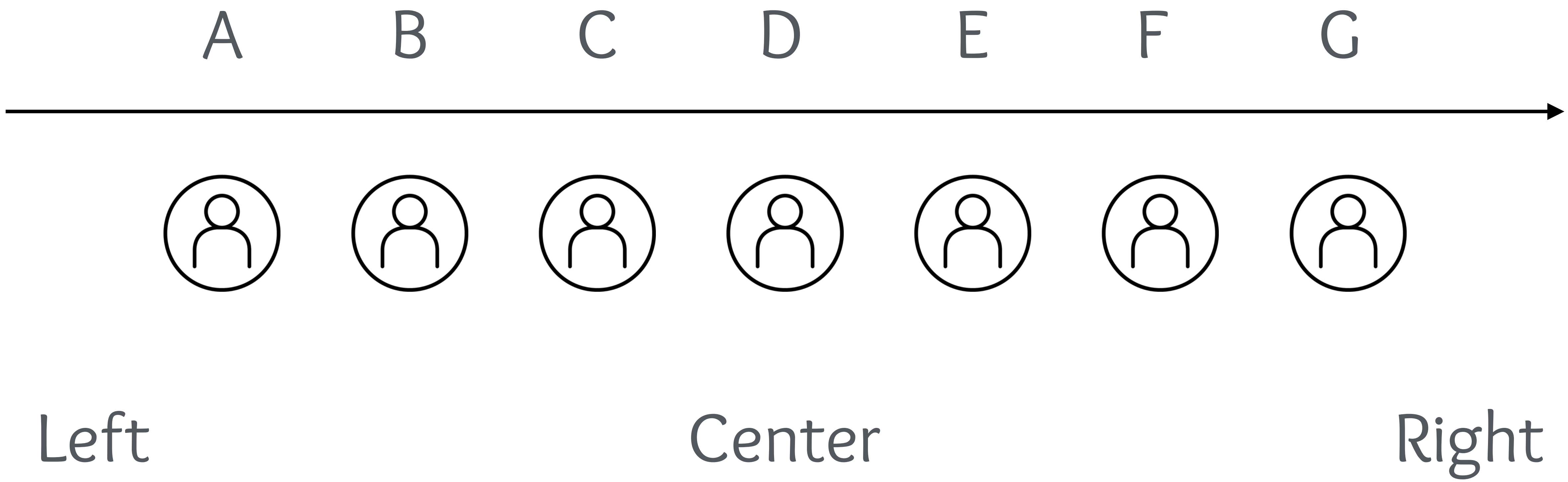
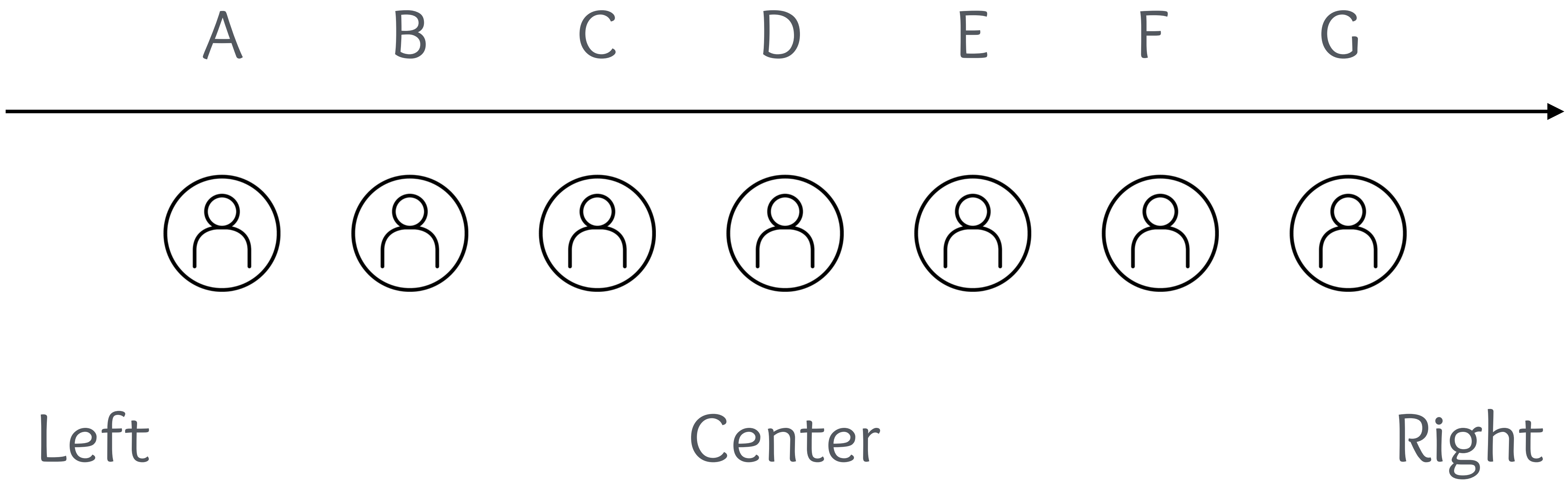


If an agent with single-peaked preferences prefers x to y, one of the following must be true:

- x is the agent's peak,
- x and y are on opposite sides of the agent's peak, or
- x is closer to the peak than y.



The notion is popular for several reasons:

- No Condorcet Cycles.
- No incentive for an agent to misreport its preferences.
- Identifiable in polynomial time.
- Reasonable (?) model of actual elections.

SINGLE PEAKED PREFERENCES

Recognition

Stable Matching with Preferences Derived from a Psychological Model
Bartholdi and Trick, Operations Research Letters, 1986

Recognising if a given profile is single-peaked with respect to some preference ordering.

↓

Checking if a 0/1 matrix has the **consecutive-ones property**.

A B C D **E** F G

A	B	C	D	**E**	F	G
E	D	F	C	B	A	G

A	B	C	D	**E**	F	G
E	D	F	C	B	A	G

A fixed permutation

	A	B	C	D	E	F	G
	E	D	F	C	B	A	G
A fixed permutation	0	0	0	0	1	0	0

A	B	C	D	E	F	G
E	D	F	C	B	A	G
0	0	0	0	1	0	0

A fixed permutation

A	B	C	D	E	F	G
E	D	F	C	B	A	G
0	0	0	0	1	0	0
0	0	0	1	1	0	0

A fixed permutation

	A	B	C	D	E	F	G
A fixed permutation	E	D	F	C	B	A	G
	0	0	0	0	1	0	0
	0	0	0	1	1	0	0

A	B	C	D	E	F	G
E	D	F	C	B	A	G
0	0	0	0	1	0	0
0	0	0	1	1	0	0
0	0	0	1	1	1	0

A fixed permutation

	A	B	C	D	E	F	G
A fixed permutation	E	D	F	C	B	A	G
	0	0	0	0	1	0	0
	0	0	0	1	1	0	0
	0	0	0	1	1	1	0

A fixed permutation

A	B	C	D	E	F	G
E	D	F	C	B	A	G
0	0	0	0	1	0	0
0	0	0	1	1	0	0
0	0	0	1	1	1	0
0	0	1	1	1	1	0

A	B	C	D	E	F	G
E	D	F	C	B	A	G
0	0	0	0	1	0	0
0	0	0	1	1	0	0
0	0	0	1	1	1	0
0	0	1	1	1	1	0

A fixed permutation

A	B	C	D	E	F	G
E	D	F	C	B	A	G
0	0	0	0	1	0	0
0	0	0	1	1	0	0
0	0	0	1	1	1	0
0	0	1	1	1	1	0
0	1	1	1	1	1	0

A fixed permutation

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A	B	C	D	E	F	G
E	D	F	C	B	A	G
0	0	0	0	1	0	0
0	0	0	1	1	0	0
0	0	0	1	1	1	0
0	0	1	1	1	1	0
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A fixed permutation

A	B	C	D	E	F	G
E	D	F	C	B	A	G
0	0	0	0	1	0	0
0	0	0	1	1	0	0
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1	1	1	1	1	1	0

A fixed permutation

A	B	C	D	E	F	G
E	D	F	C	B	A	G
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0	0	0	1	1	1	0
0	0	1	1	1	1	0
0	1	1	1	1	1	0
1	1	1	1	1	1	0

A fixed permutation

A	B	C	D	E	F	G
E	D	F	C	B	A	G
0	0	0	0	1	0	0
0	0	0	1	1	0	0
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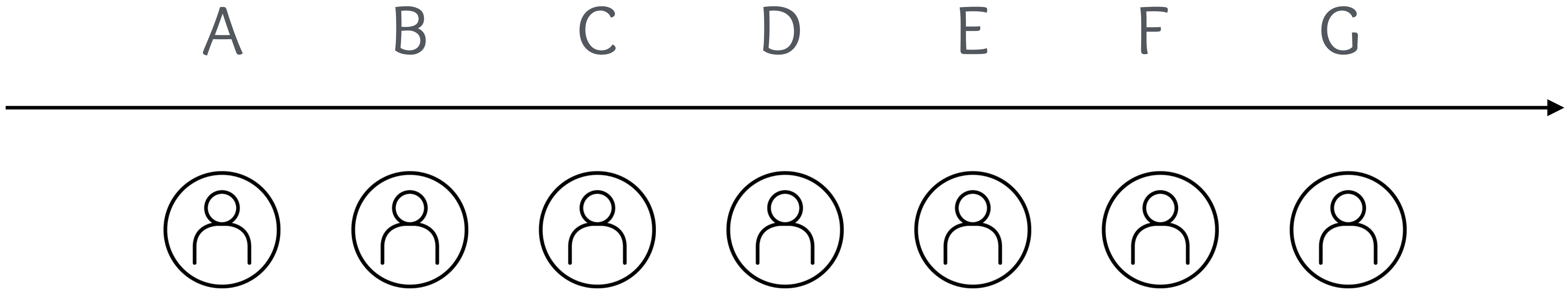
SINGLE PEAKED PREFERENCES

Strategyproofness

The Theory of Committees and Elections.
Black, D., New York: Cambridge University Press, 1958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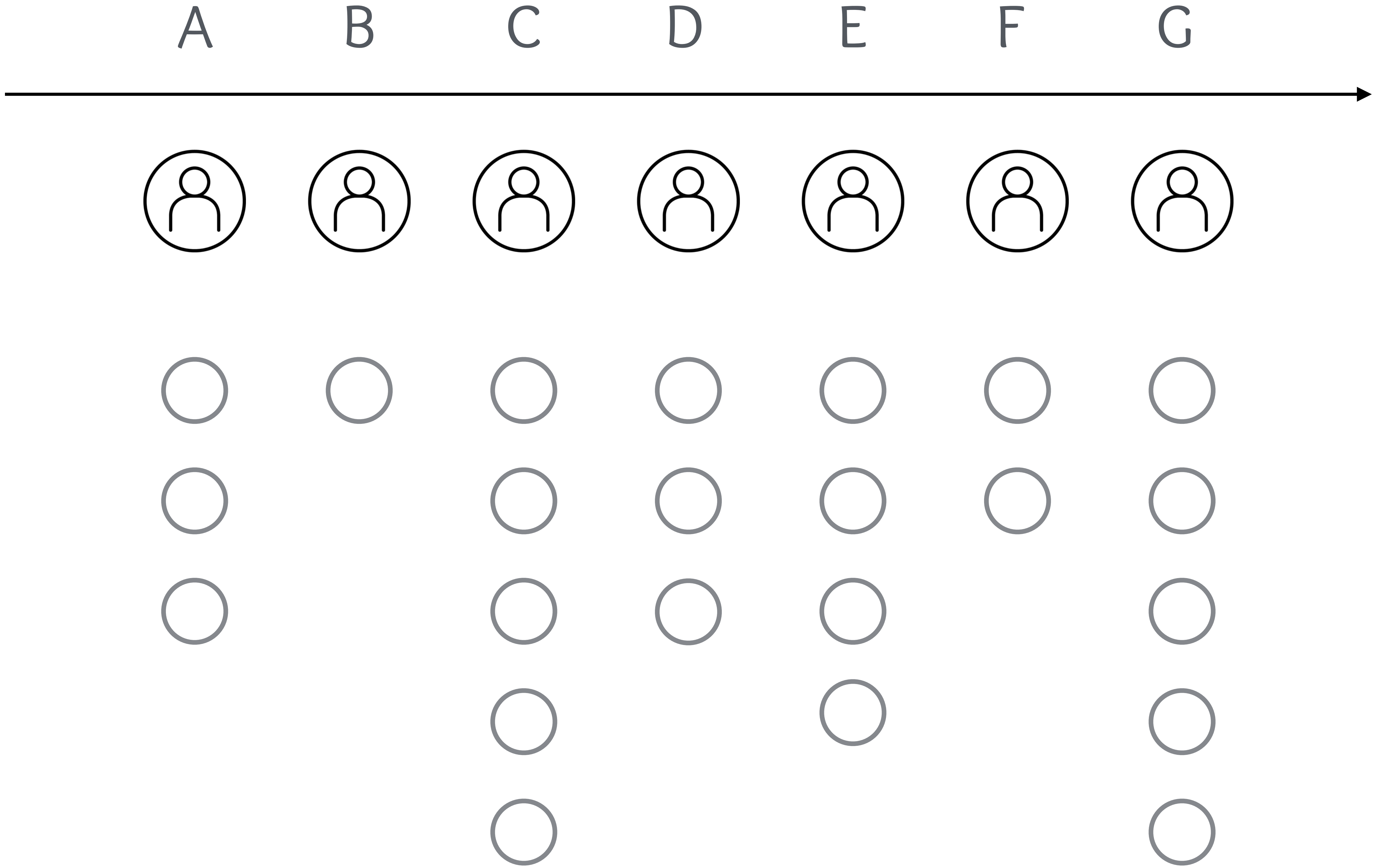
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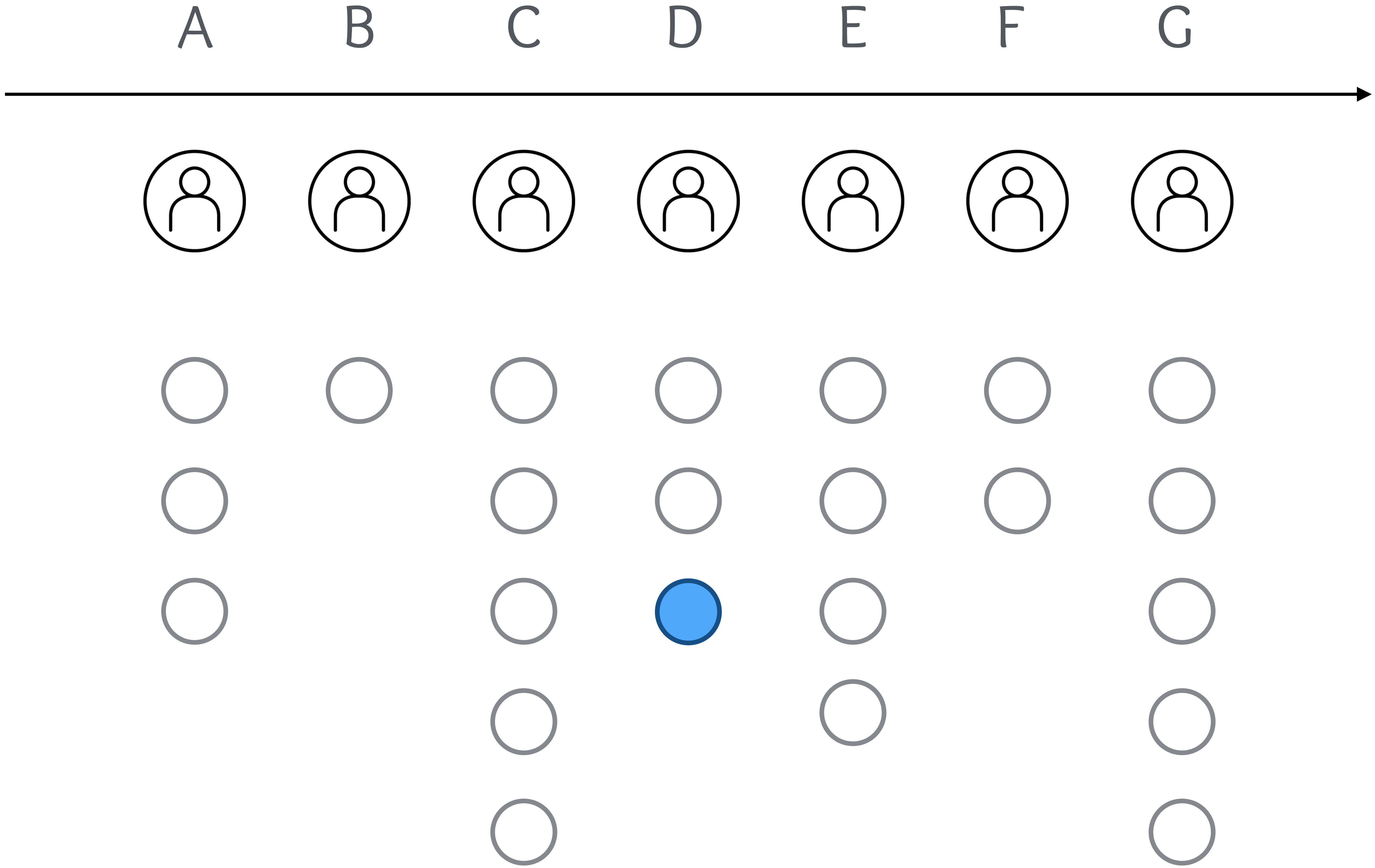
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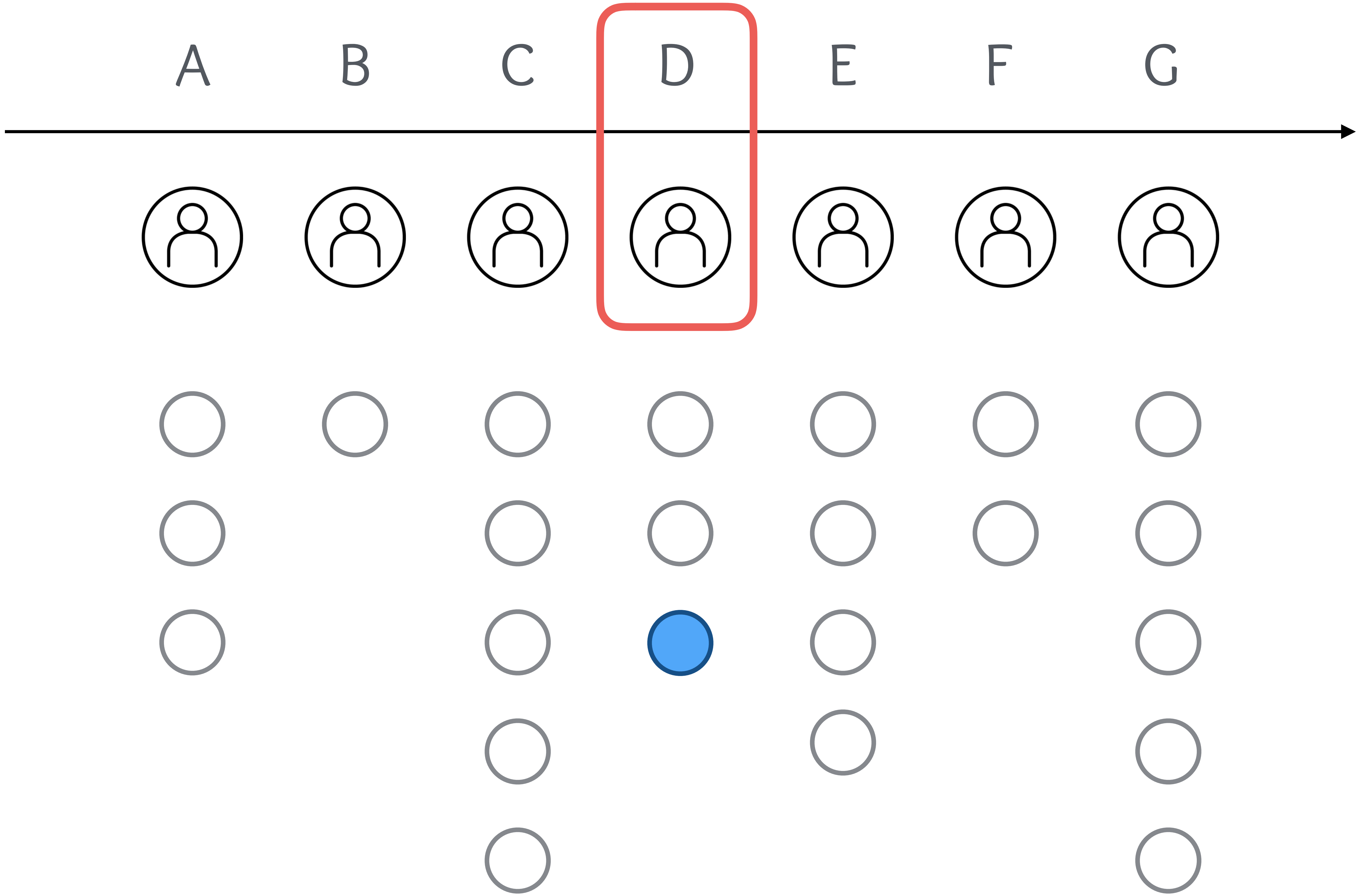
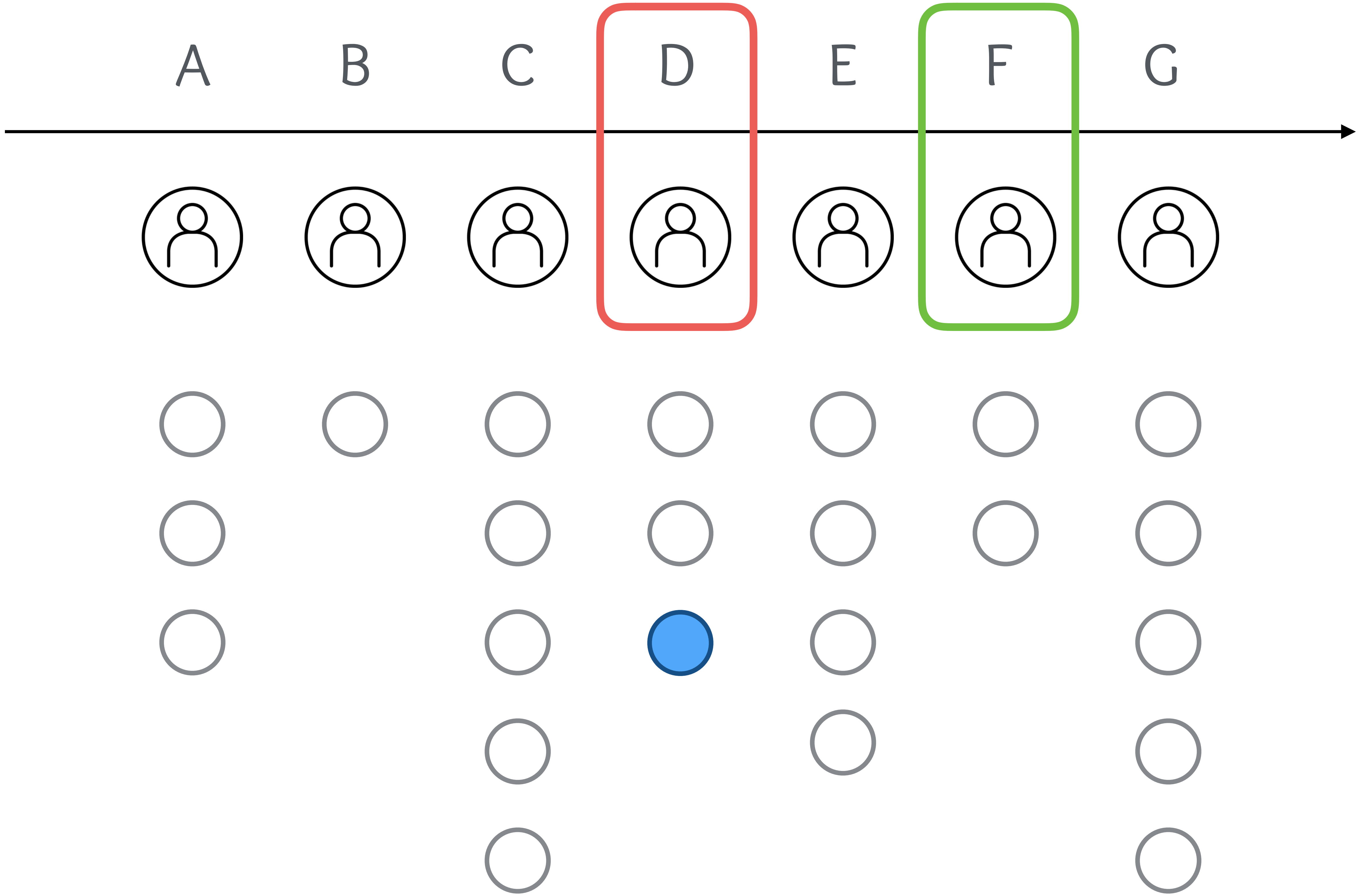


Claim: D beats all other candidates in pairwise elections.



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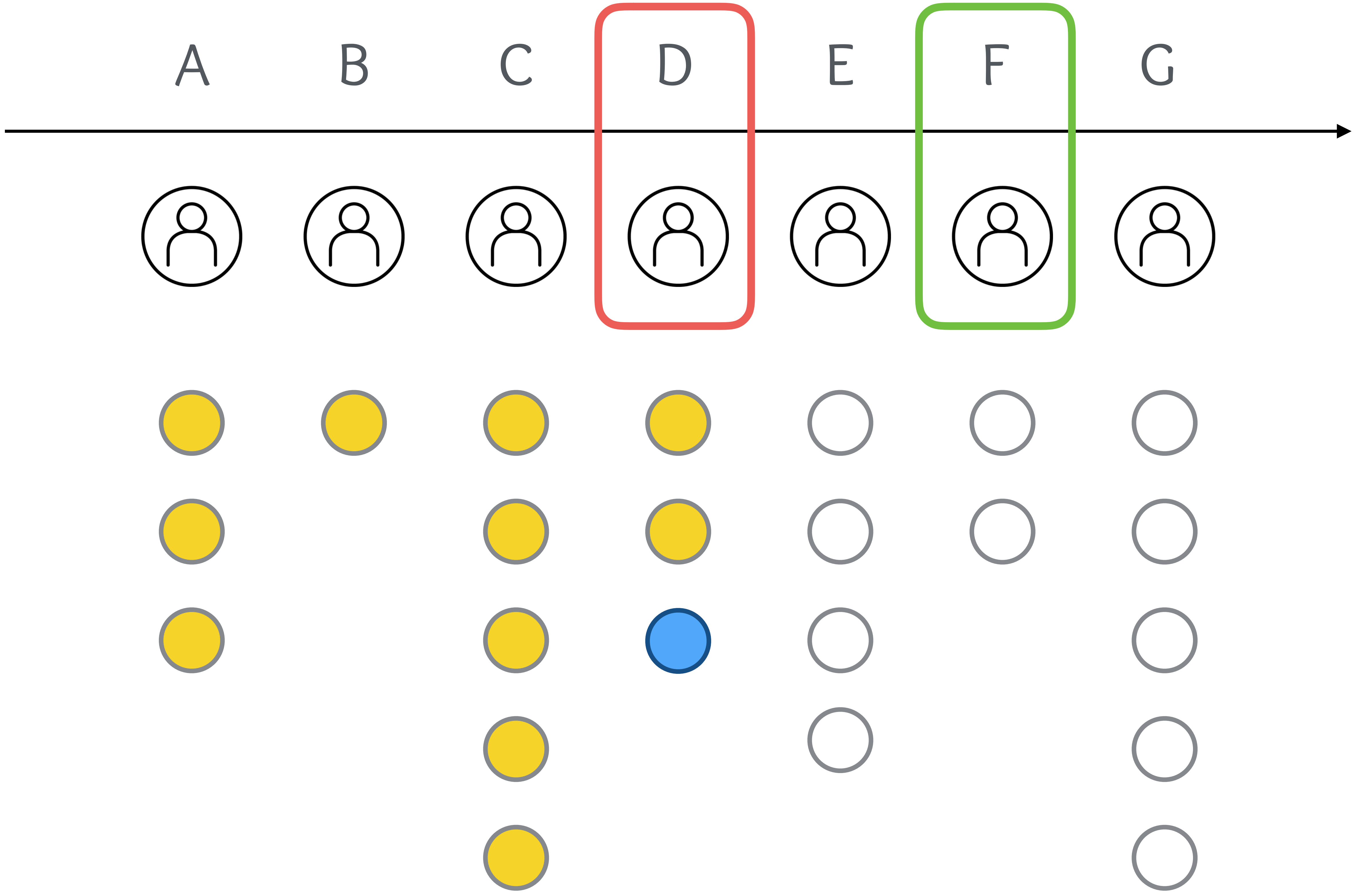

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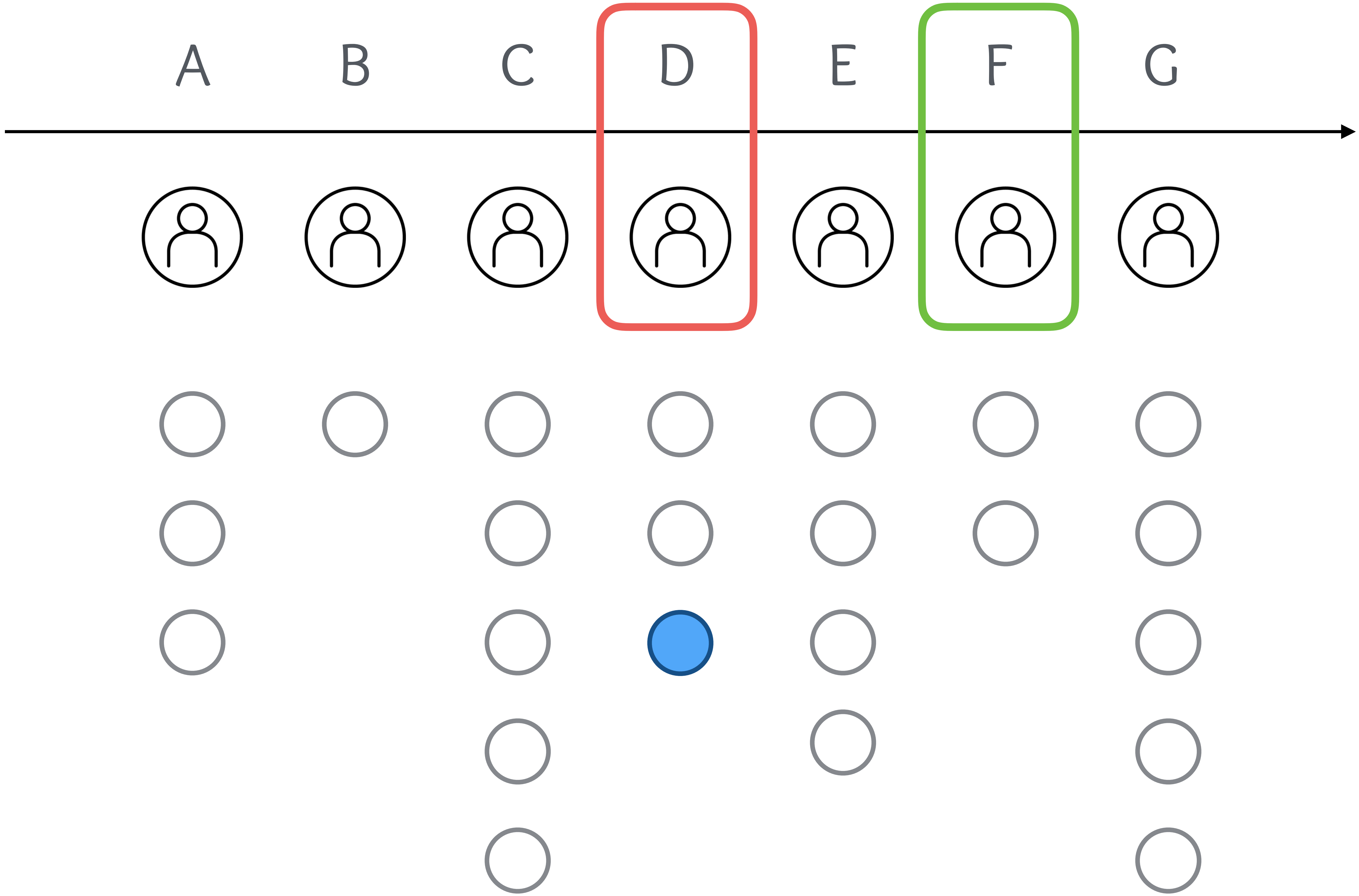

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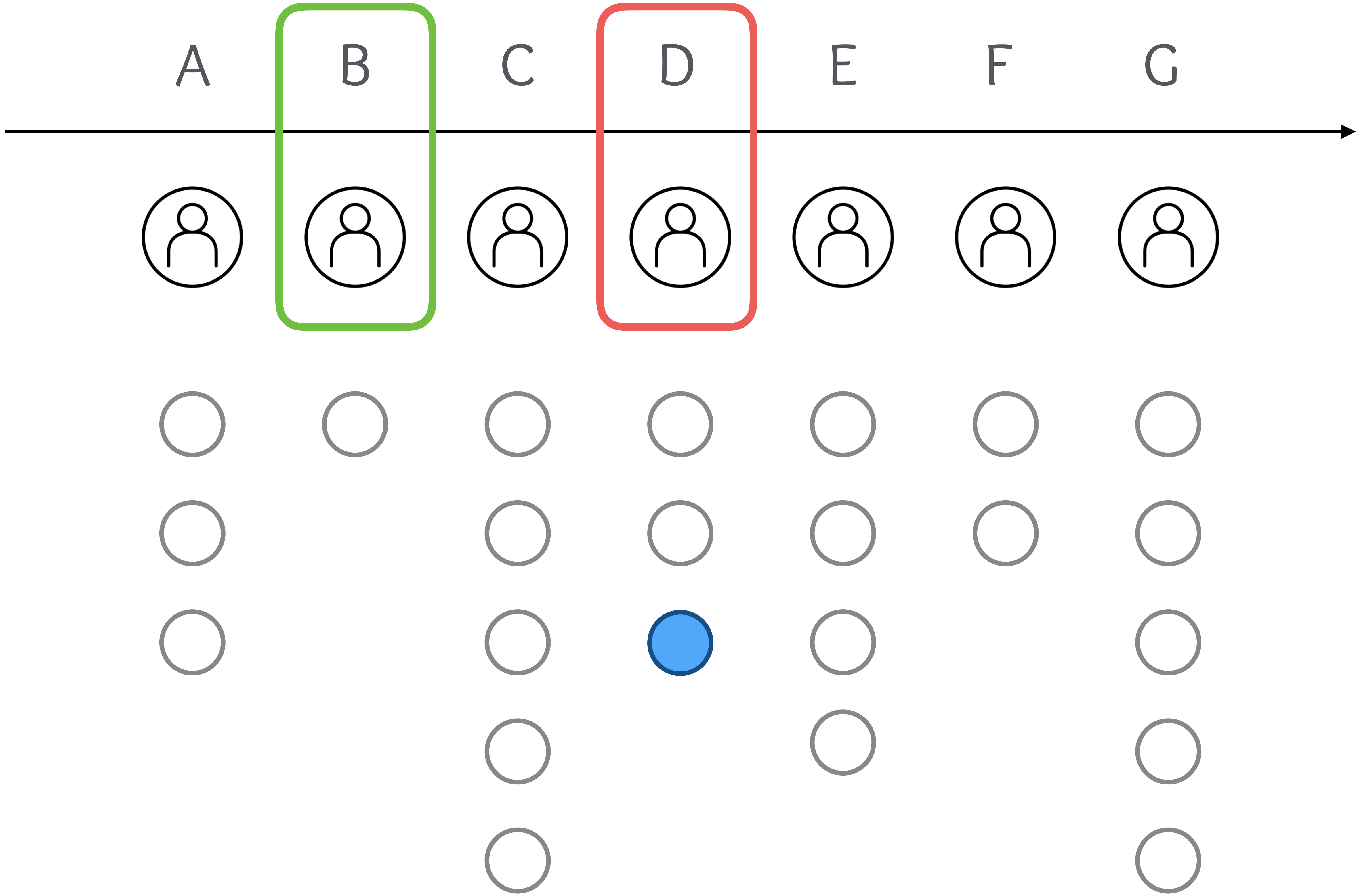

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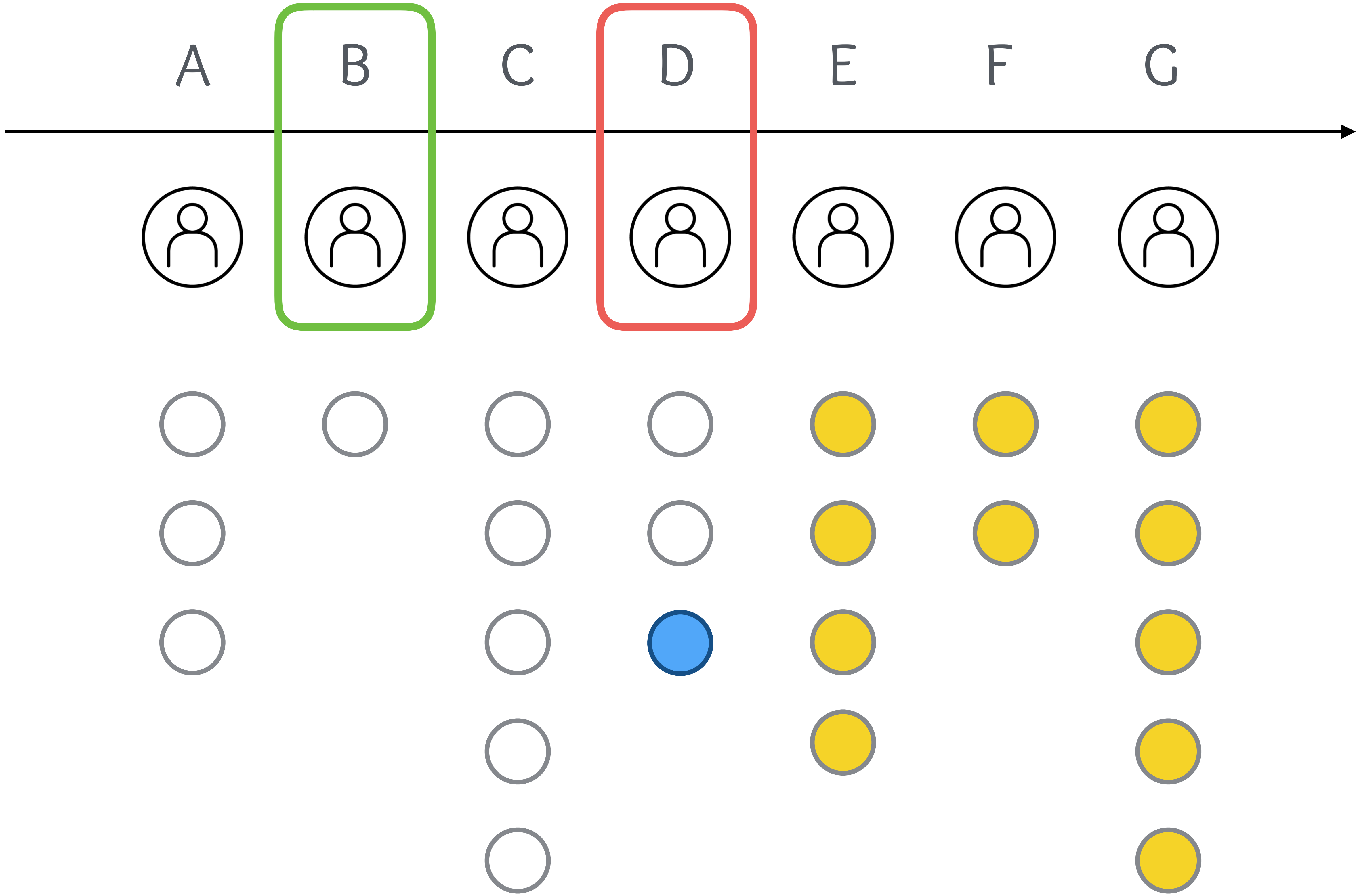

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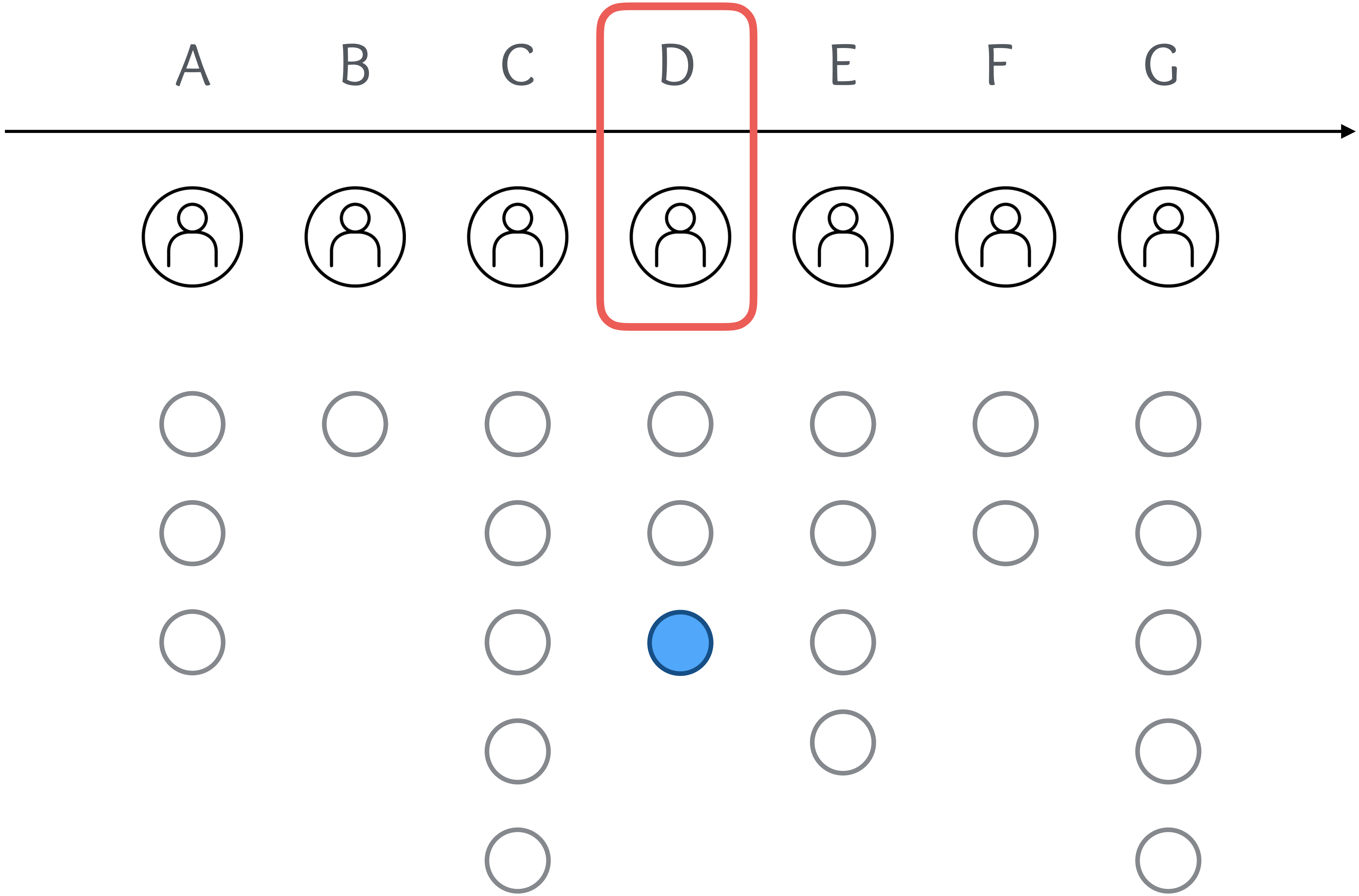

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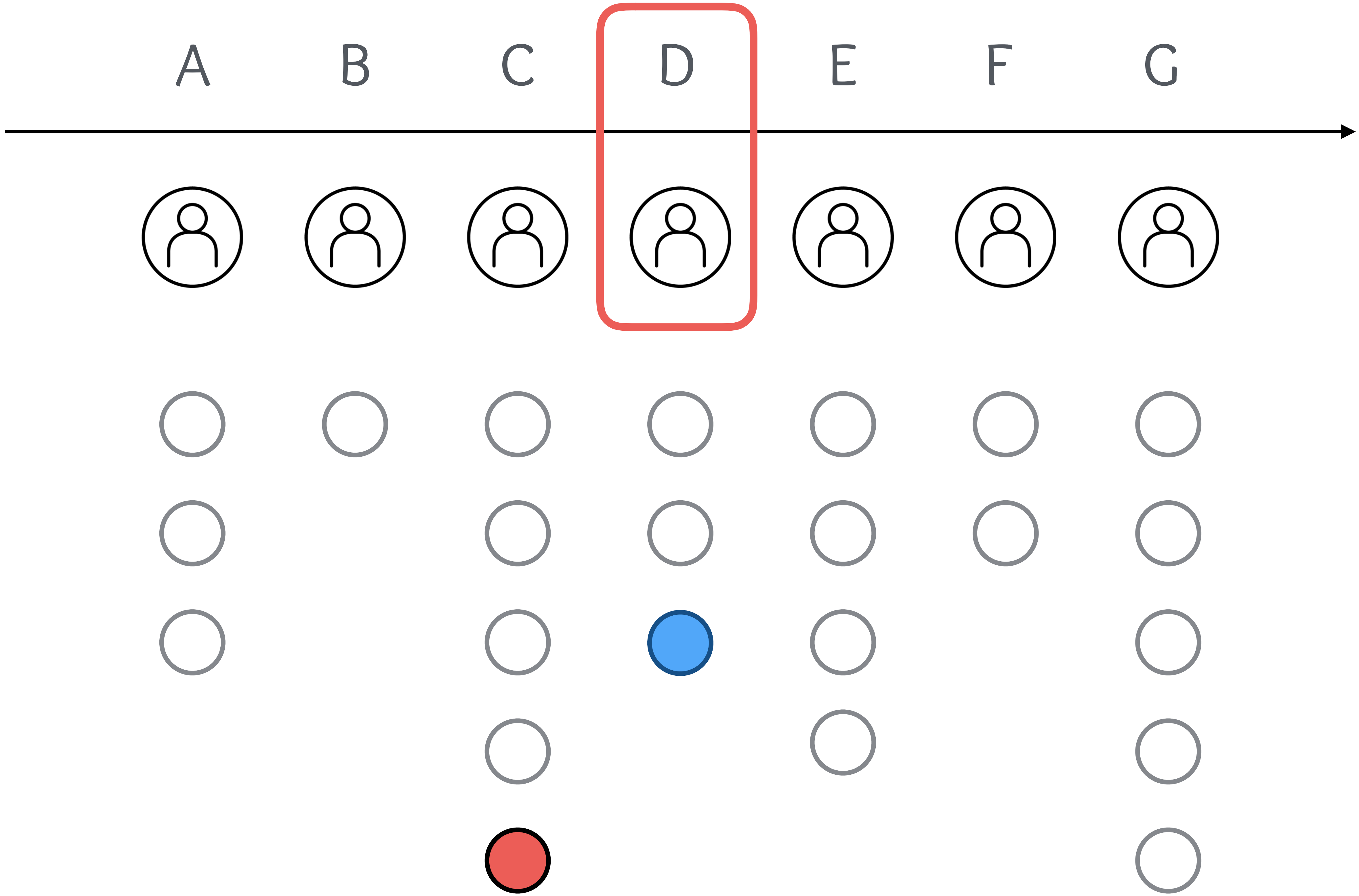

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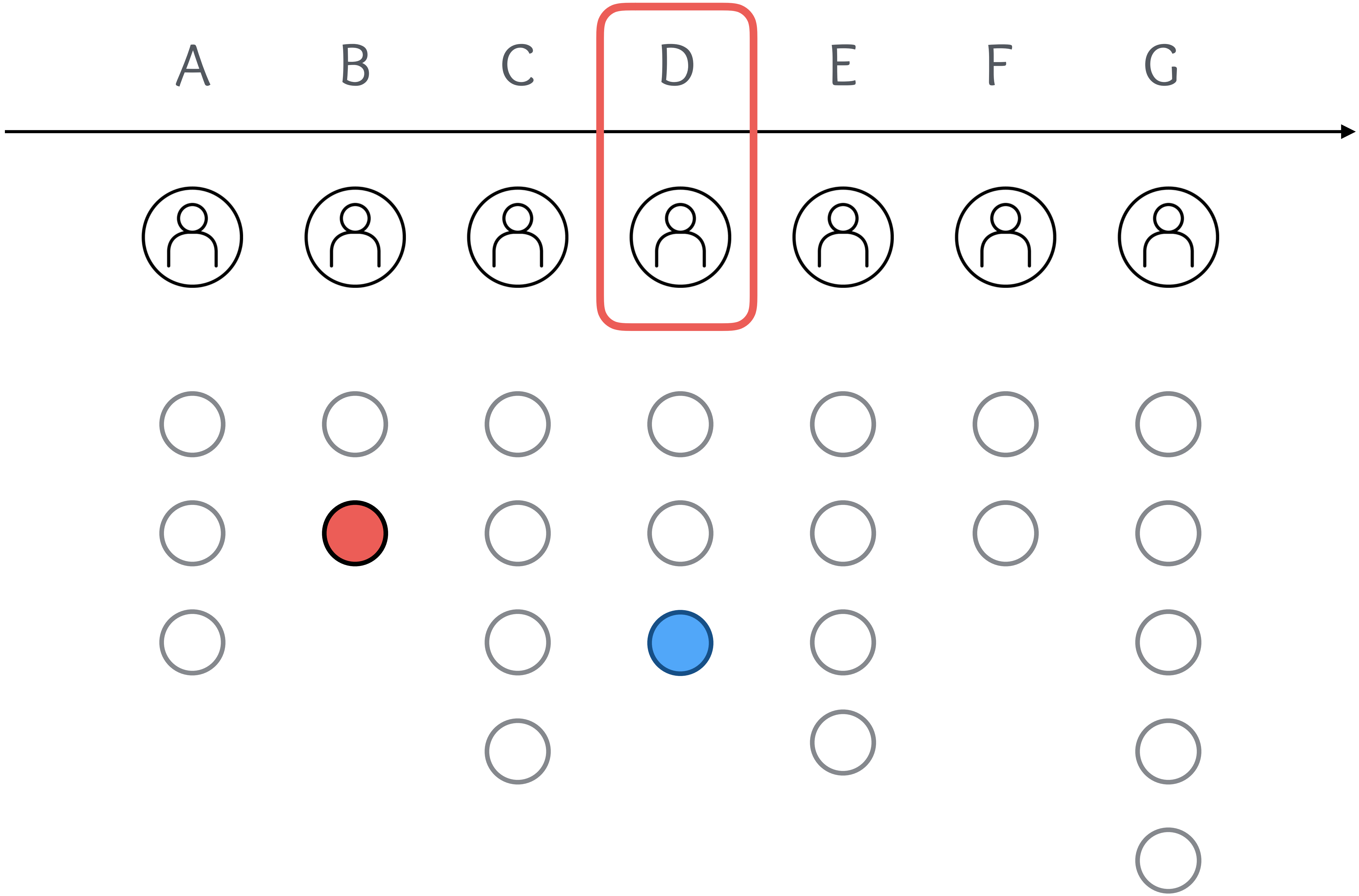

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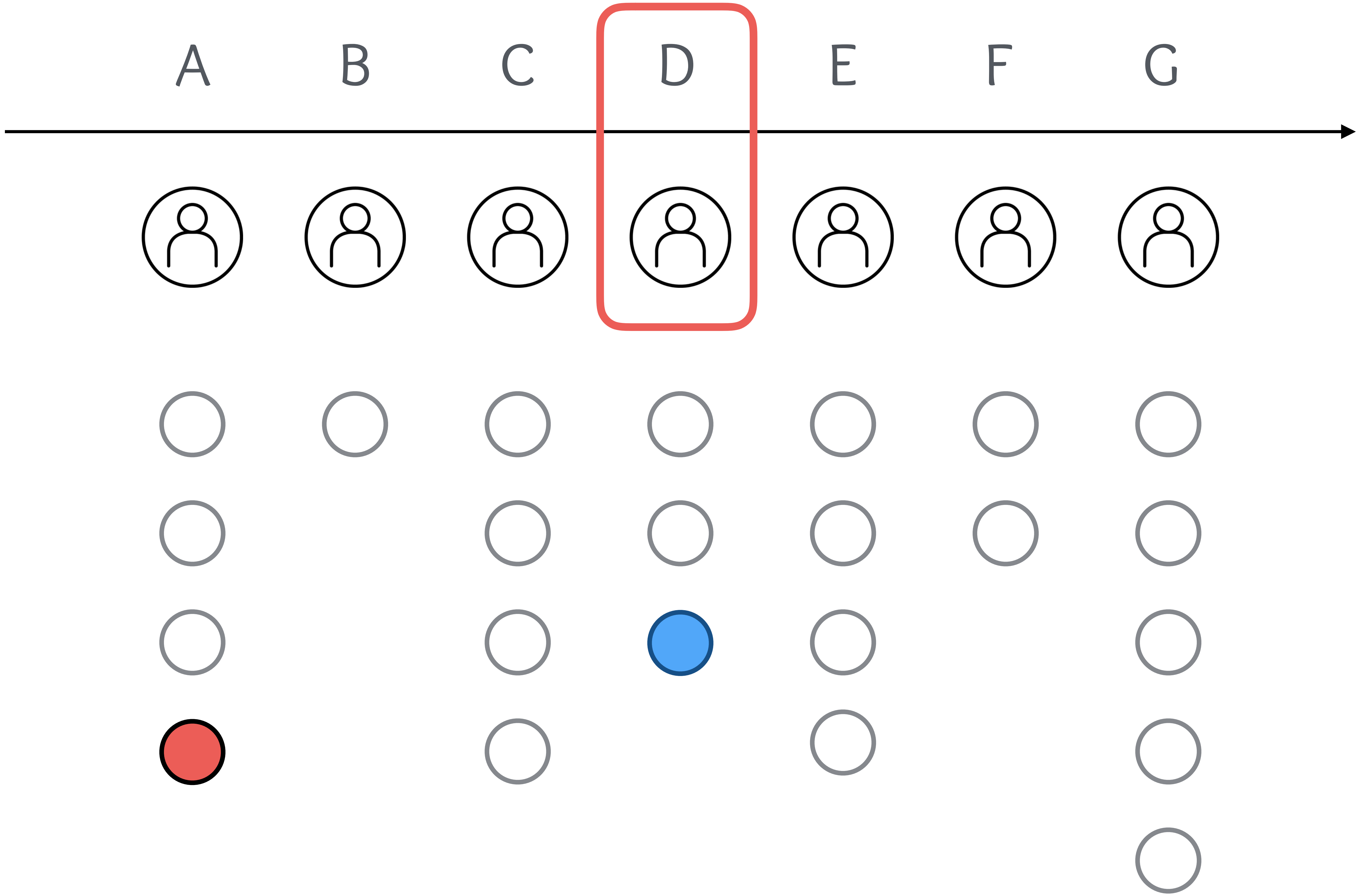

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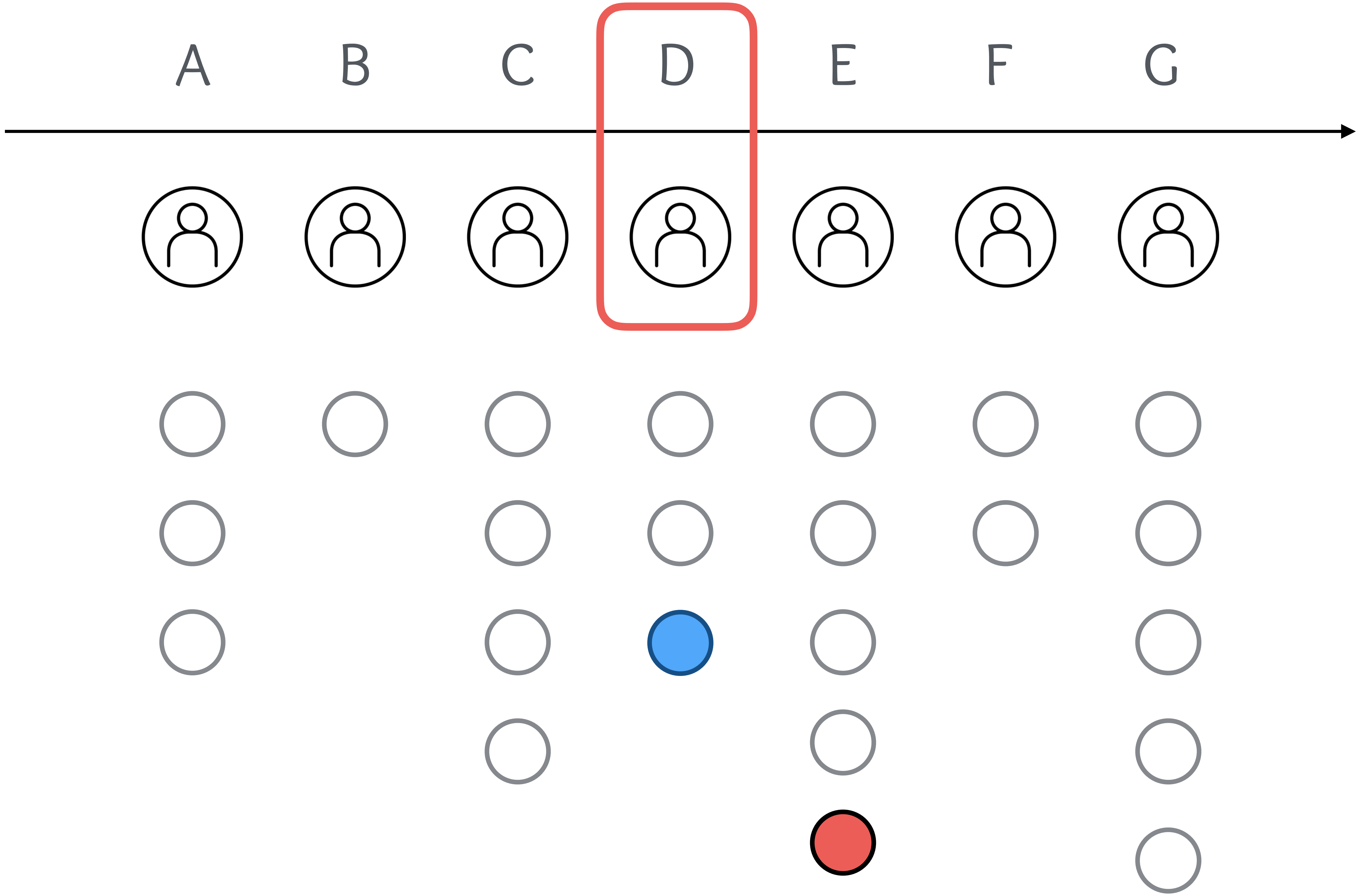

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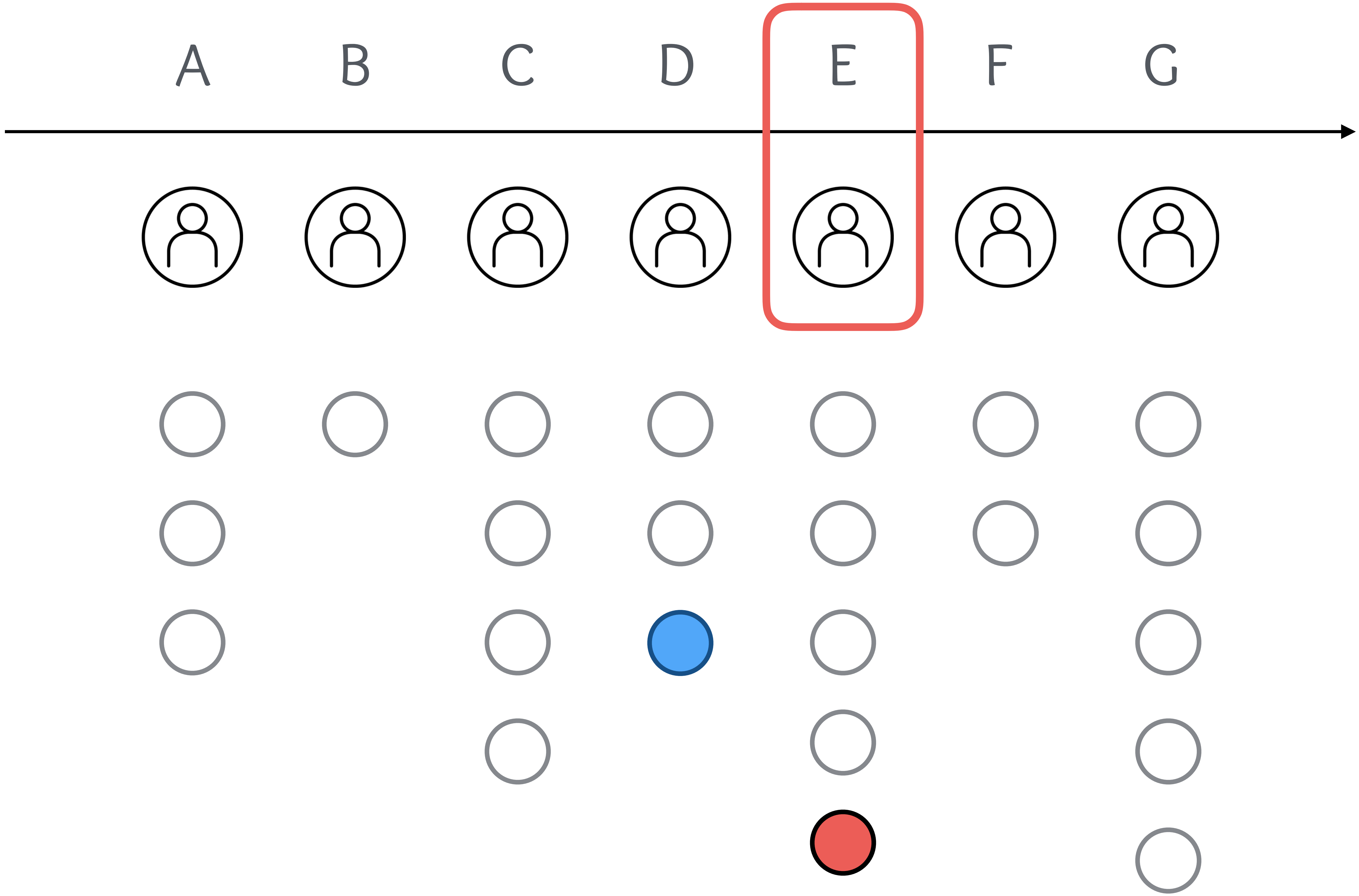

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SINGLE PEAKED PREFERENCES

Preference Elicitation

Eliciting single-peaked preferences using comparison queries,
Conitzer, J. Artif. Intell. Res.; 2016

When the number of candidates is large,
soliciting a full ranking can be a little unmanageable.

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soliciting a full ranking can be a little unmanageable.

Voters typically find it easier to answer "comparison queries":

Would you rather hang out over coffee or join me for a concert?

How many such queries to do we need to make to be able to reconstruct the full preference?

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Just like the weighing scale puzzles, except you can only compare two single options at a time.

Using a “merge sort” like idea,
O(m log m) comparisons are enough.

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Recursively order half the alternatives.

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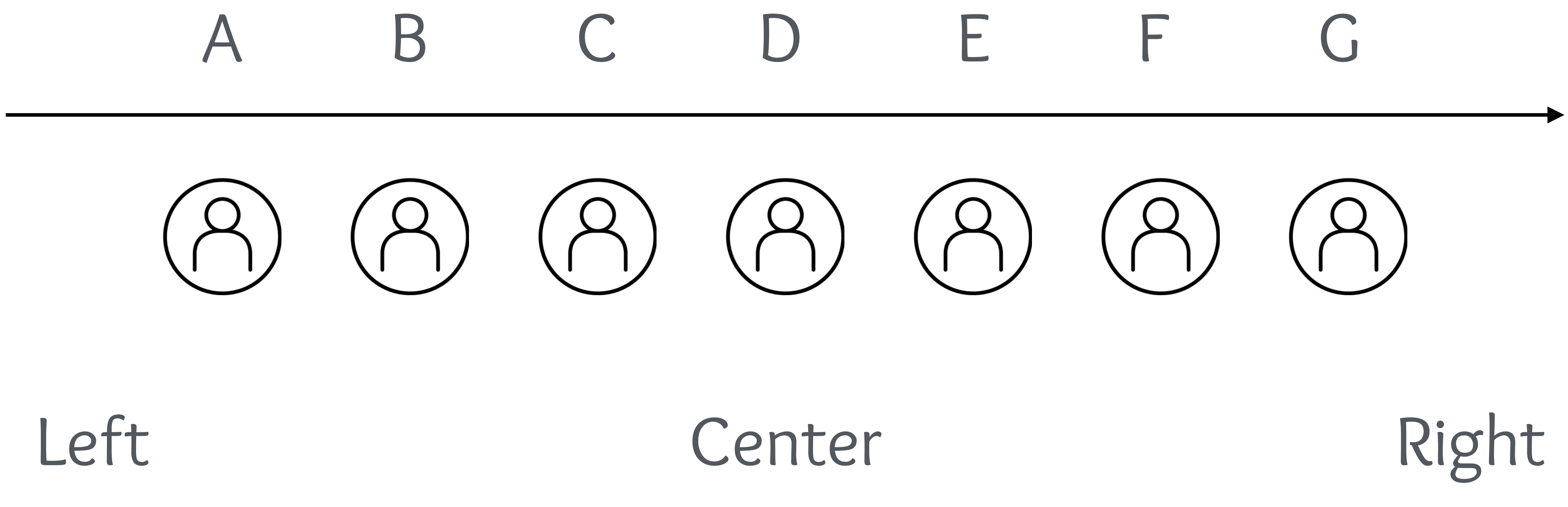
Using a "merge sort" like idea,
O(m log m) comparisons are enough.

Merge the two lists with a linear number of queries.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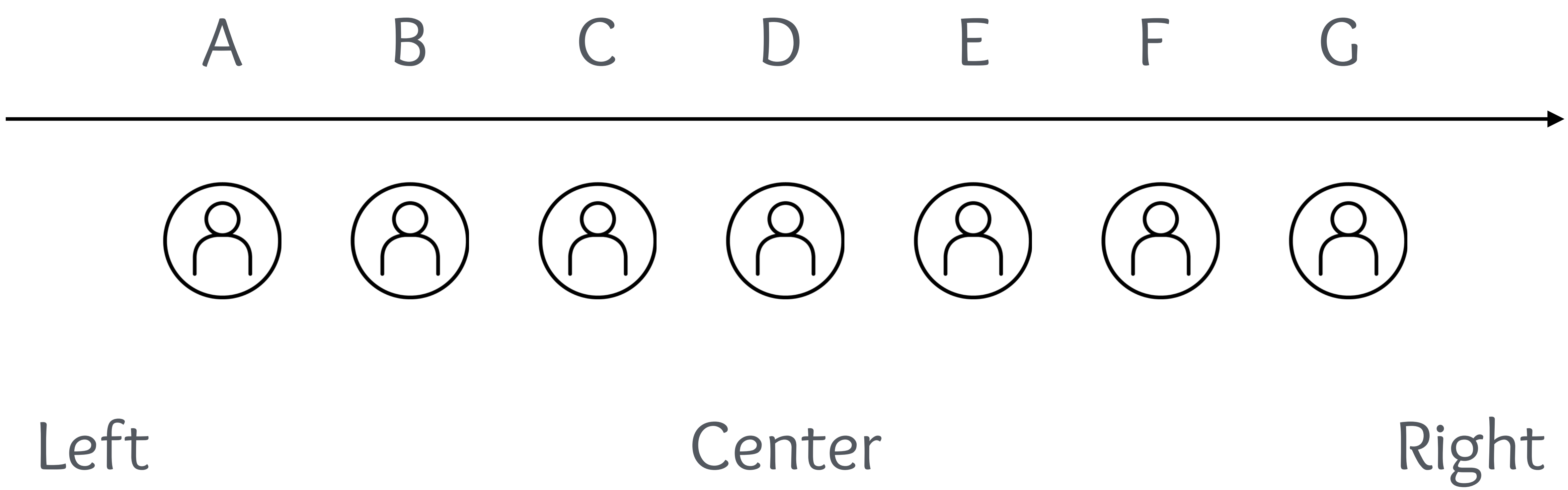
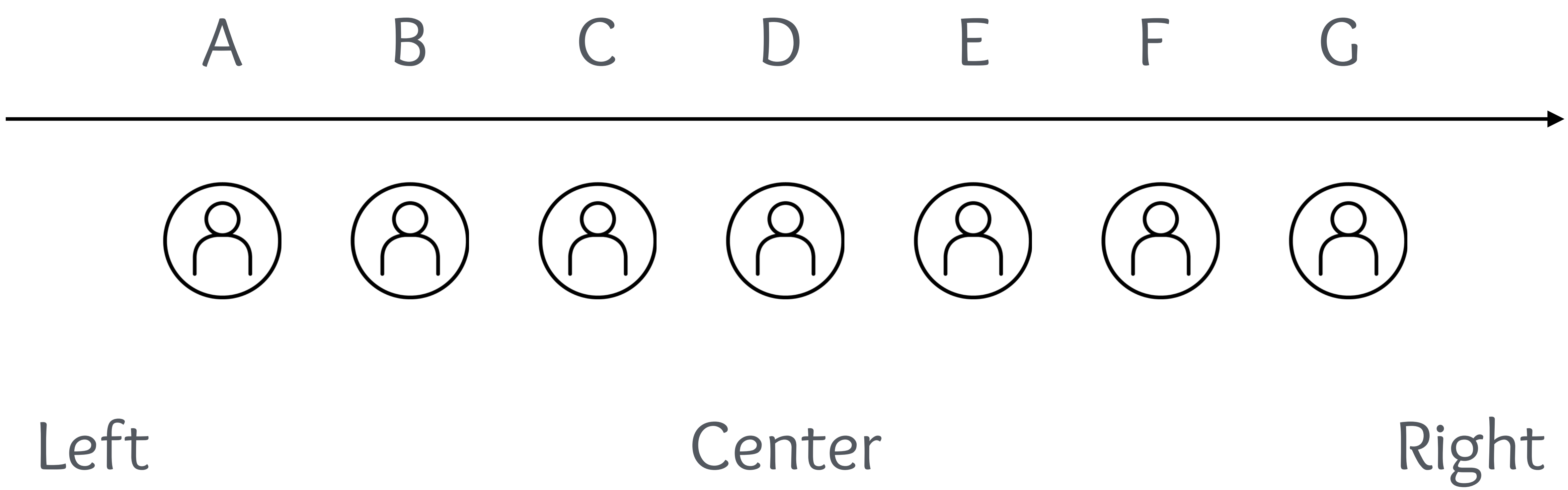
…and we can do better if the preferences are single-peaked!



…and we can do better if the preferences are single-peaked!

(i) Identify the peak: use binary search.



[Better query complexities for elicitation.]

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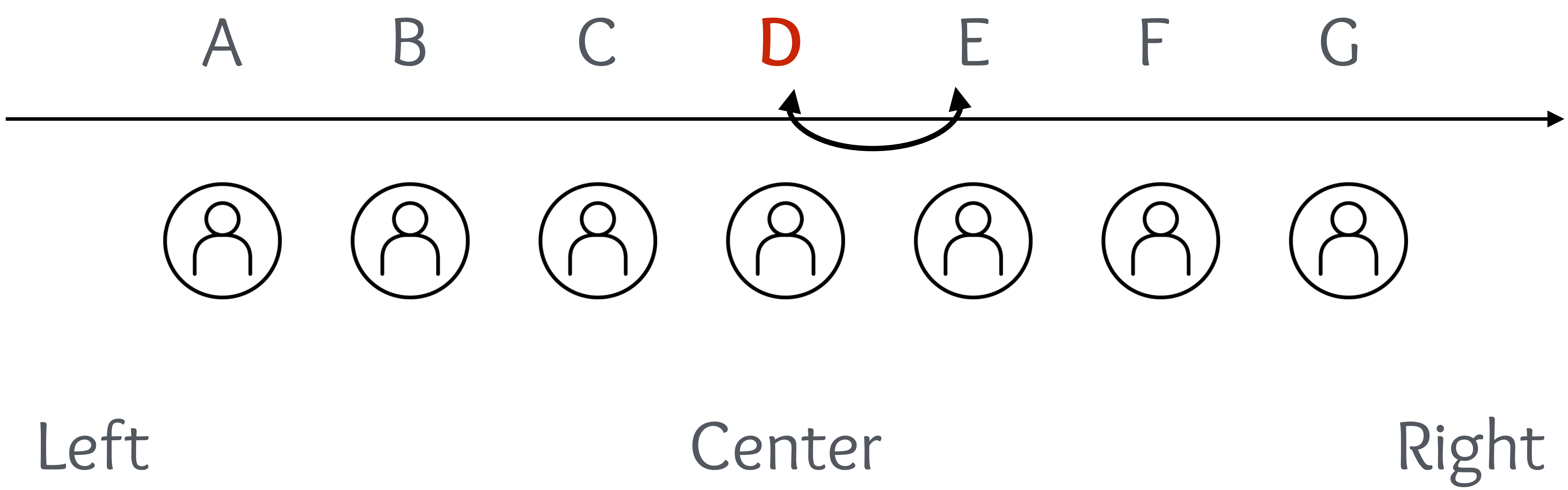

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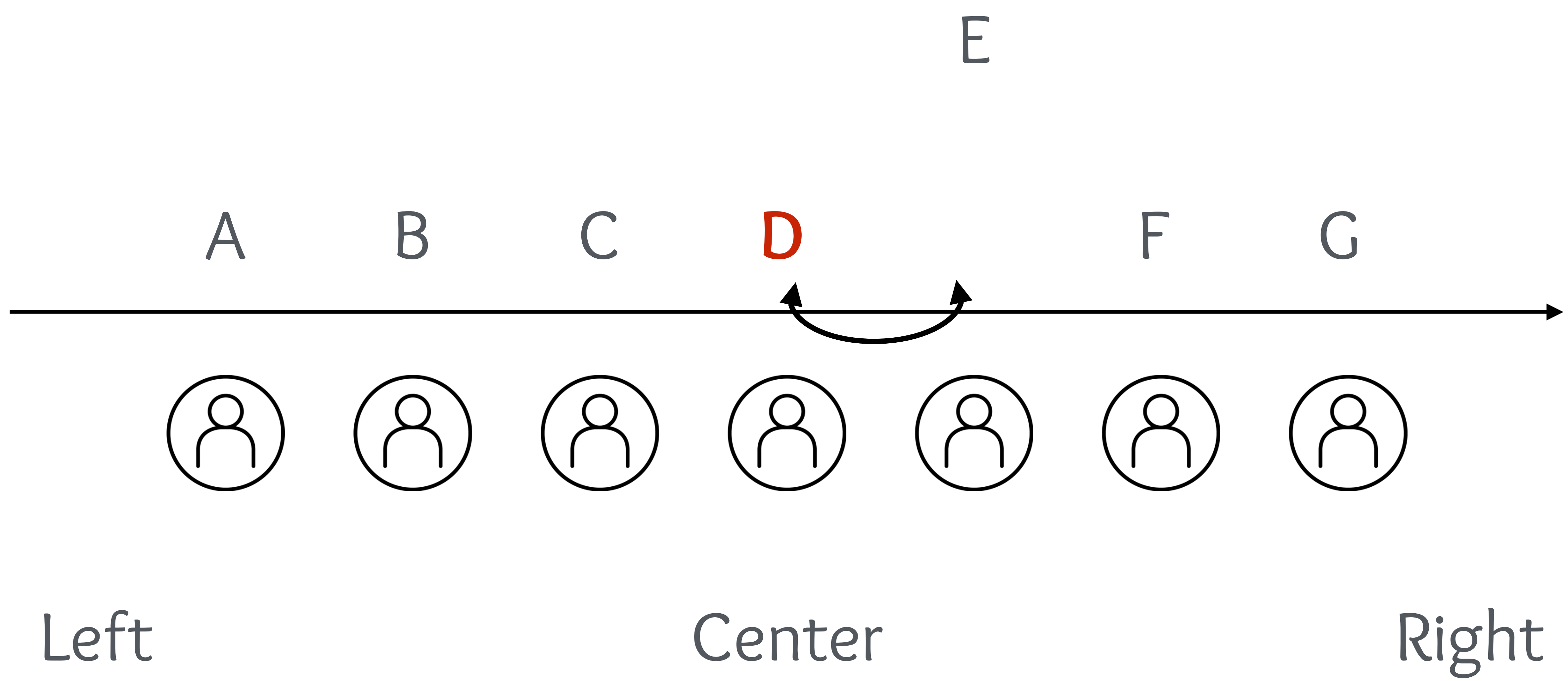

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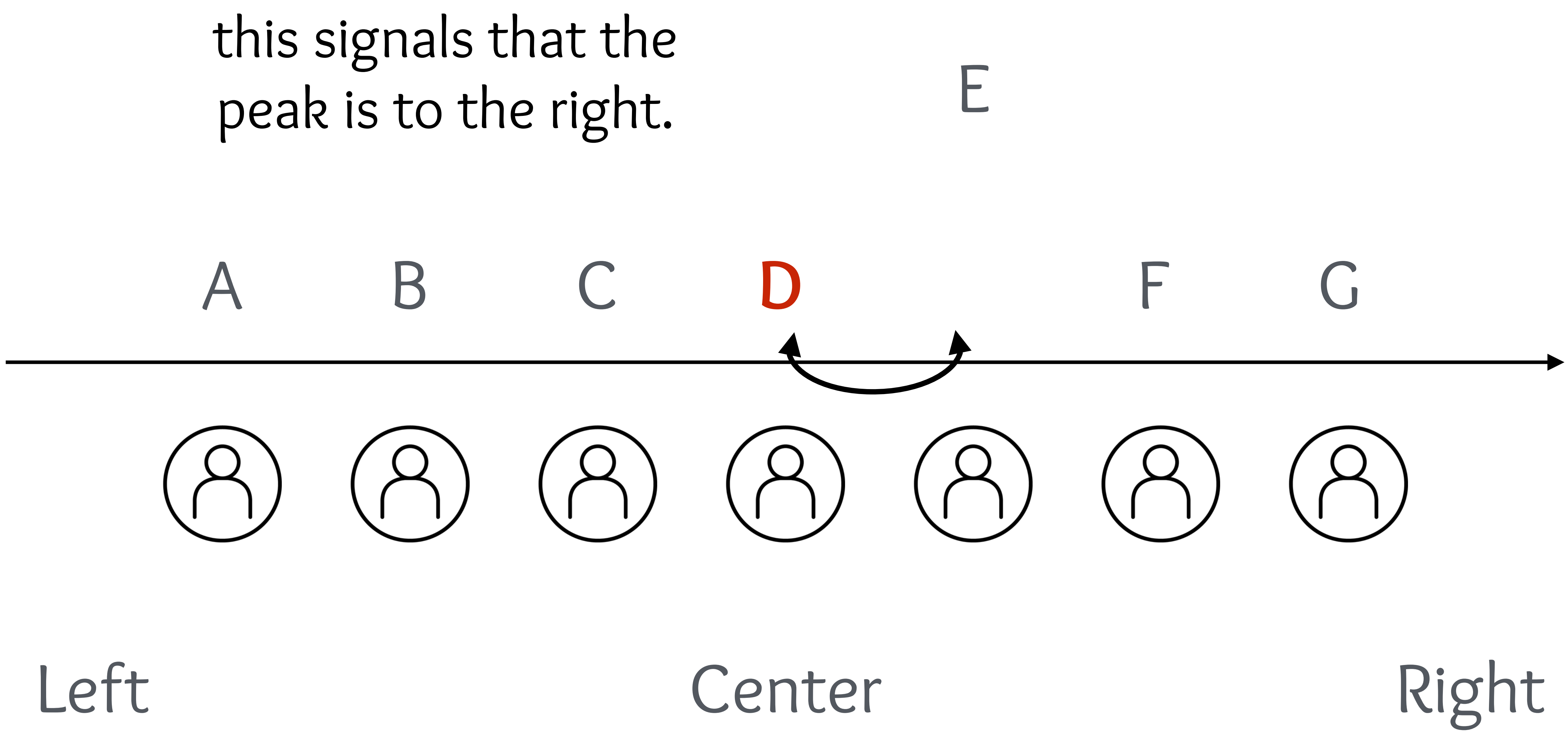

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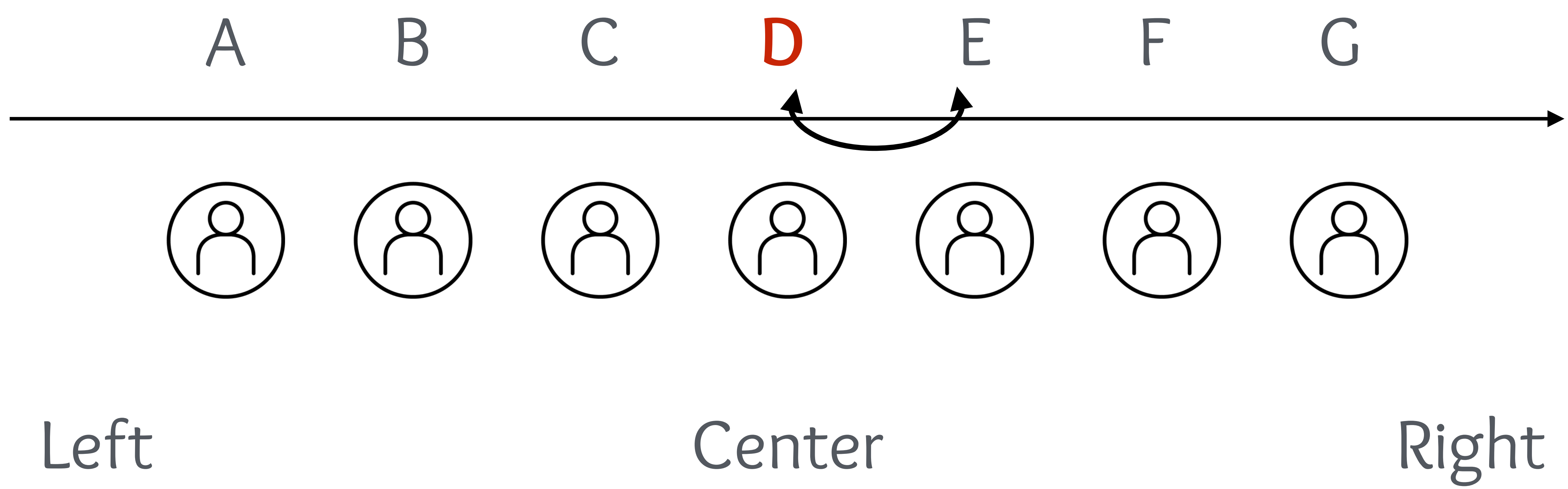

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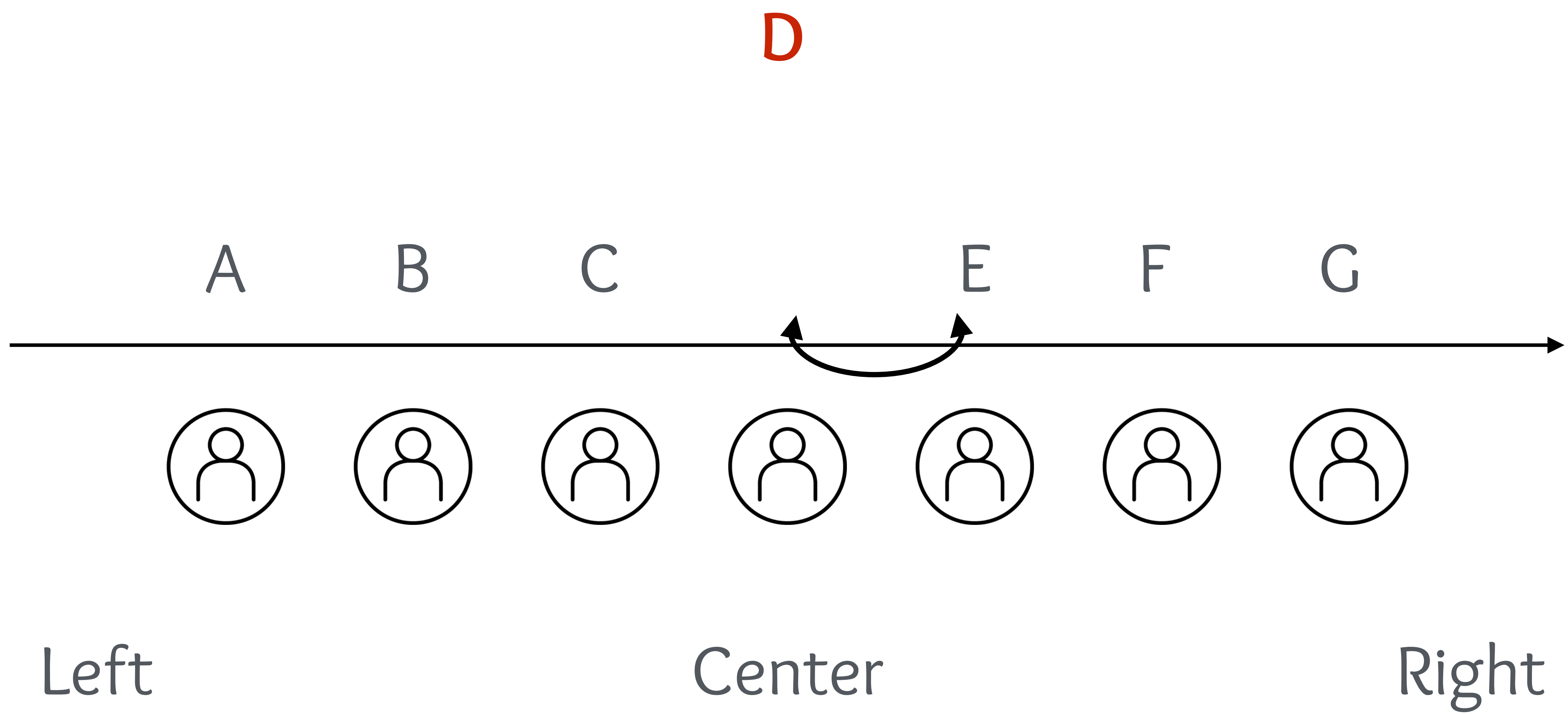

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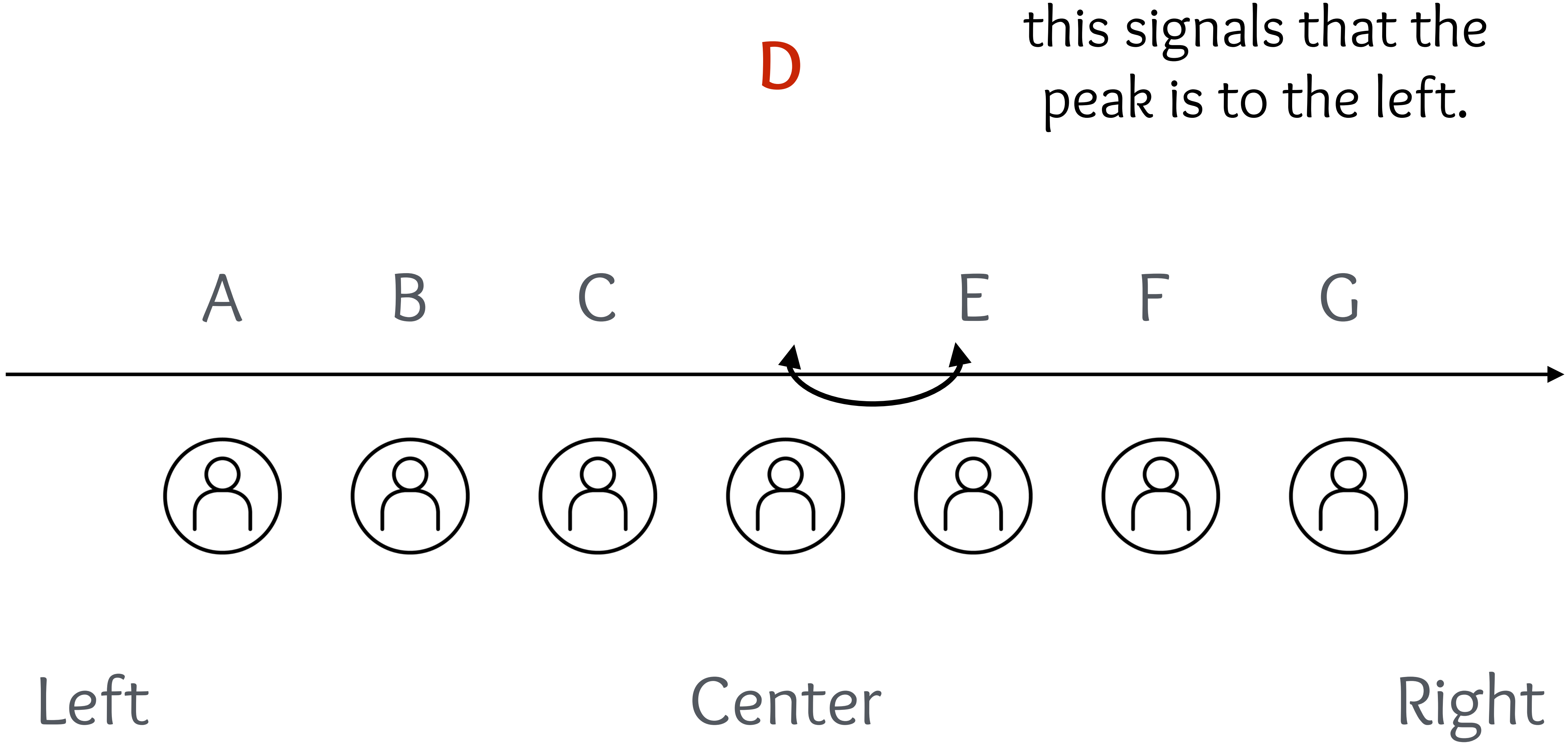

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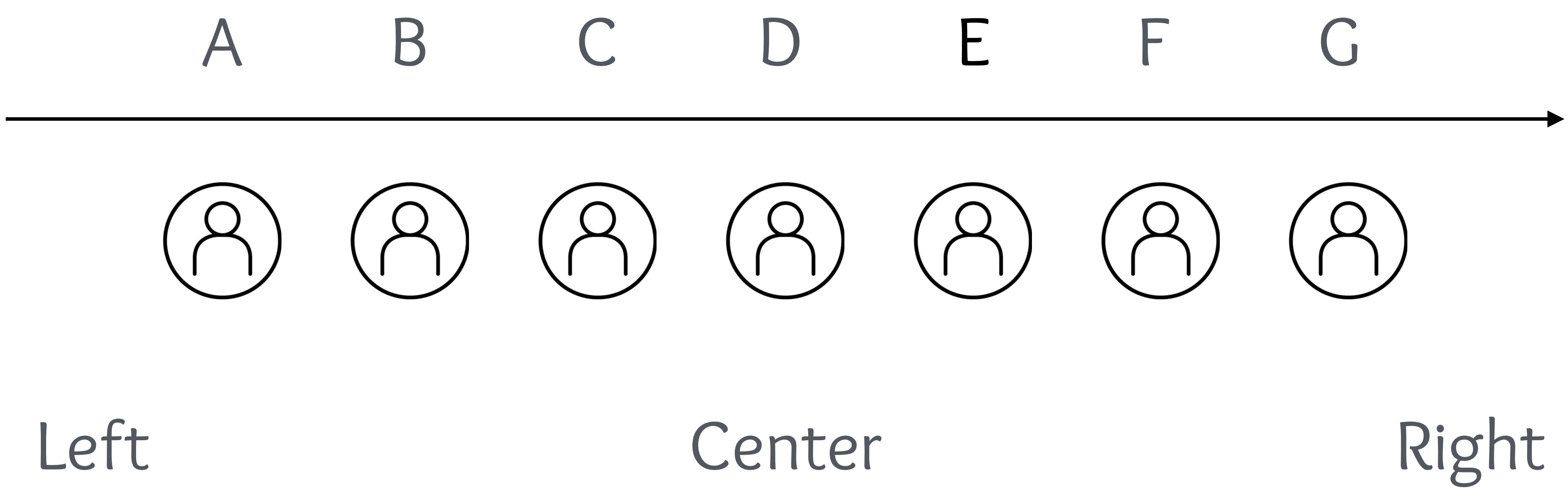

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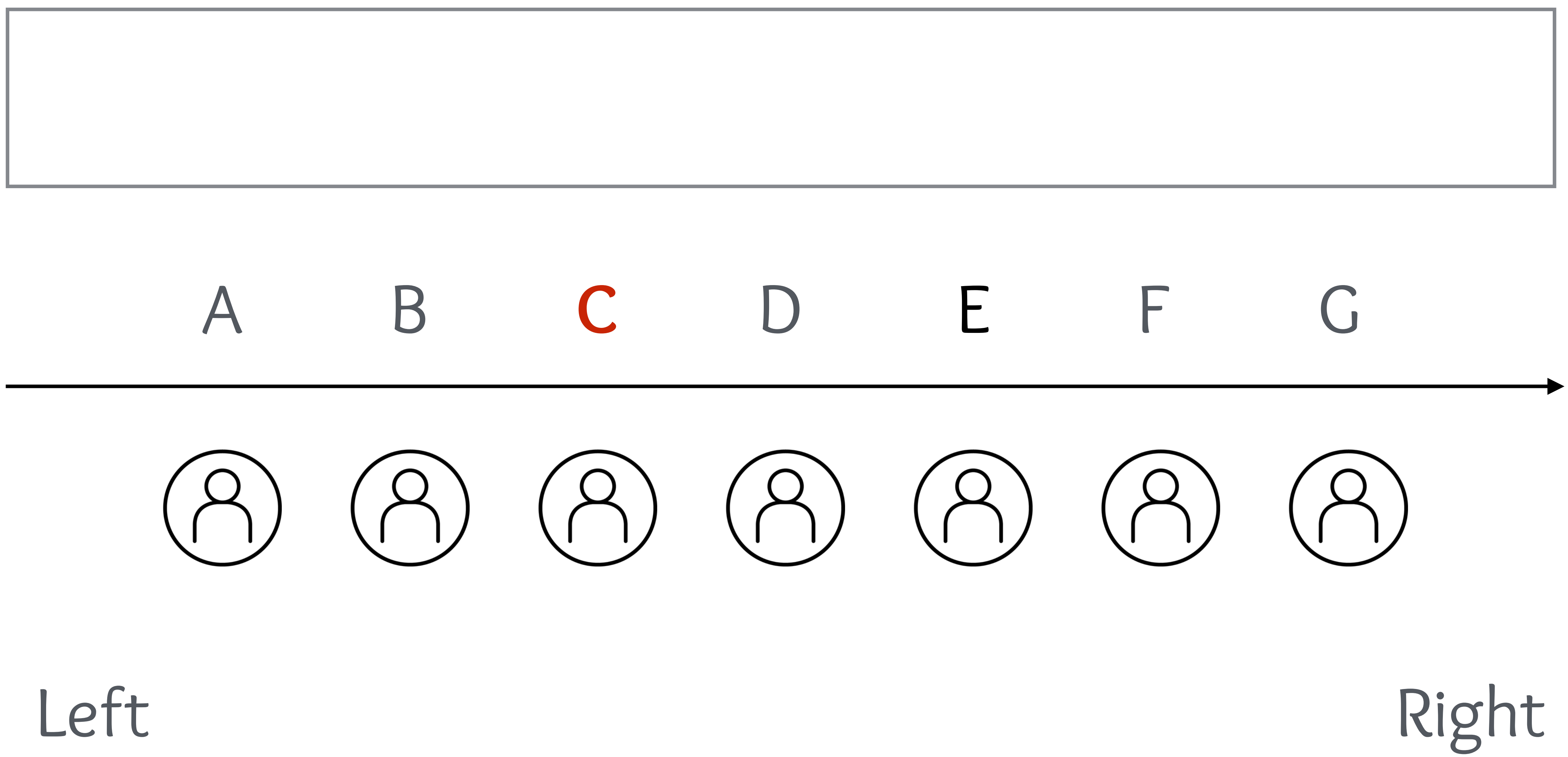

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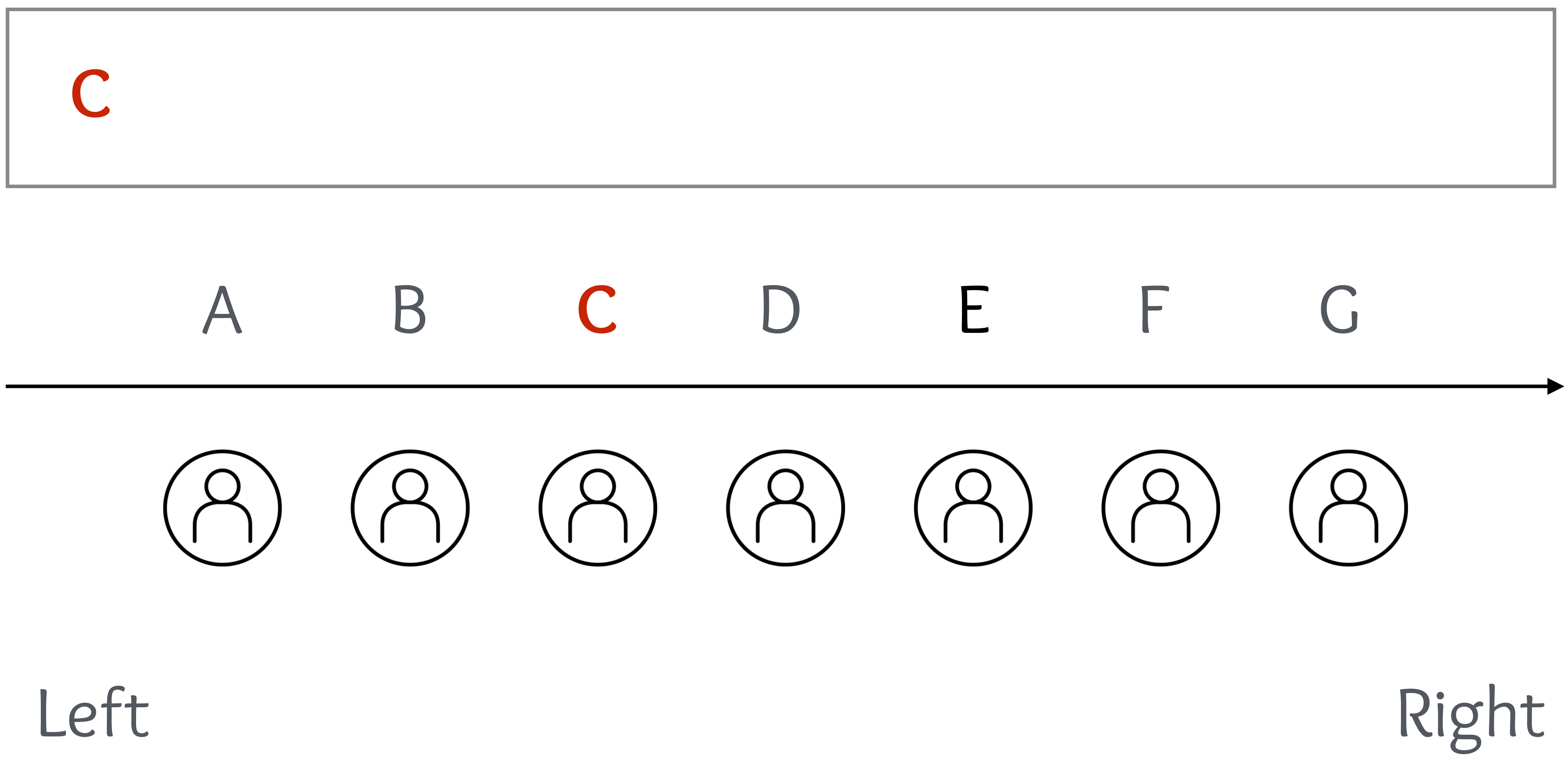

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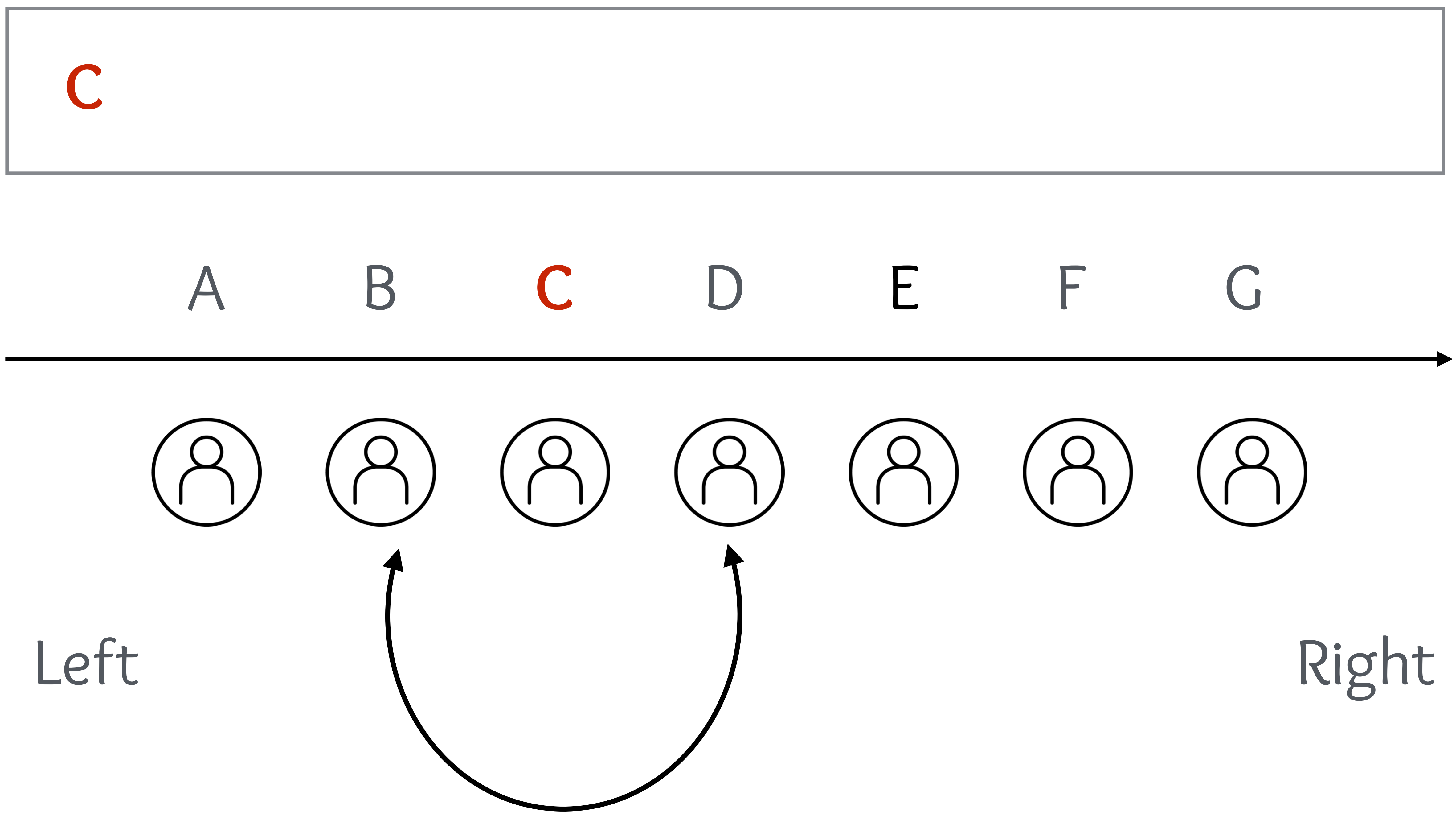

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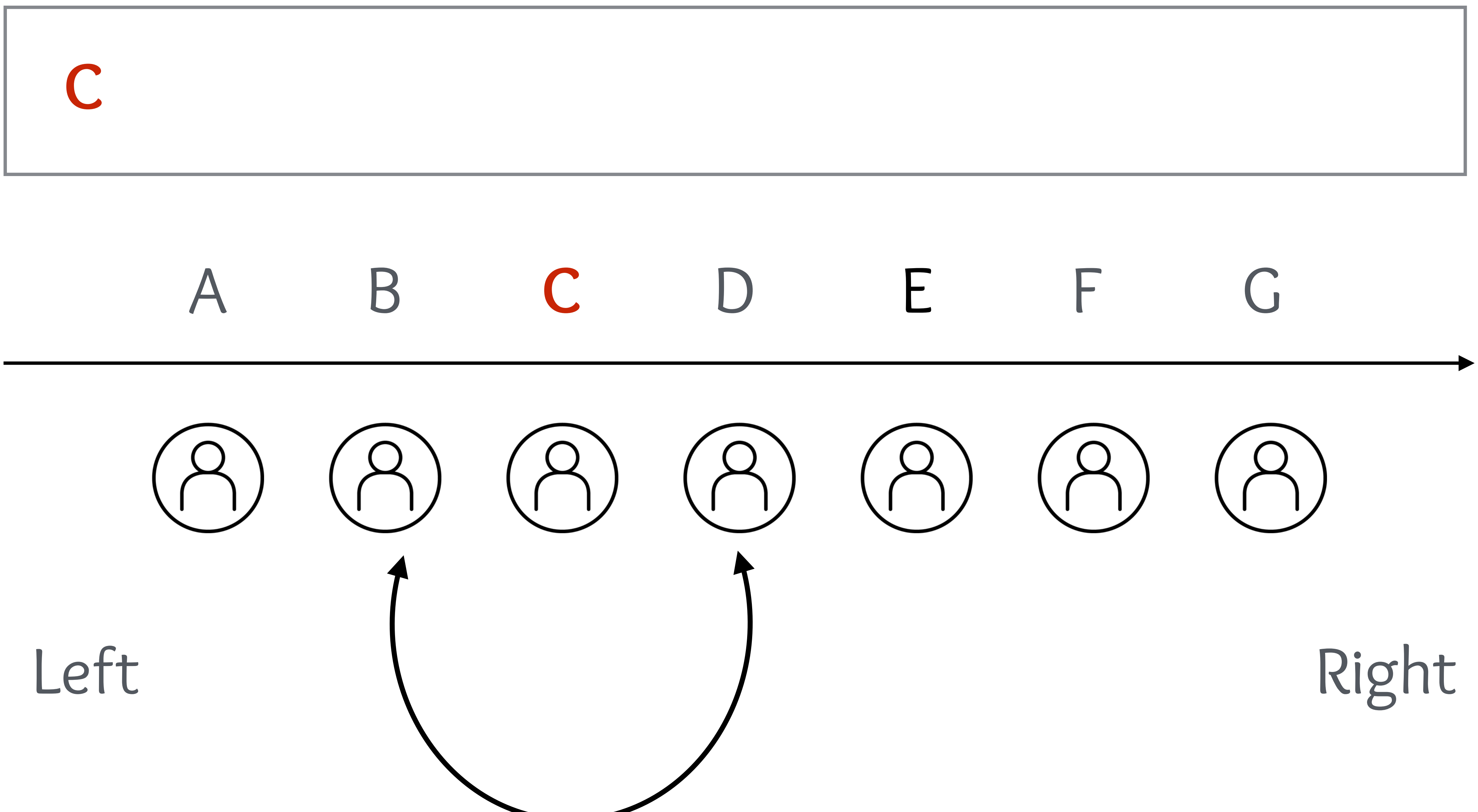

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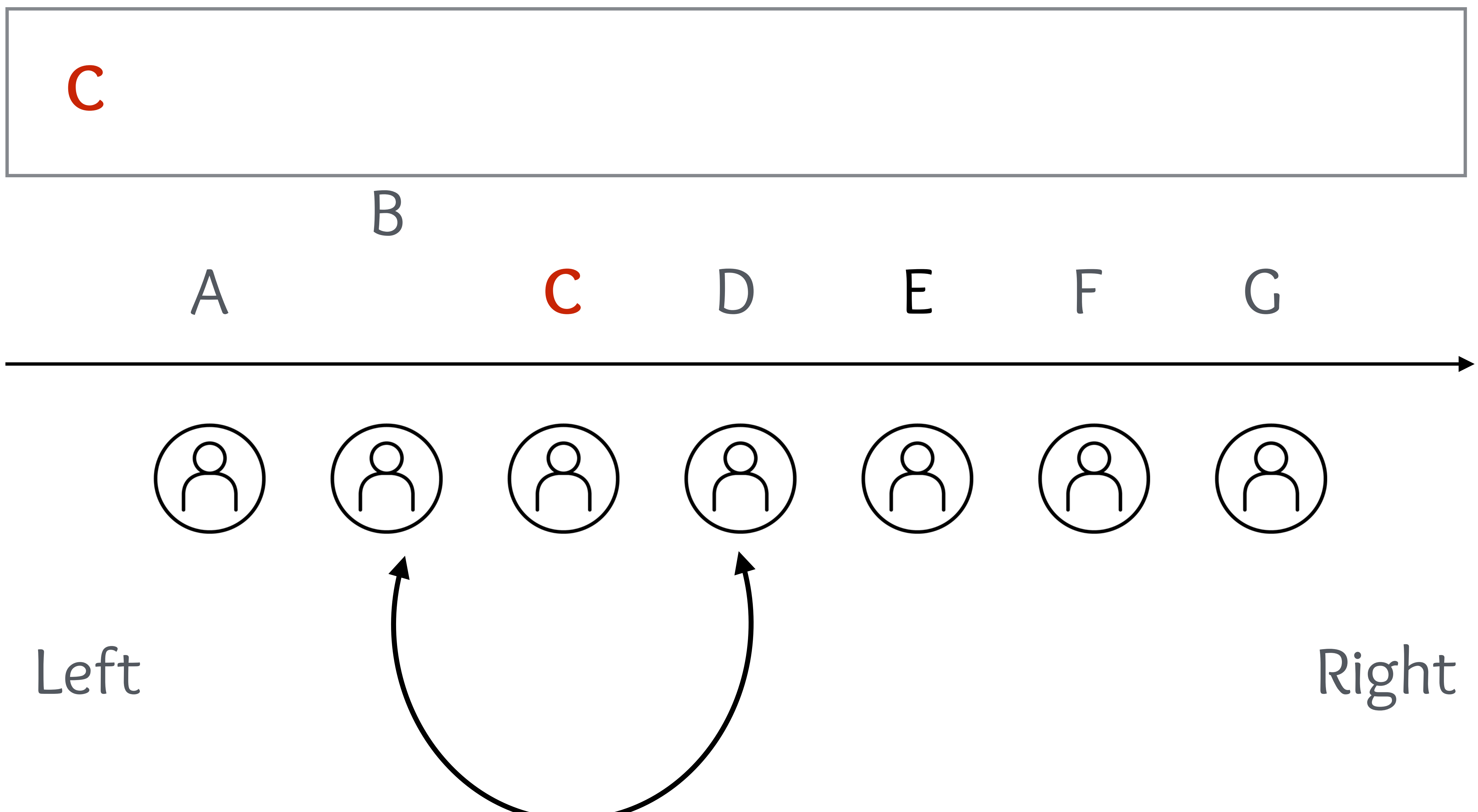

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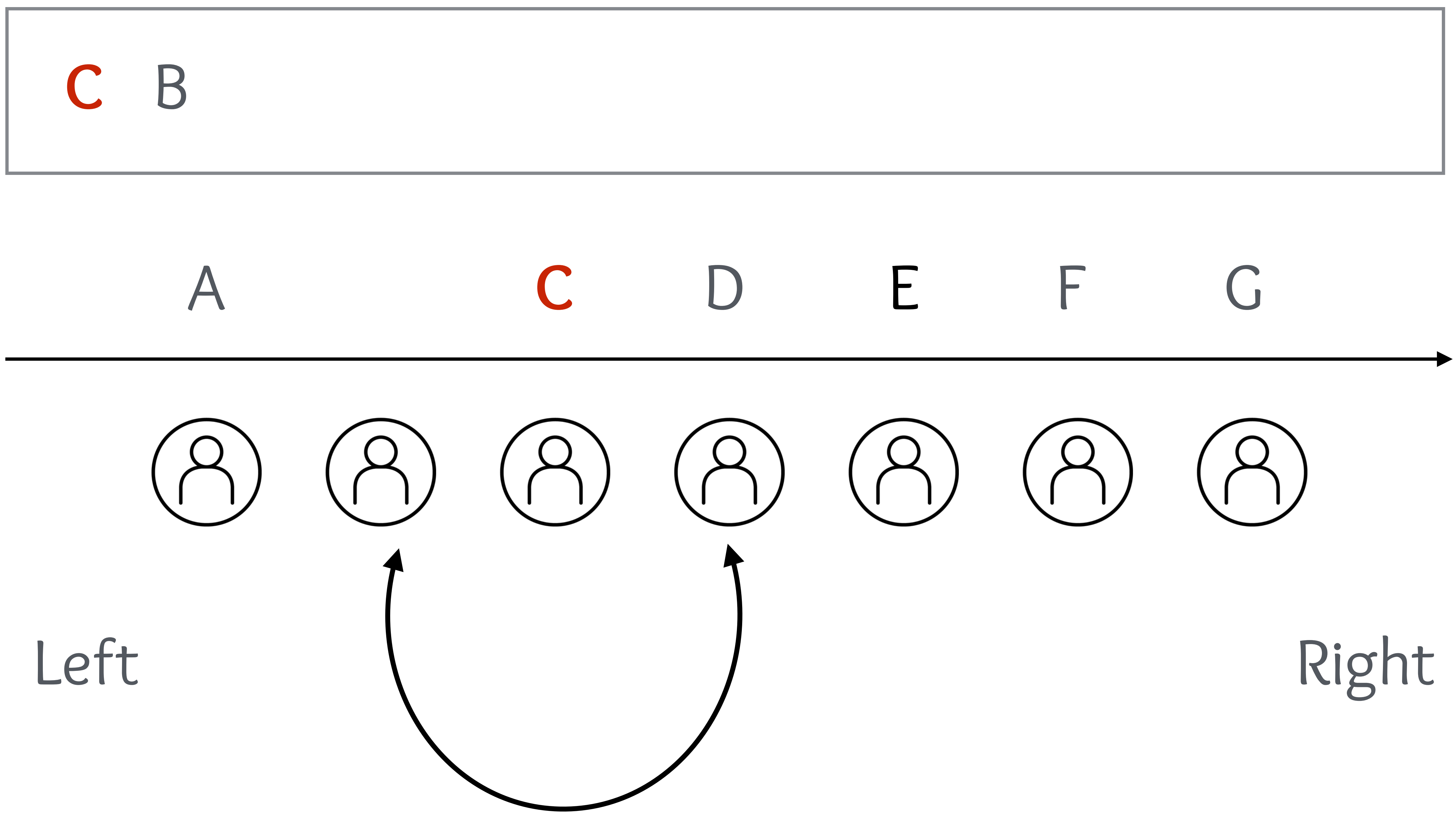

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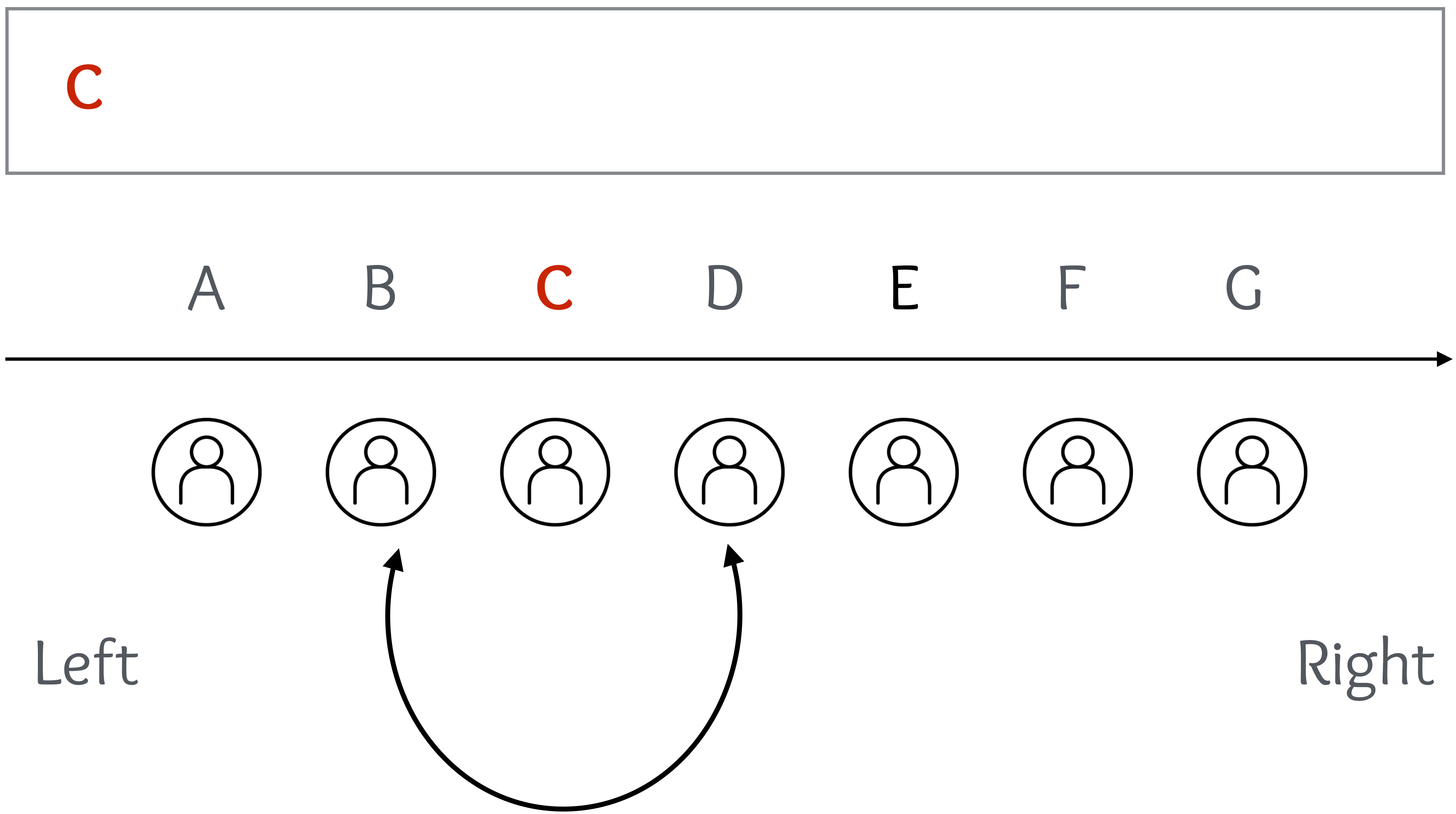

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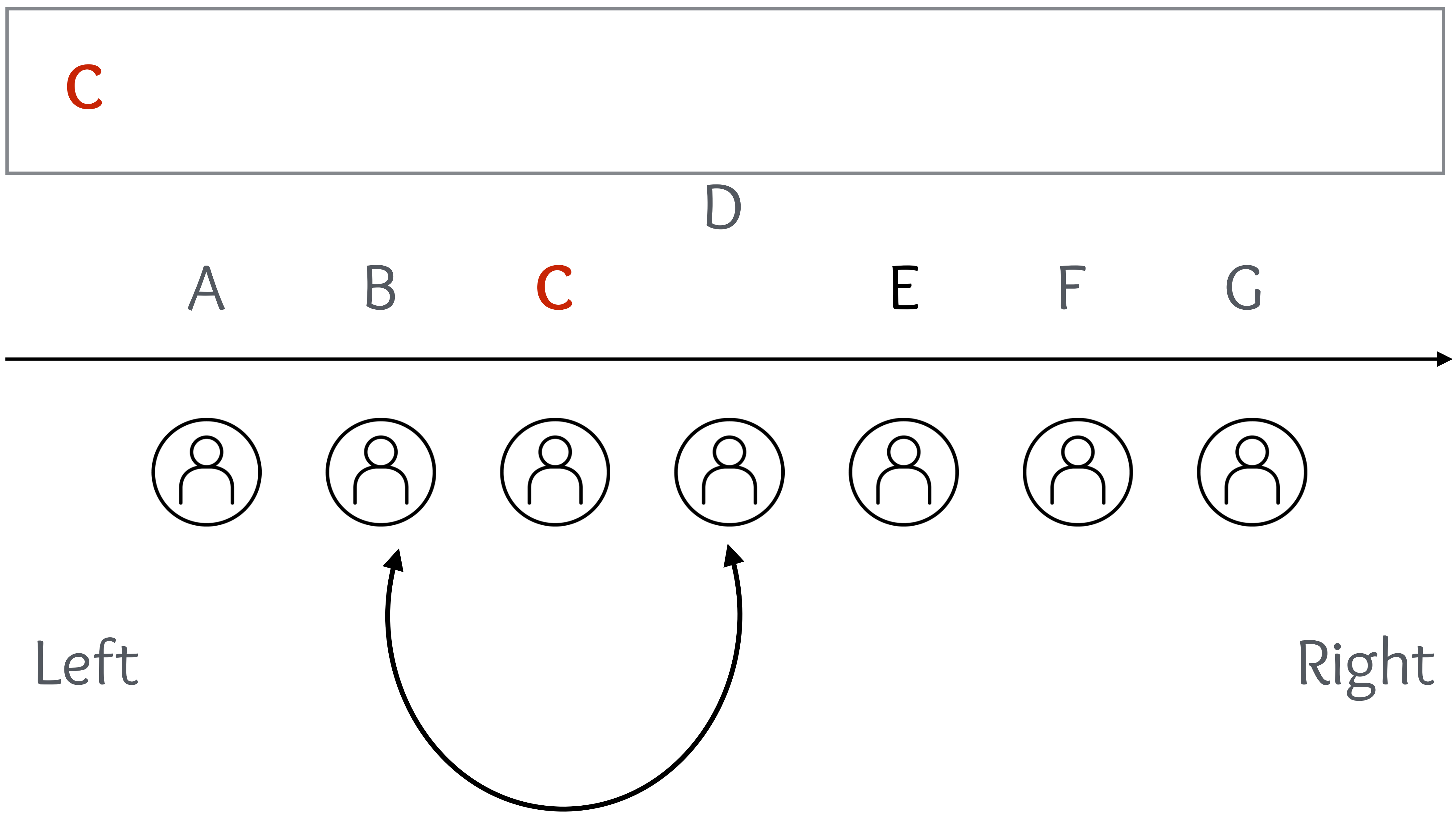

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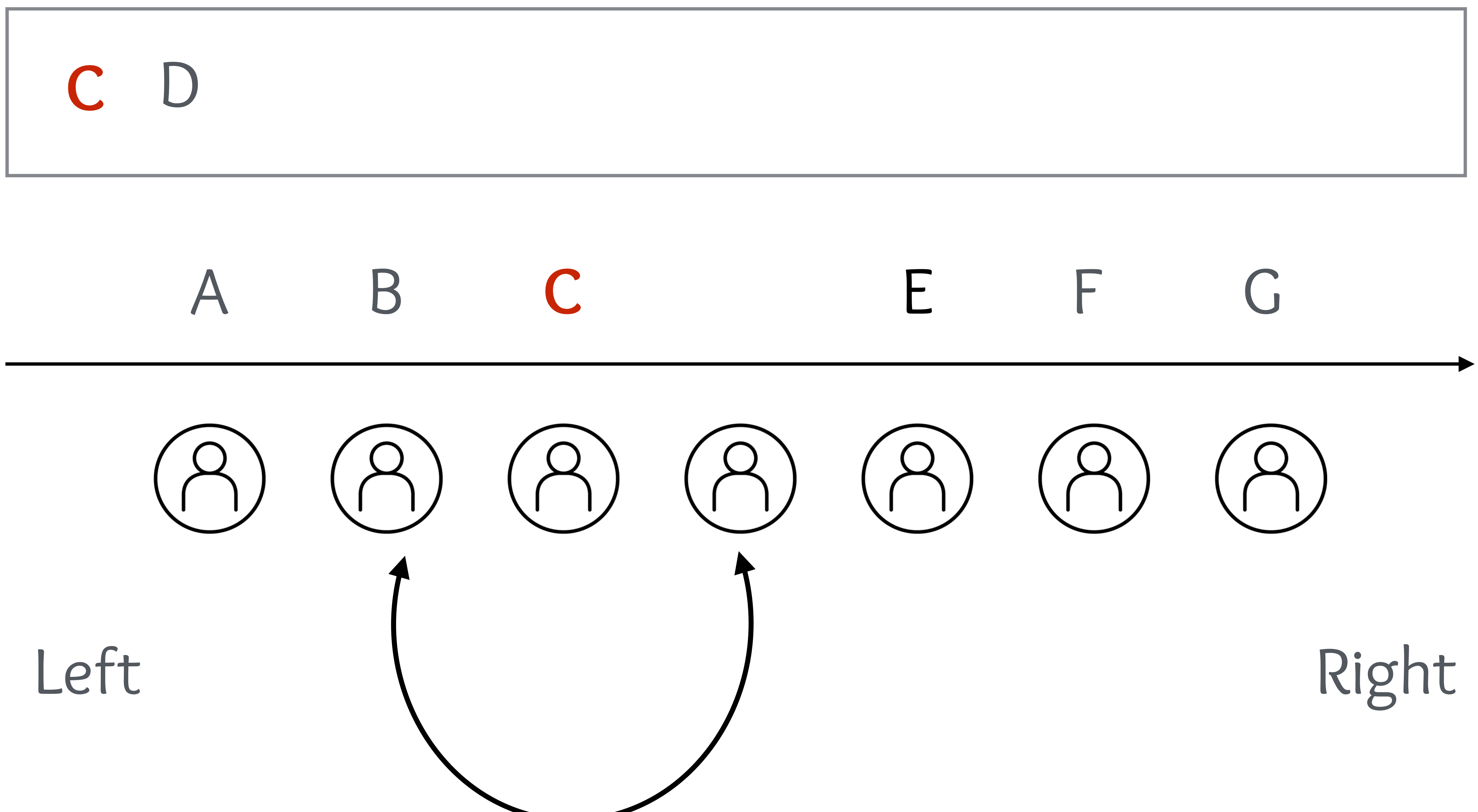

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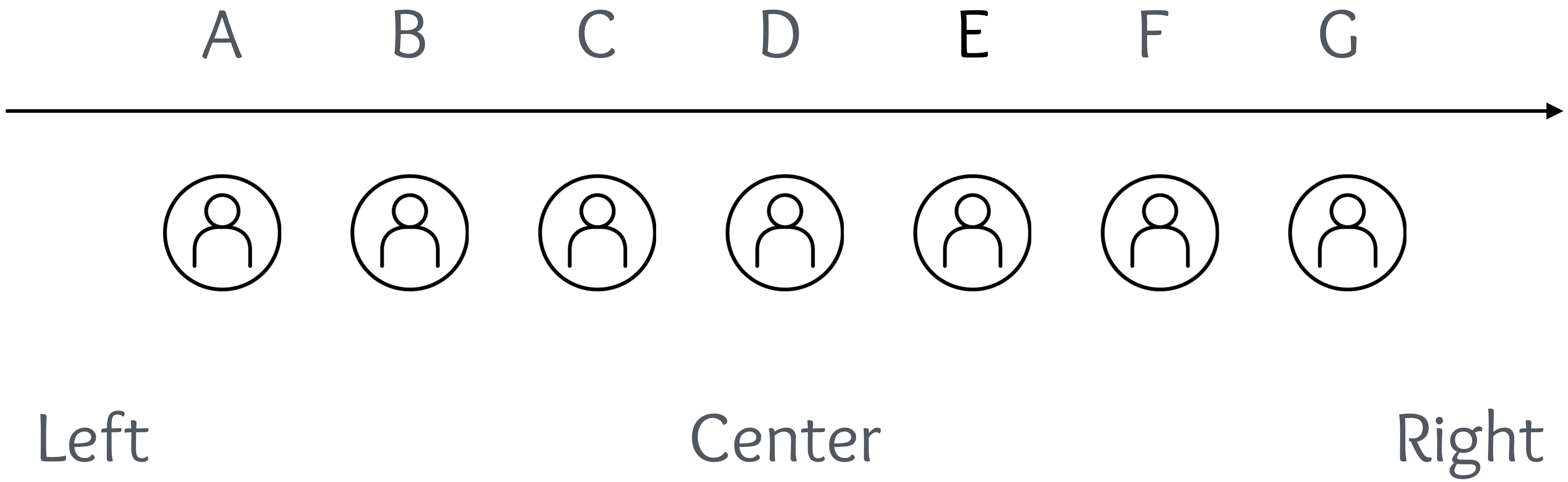
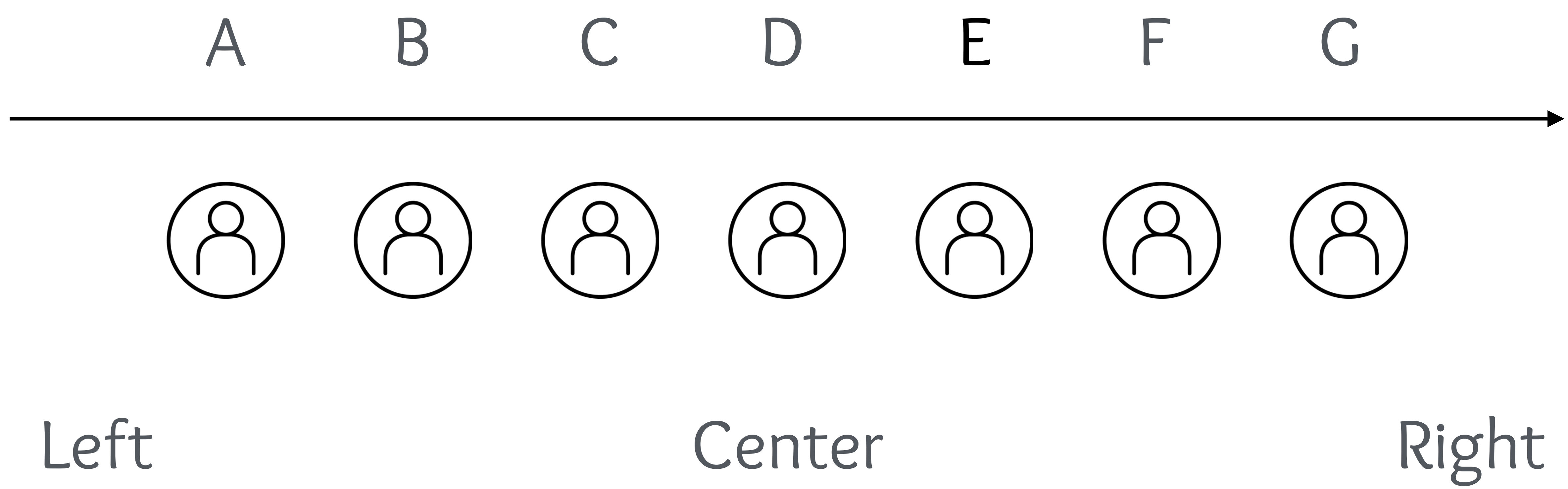


[Better query complexities for elicitation.]

(i) Identify the peak: use binary search (O(log m) queries).

(ii) Once we know the peak, the rest is O(m) queries.



SINGLE CROSSING PREFERENCES

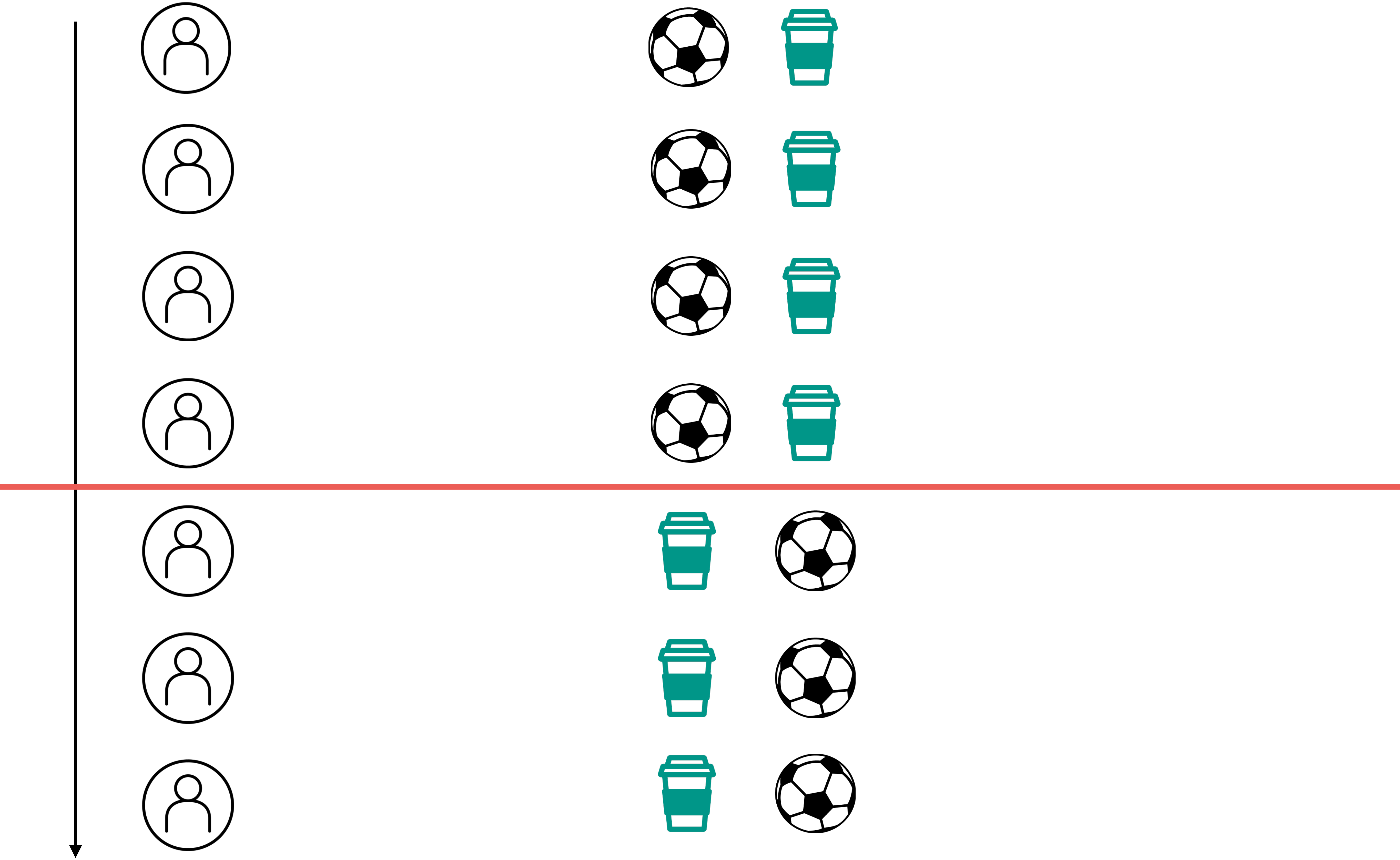
definition, recognition and Condorcet winners

SINGLE CROSSING PREFERENCES

Definition

A profile is **single-crossing** if it admits an ordering of the voters such that for *every* pair of candidates (a,b), either:

a) all voters who prefer a over b appear before all voters who prefer b over a, or,
b) all voters who prefer a over b appear after all voters who prefer b over a, or,



The notion is popular for several reasons:

- No Condorcet Cycles.
- Identifiable in polynomial time.
- Reasonable (?) model of actual elections.

SINGLE CROSSING PREFERENCES

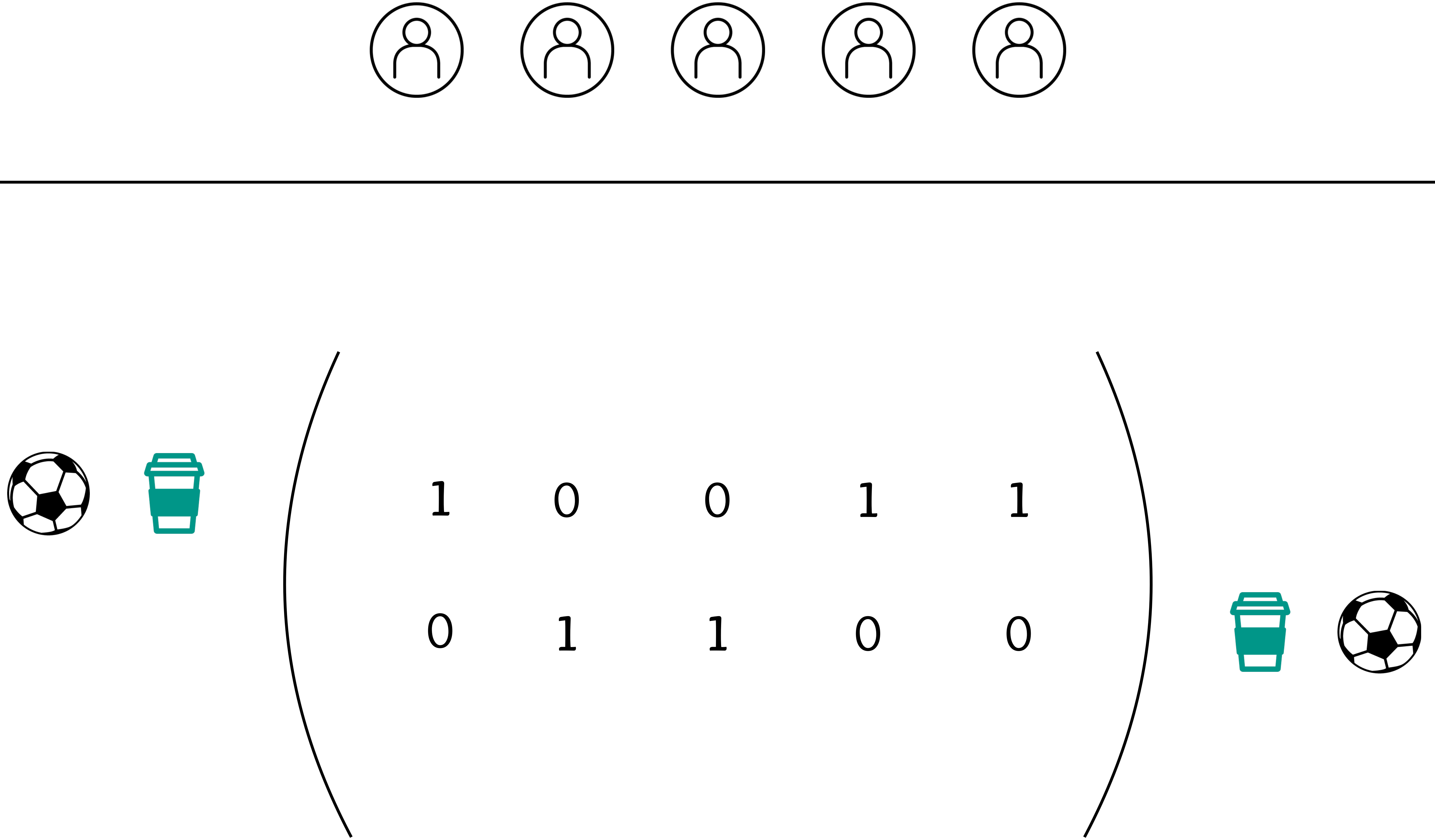
Recognition

A Characterization of the Single-Crossing Domain
Bredereck, Chen and Woeginger, Social Choice and Welfare, 2013

Recognising if a given profile is single-crossing with respect to some preference ordering.

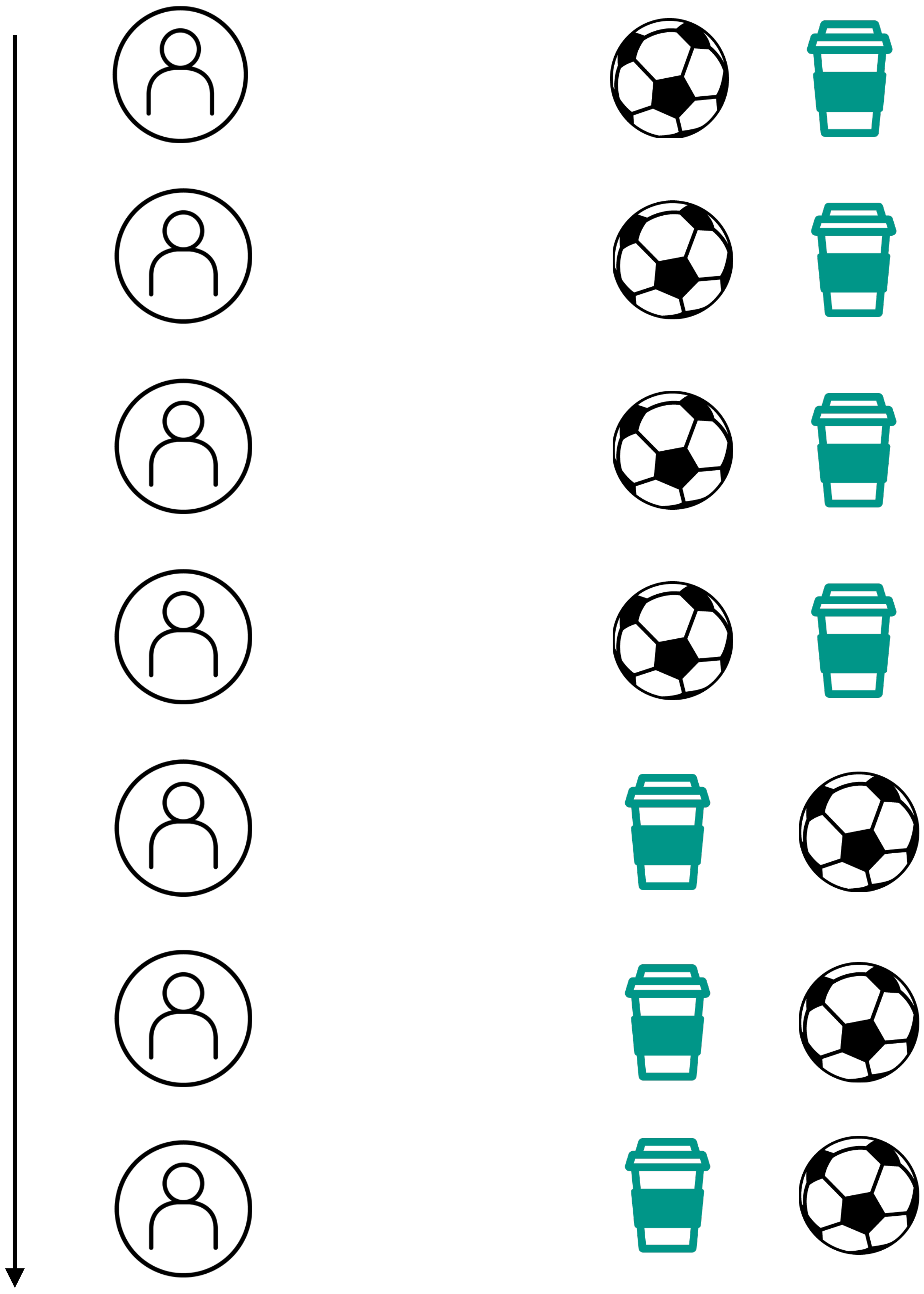
↓

Checking if a 0/1 matrix has the **consecutive-ones property**.


1 0 0 1 1
0 1 1 0 0

SINGLE CROSSING PREFERENCES

No Condorcet Cycles

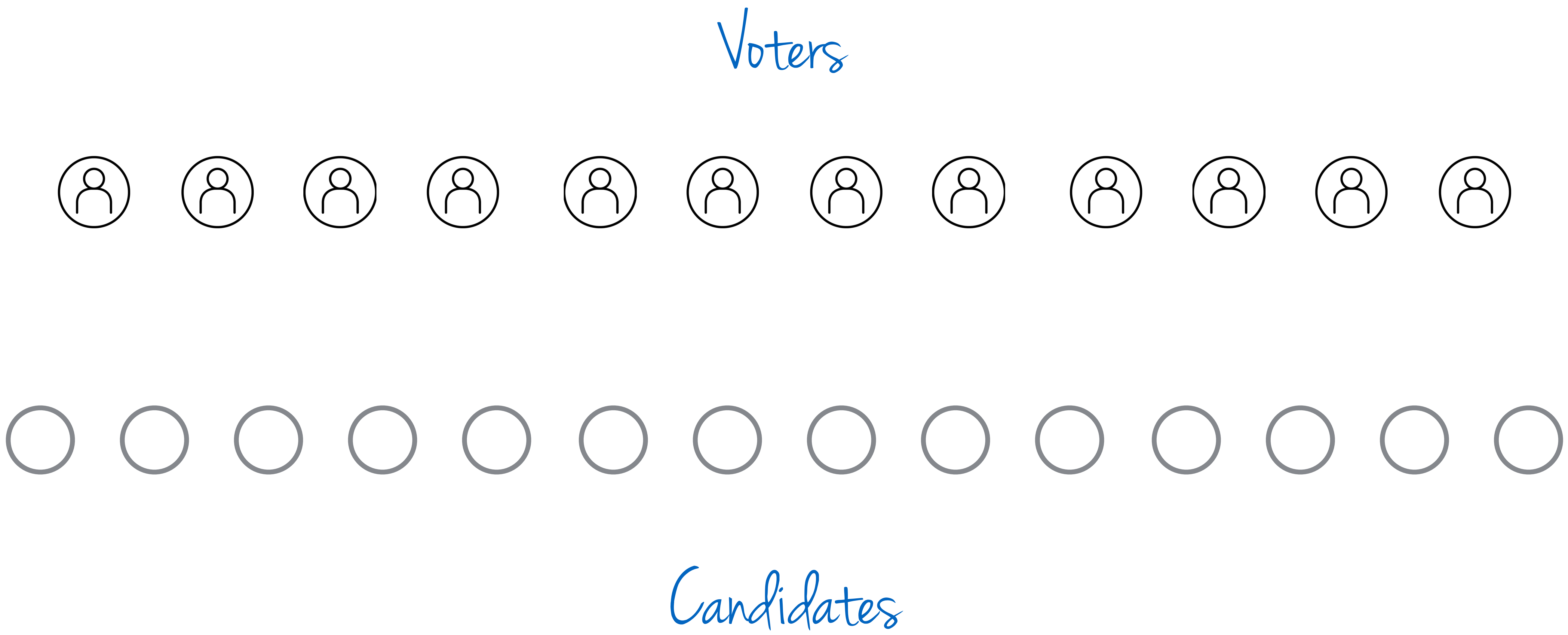


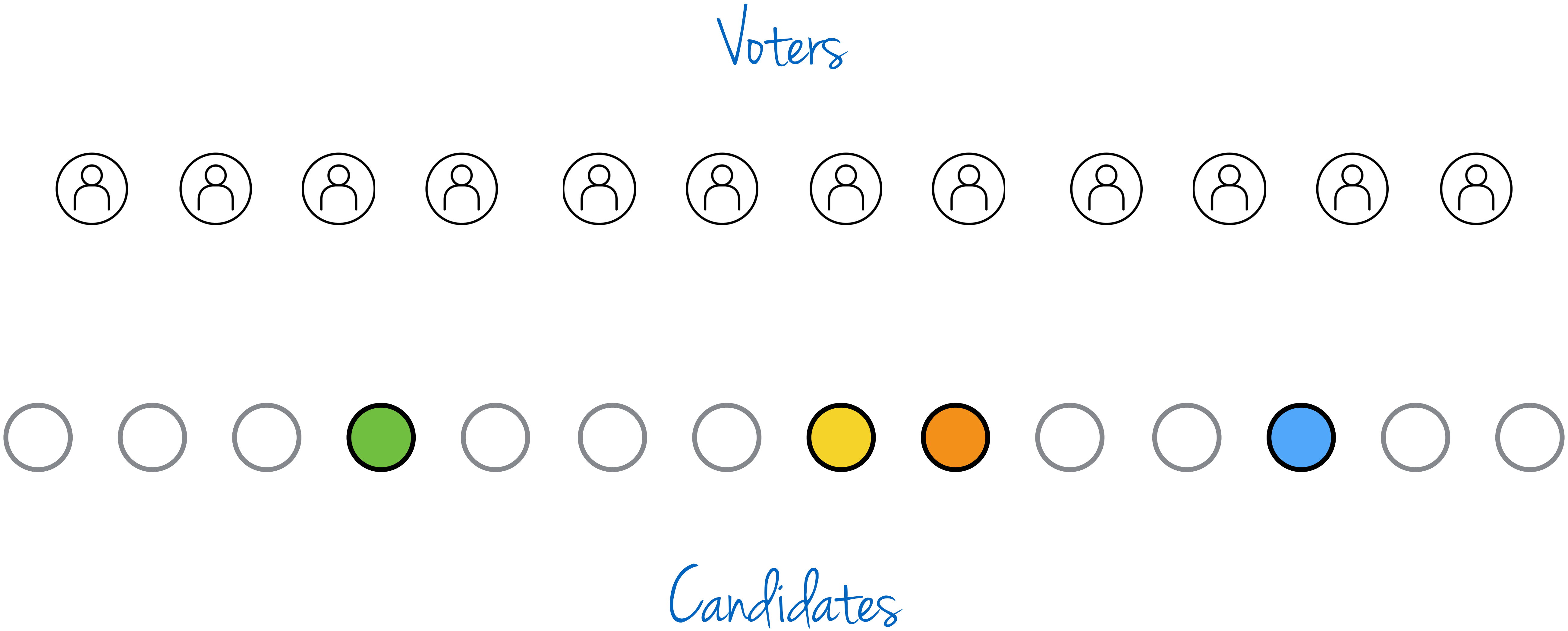
For any pair of candidates, the opinion of the middle voter is also the majority opinion.

SINGLE CROSSING PREFERENCES

Chamberlin-Courant

The Complexity of Fully Proportional Representation for Single-Crossing Electorates
Skowron, SAGT, 2013


Voters
Candidates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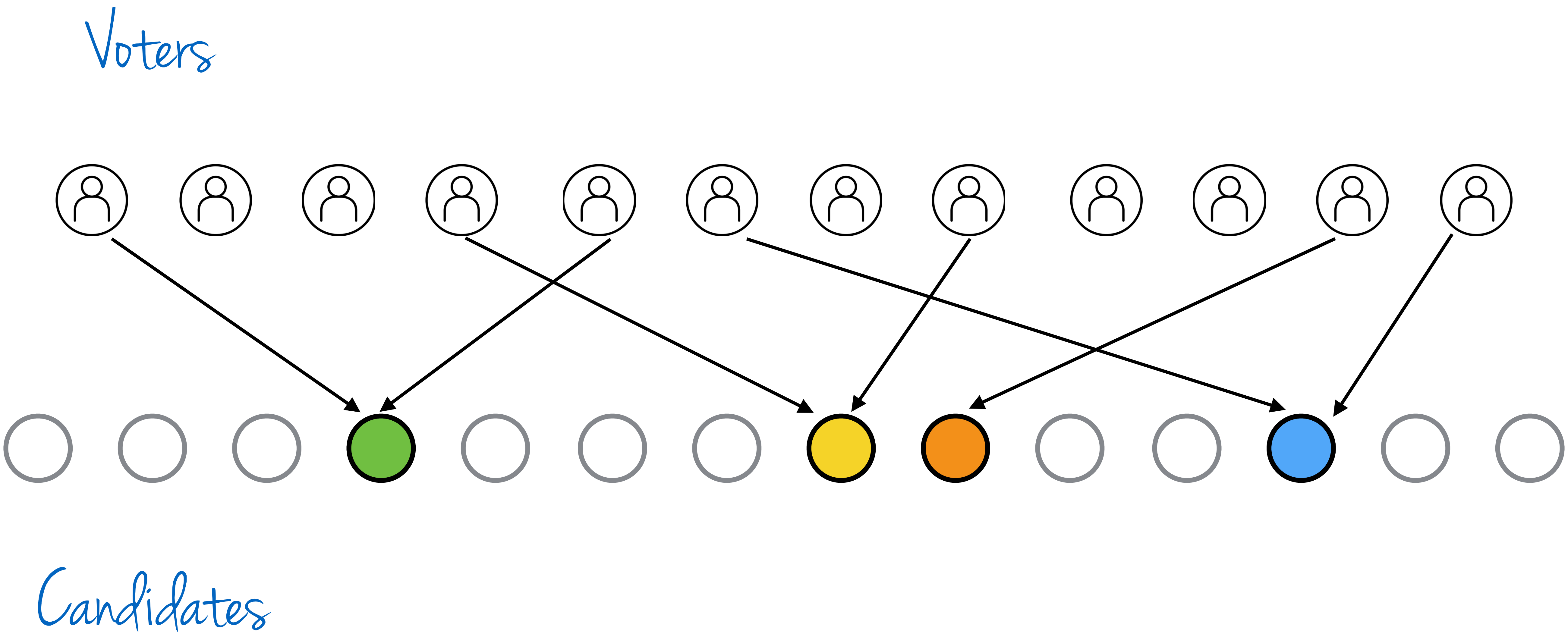
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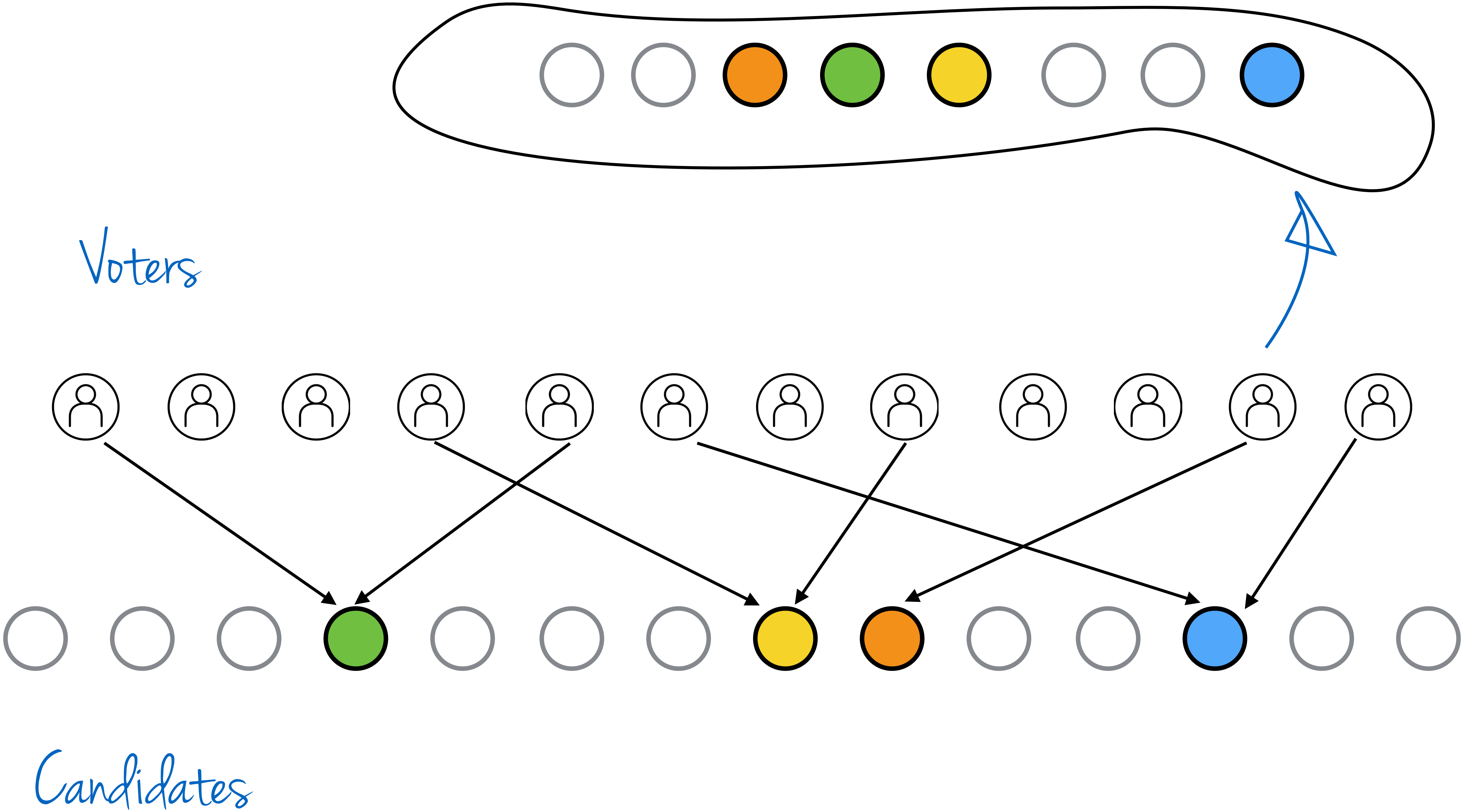
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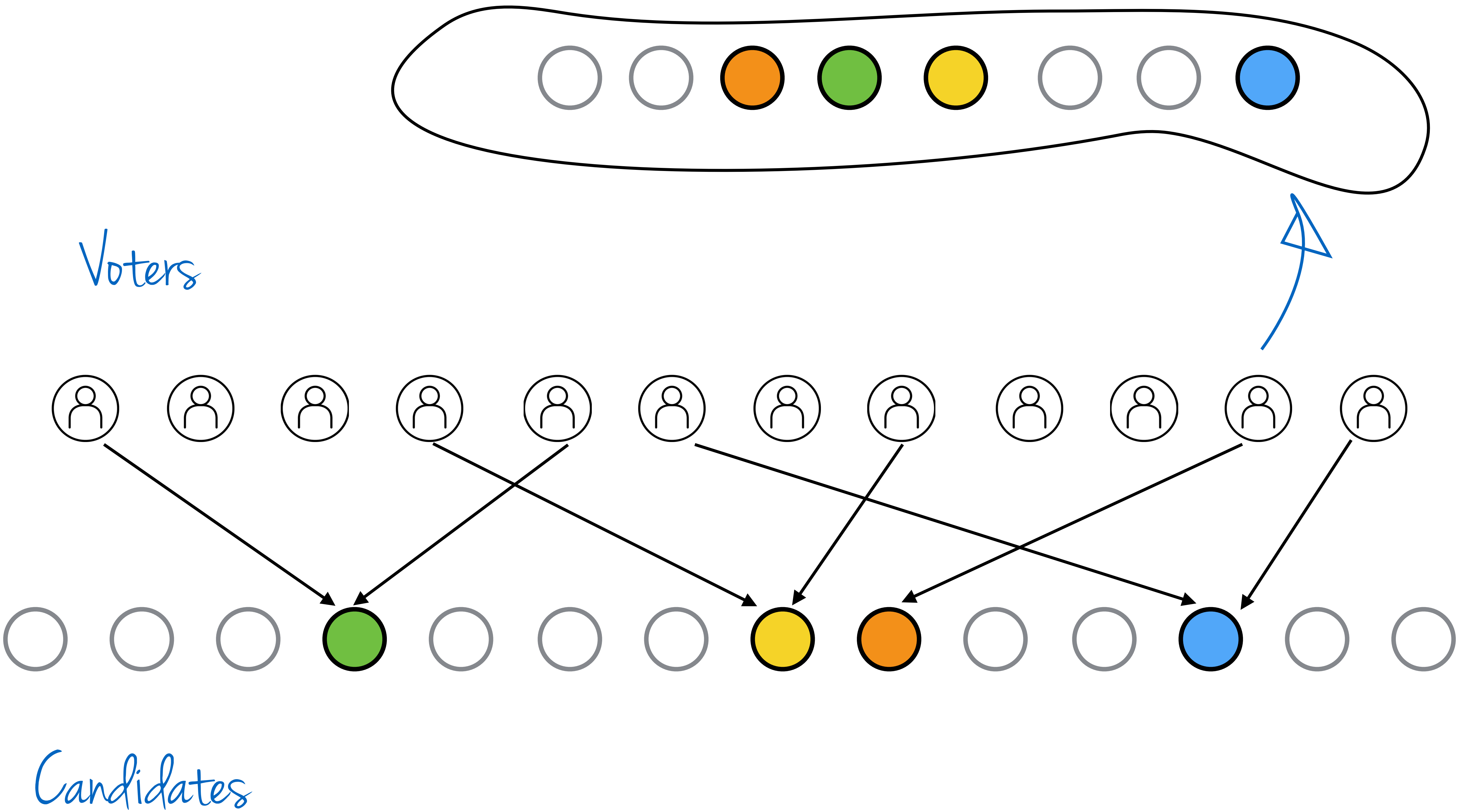
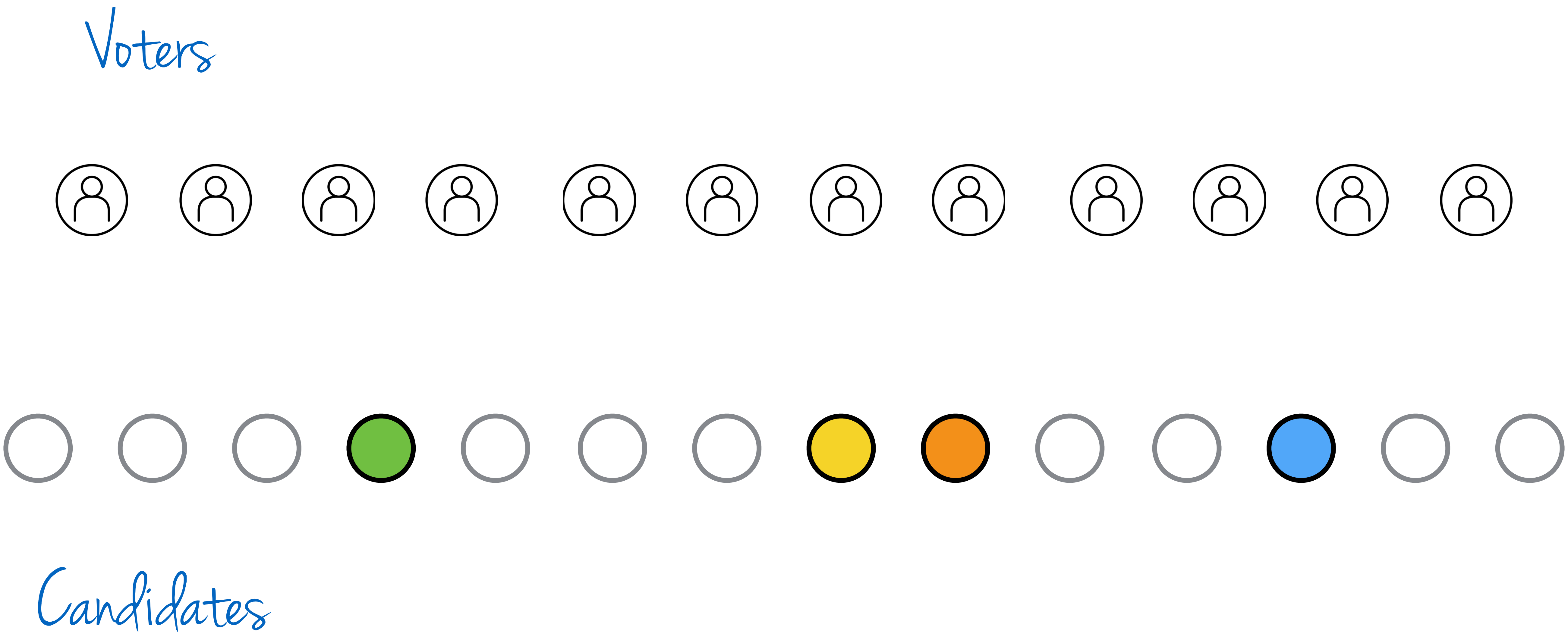


dissatisfaction of voter v =
rank of best candidate in the committee in his vote



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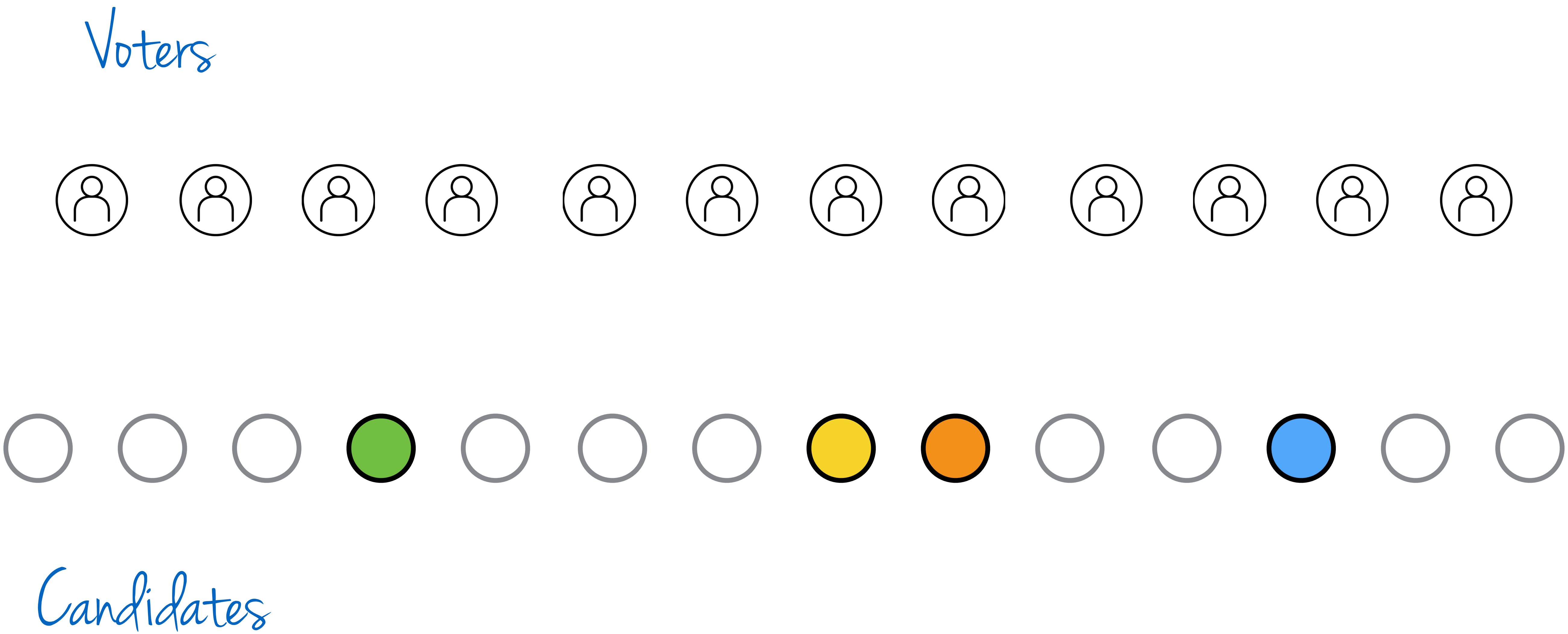
On single-crossing profiles, optimal CC solutions exhibit a "contiguous blocks property".



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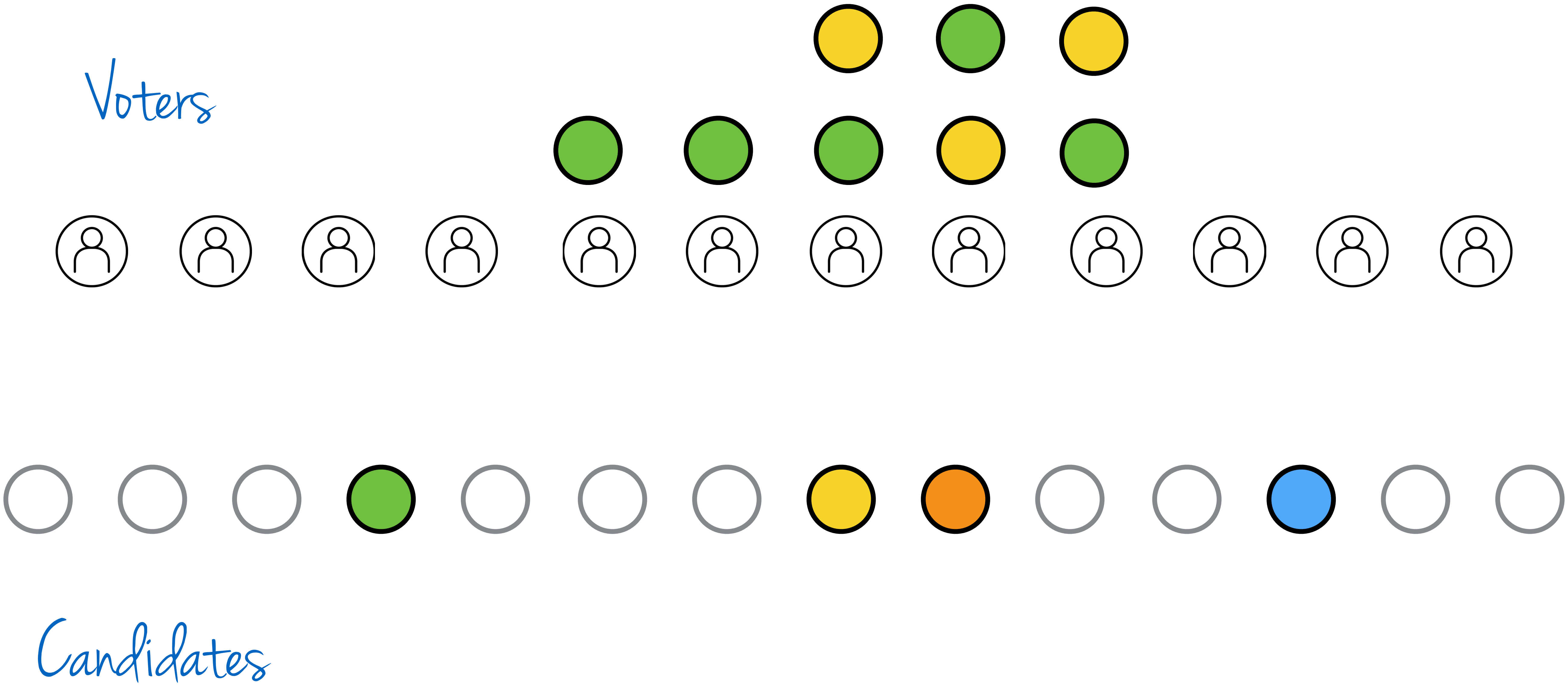


dissatisfaction of voter v =
rank of best candidate in the committee in his vote

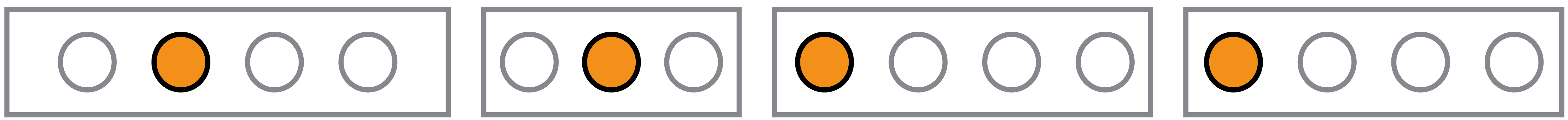
CONCLUDING REMARKS

nearly structured preferences, dichotomous preferences

The single-peaked and single-crossing domains have been generalised to notions of **single-peaked and single-crossing on trees**. The generalised domains continue to exhibit many of the nice properties we saw today.

Single-peaked orders on a tree, *Gabrielle Demange,* Math. Soc. Sci, 3(4), 1982.

Generalizing the Single-Crossing Property on Lines and Trees to Intermediate Preferences on Median Graphs, *Clearwater, Puppe, and Slinko,* IJCAI 2015



The single-peaked and single-crossing domains have been generalised to notions of **single-peaked-width and single-crossing-width.**

Here, it is common that algorithms that work in the single-peaked or single-crossing settings can be generalised to profiles of width w at an expense that is exponential in w.

Kemeny Elections with Bounded Single-peaked or Single-crossing Width,
Cornaz, Galand, and Spanjaard, IJCAI 2013

Profiles that are “close” to being single-peaked or single-crossing (closeness measured usually in terms of candidate or voter deletion) have also been studied.

It’s typically NP-complete to determine the optimal distance, but FPT and approximation algorithms are known.

On Detecting Nearly Structured Preference Profiles
Elkind and Lackner, AAAI 2014

Computational aspects of nearly single-peaked electorates,
Erdélyi, Lackner, and Pfandler, AAAI 2013

Are There Any Nicely Structured Preference Profiles Nearby?
Bredereck, Chen, and Woeginger, AAAI 2013

Γ	VDel		CDel
	$k < n/2$	$k \geq n/2$	
Single-peaked / Single-caved	$\mathcal{O}^*(1.28^k)$	$\mathcal{O}^*(2.08^k)$	P
Single-crossing	P	P	$\mathcal{O}^*(5.07^k)$
Best-/Medium-/Worst-restricted	$\mathcal{O}^*(1.28^k)$	$\mathcal{O}^*(2.08^k)$	$\mathcal{O}^*(2.08^k)$
Value-restricted	$\mathcal{O}^*(2.08^k)$	$\mathcal{O}^*(2.08^k)$	$\mathcal{O}^*(2.08^k)$
Group-separable	$\mathcal{O}^*(1.28^k)$	$\mathcal{O}^*(2.08^k)$	$\mathcal{O}^*(3.15^k)$

Γ	VDel	CDel
Single-peaked / Single-caved	2	P
Single-crossing	P	6
Best-/Medium-/Worst-restricted	2	3
Value-restricted	3	3
Group-separable	2	4

Summary from:
On Detecting Nearly Structured Preference Profiles *Elkind and Lackner,* AAAI 2014

While the domain restrictions we saw today apply to votes given as linear orders, similar restrictions have also been studied in the context of votes that are given by approval ballots.

As was the case here, there are close connections with COP and many hard problems become easy on these structured (dichotomous) profiles.

Structure in Dichotomous Preferences,
Elikind and Lackner; IJCAI 2015.

Euclidean preferences capture settings where voters and alternatives can be identified with points in the Euclidean space, with voters' preferences driven by distances to alternatives.

Psychological scaling without a unit of measurement, *Coombs;* Psychological review, 1950

The Dark Side: Domain restrictions also have some side-effects: problems like manipulation, bribery, and so forth also become easy!

The Shield that Never Was: Societies with Single-Peaked Preferences are More Open to Manipulation and Control, *Faliszewski et al;* TARK 2009

Bypassing Combinatorial Protections: Polynomial-Time Algorithms for Single-Peaked Electorates, *Brandt et al;* AAAI 2010

THANK YOU!

The Handbook of Computational Social Choice,
Brandt, Conitzer, Endriss, Lang and Procaccia; 2016

Preference Restrictions in Computational Social Choice: Recent Progress, *Elikind, Lackner, and Peters;* IJCAI 2016.